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Analysis of Surface Crack in Pelton Turbine using Kitagawa Takahashi Diagram

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Abstract
Pelton turbine is widely used for its excellent performance for low discharge with high head however, fluctuation in force exerted by water jet leads to fatigue and subsequently resulting in the formation of cracks at the root of runner. Study on stress distribution along with estimation of stress intensity factor of Pelton runner has not been the subject of interest so, this paper focuses on study of crack transition and the stress distribution of Pelton turbine runner with semi elliptical surface crack front using Linear Elastic Fracture Mechanics principle. Code is developed and implemented for calculation of stress intensity factor and corresponding crack length incorporating Iso- stress intensity factor model in Mechanical APDL and result obtained was employed to Kitagawa-Takahashi diagram. Iso-stress intensity factor obtained from finite element analysis result for different surface crack was found to have maximum value of 1.85E-03 Mpa $\sqrt{mm}$ and minimum value being 1.09E-03 Mpa $\sqrt{mm}$.

Keywords
Stress Intensity Factor,Pelton runner ,Semi-elliptical Surface Crack, Iso-KI

1. Introduction

Pelton turbine are classified as impulse turbine which runs through transfer of momentum from water to the runner and is widely used for high head and low flow rate condition. Kinetic energy of water is injected through jet and energy gets transferred into runner however, huge load creates bending stress at the root of runner and small cracks starts to appear after certain time. Linear Elastic Fracture Mechanics approach is widely adopted for analysis of surface flaw and stress associated with it where every material are considered to have a flaw inside them. These flaw with applied force increase the amount of stress near crack field and discontinuity in material grows with time. Surface crack problems can be solved according to [1] reference by incorporating special singular element. Surface crack problems was identified by [1] during their study of Pelton turbine at Khimti Hydropower Plant.Attempt to relate damage tolerance and conventional S-N approach was done by [2]introducing stress versus crack length plot relating stress difference and crack length.

2. Methodology

2.1 Governing Equations

In this work, crack growth behavior of runner is assumed to be of tensile opening mode with stress ratio R=0 which in general is termed as mode I . Stress intensity factor and displacement field near crack tip for mode I is given [3] as:

$$
\begin{aligned}
\sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] \\
\sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1+\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right] \\
\tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)\right] \\
\sigma_{zz} &= \nu\left(\sigma_{xx}+\sigma_{yy}\right) \\
u_x &= \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}} \cos\left(\frac{\theta}{2}\right)\left[\kappa-1+2\sin^2\left(\frac{\theta}{2}\right)\right] \\
u_y &= \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}} \cos\left(\frac{\theta}{2}\right)\left[\kappa+1-2\cos^2\left(\frac{\theta}{2}\right)\right] \\
\kappa &= 3-4\nu(\text{plane strain})
\end{aligned}
\tag{1}
$$

Pelton turbine structure is first simplified as all the modelling is carried out through script resulting in rejection of curves being accounted in the model and therefore simple rectangular bar is assumed as a single bucket. A sector of a turbine is considered and a single bucket of runner is under study with all the simplification process shown in Fig.1.

2.2 Simplification and Finite Element Modelling

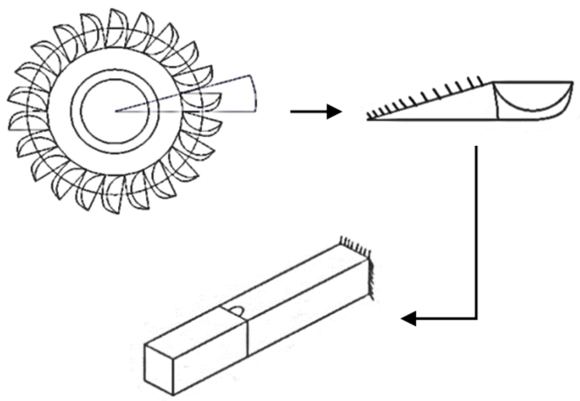

Figure 1: Runner Simplification

Finite element method technique is a flexible way to model complex structures especially when it comes to addressing discontinuities within a structure. Stress intensity factor estimation using Finite Element Method is done as per suggested by [4, 5, 6, 7, 8] to achieve accuracy. Nomenclature of surface crack is done using major axis and minor axis as two variable parameters shown in the figure 2. Modelling of semi elliptical surface crack in FEM is done as per [9] by angular discretization around the crack tip. Stress field near the crack tip becomes infinite as the distance 'r' is decreased and rise in stress level is increased exponentially so, in order to give good estimation of stress [10] highlighted the use of quadratic iso-parametric elements in triangular form with mid-size nodes moved at quarter points asserting

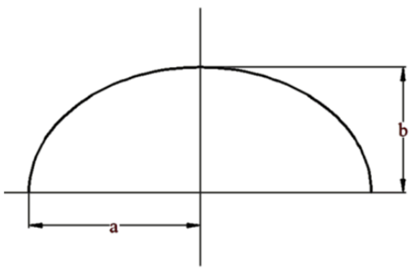

Figure 2: Crack Nomenclature

theoretical stress singularity. Any linear isotropic material cannot withstand infinite stress so, representation of plastic zone at crack tip as well as retaining elastic behavior elements was suggested by [11] and is available as SOLID 95 in [12]. Crack-tip is modeled as SOLID 95 where as other element type is SOLID 45 and PLANE 182 as shown in Fig.3 below:

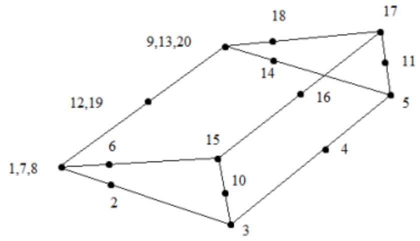

Figure 3: 20 Node Iso-parametric Element

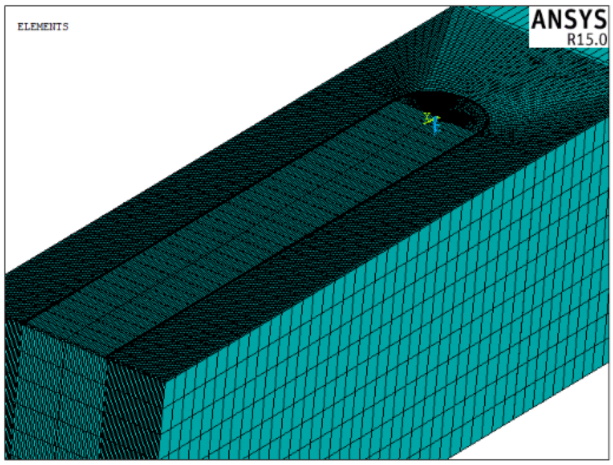

Figure 4: Crack Region Mesh

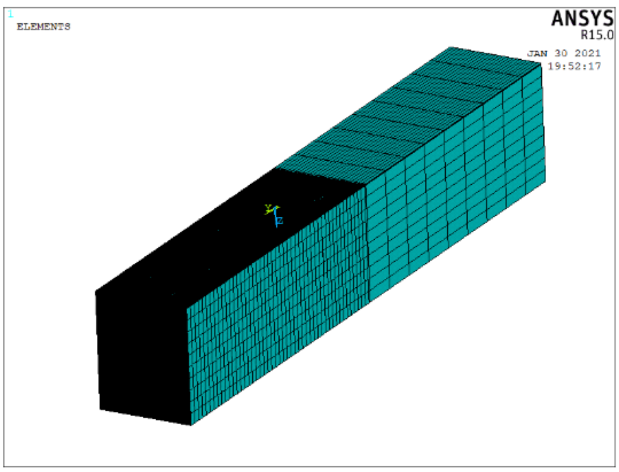

Figure 5: Bucket Mesh

2.3 Boundary Conditions

Pelton runner in this work is modeled as cantilever beam with jet force of unit magnitude acting on the region without crack. The rear end of the model is fixed as well as the bucket root region excluding crack has been fixed at the upper end as shown in Fig. 6 below:

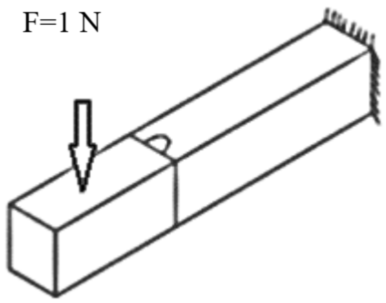

Figure 6: Bucket as cantilever beam

2.4 Iso-KI Crack Front Propagation

Theoretical fatigue crack propagation model adopted is Paris Erdogan Law where the rate of crack growth is directly proportional to the stress intensity factor (ΔK) which is estimated in every node along the crack front based on displacement extrapolation technique [9] using KCALC command in Mechanical APDL. Stress level along the specific aspect ratio of crack front is constant [13] for semi elliptical crack front and is also inferred as Iso-KI criteria which states that flaw can only grow when a constant value of stress intensity factor is achieved along the crack front. Similarly work on fatigue life estimation of pipe structure by [14] defined the Iso-KI criteria as equation (2) which is incorporated in this study.

$$K_I = \frac{[K_{Ii}]_{\max} - [K_{Ii}]_{\min}}{[K_{Ii}]_{\max}} \times 100 \qquad (2)$$

Where i=1, 2, 3 assigned for respective number of nodes along the crack front. Calculation of stress intensity factor as per the criteria in equation (8), APDL script with geometry variables 'a' and 'b' as shown in figure is created and applied structure of program is shown in Fig.7

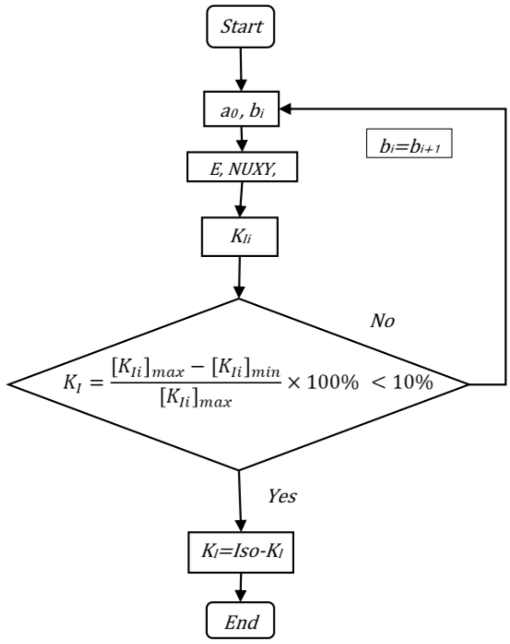

Figure 7: Iso-KI Algorithm

3. Finite Element Result

3.1 Iso- Stress Intensity Factor

Stress intensity factor is calculated using displacement extrapolation technique using singularity elements. All values of stress intensity factor within the crack front was listed in an array and then calculation of mean and standard deviation was done. The surface flaws whose difference in percentage of mean and standard deviation less than 10 was then stored and called as Iso-Stress intensity factor which are summarized below in the table 1:

Table 1: Iso-KI data from simulation

Surface crack	Depth (mm)	Iso-KI(Mpa $\sqrt{mm}$)
2	4.7	1.16E-03
3	6.75	1.10E-03
10	23	1.12E-03
20	41	1.09E-03
30	52.5	1.12E-03
40	62	1.13E-03
50	70	1.16E-03
60	78	1.18E-03
70	87.5	1.19E-03
80	92	1.30E-03
90	99	1.41E-03
100	110	1.51E-03
110	115.5	1.85E-03

Iso-stress intensity factor is an average value of stress along the crack front under study and the growth of crack front is represented in Fig.8 below:

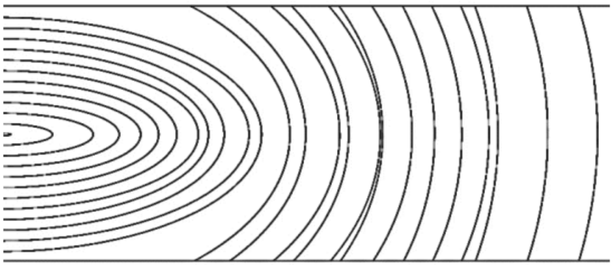

Figure 8: Crack Shapes of various Depth

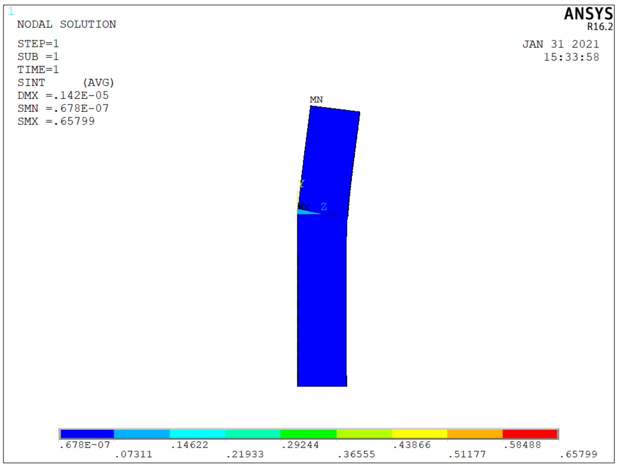

Figure 9: Deformed shape of Pelton bucket

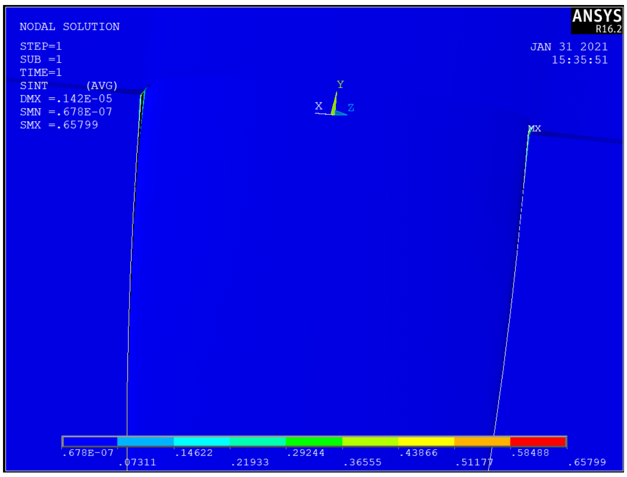

Figure 10: Stress distribution along crack ends

3.2 Crack Length and Depth

Simulation result showed the transition of crack where increment in depth is relatively high in comparison to its length so relation between length and depth is summarized in the figure 11:

Figure 11 shows the relation of surface crack length and crack depth as the transition of initial crack is observed to change from elliptical to circular and into straight line at later stages of propagation. Length of

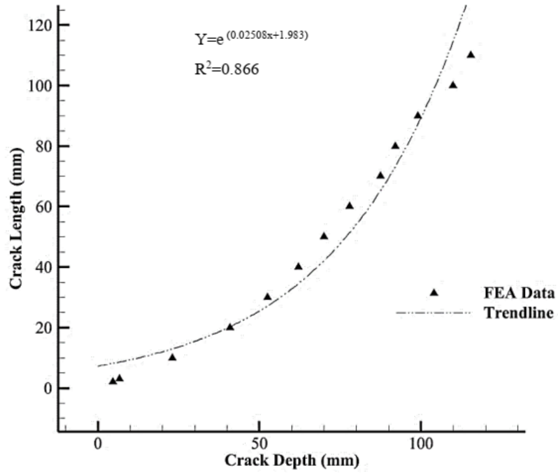

Figure 11: Surface Crack vs Crack Depth

surface crack is smaller with respect to crack depth during the initial stage where as it increases at the final stages.

3.3 Kitagawa-Takahashi Diagram

Stress distribution along the crack front and the corresponding crack length obtained from the simulation is used to plot the variation of stress difference and the crack depth shown in figure 12:

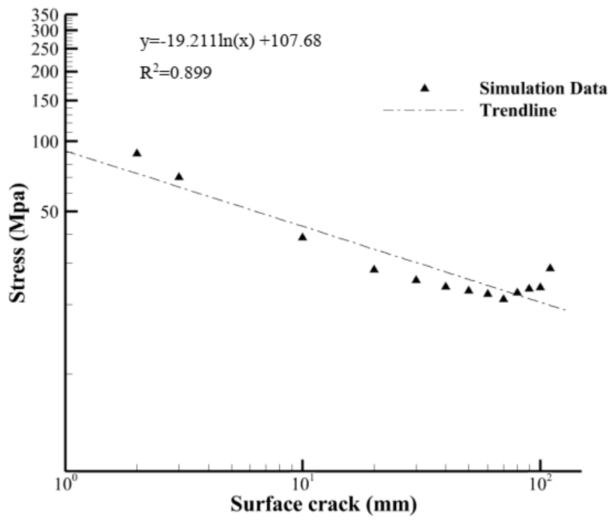

Figure 12: Kitagawa-Takahashi Diagram

Traditional S-N approach relates the stress amplitude and the cycles of failure whereas LEFM approach provides about the information of crack growth and the life of such cracks. In order to relate these two approach Kitagawa-Takahashi diagram is plotted between the stress amplitude and crack length. Figure 12 shows the relation of stress difference and the crack length which can be used as a tool for rough estimate of stress amplitude required to propagate the crack length with specific depth in case of Pelton Turbine. All the stress amplitude values lying below the line will not lead to fatigue failure. However, magnitude of stress above the line at a specific crack length will lead to fatigue failure.

4. Conclusion

Iso-stress intensity factor criteria can be successfully adopted for the study of crack propagation in Pelton turbine with semi elliptical profiles representing the crack fronts. Initial shape of crack can be used for the study as the crack fringes differ during the latter stages. Iso stress intensity factor obtained for the crack from the FEA result vary from 1.09E-03 to 1.85E-03 for the crack lengths of 2mm and 110 mm respectively. The maximum amount of stress acting on the crack front end was 351.91 Mpa whereas minimum value being 208.120 Mpa after the application of jet force equal to 1.90E+05.Kitagawa-Takahashi diagram can be used for the rough estimate of limiting stress value for a given crack length.

Acknowledgments

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References

[1] Amod Panthee, Bhola Thapa, and Hari Prasad Neopane. Quality control in welding repair of pelton runner. *Renewable Energy*, 79:96–102, 2015.

[2] H Kitagawa. Applicability of fracture mechanics to very small cracks or the cracks in the early stage. *Proc. of 2nd ICM, Cleveland, 1976*, pages 627–631, 1976.

[3] Ted L Anderson. *Fracture mechanics: fundamentals and applications*. CRC press, 2017.

[4] IS Raju and JC Newman Jr. Stress-intensity factors for a wide range of semi-elliptical surface cracks in finite-thickness plates. *Engineering fracture mechanics*, 11(4):817–829, 1979.

[5] IS Raju. Improved stress-intensity factors for semi-elliptical surface cracks in finite-thickness plates. *Proceedings of Structural Mechanics in Reactor Technology*, 1977.

[6] JC Newman Jr and IS Raju. Analyses of surface cracks in finite plates under tension or bending loads. Technical report, National Aeronautics and Space Administration Hampton VA Langley Research . . . , 1979.

[7] Andrea Carpinteri. Stress-intensity factors for semi-elliptical surface cracks under tension or bending. *Engineering fracture mechanics*, 38(4-5):327–334, 1991.

[8] JC Newman Jr and IS Raju. An empirical stress-intensity factor equation for the surface crack. *Engineering fracture mechanics*, 15(1-2):185–192, 1981.

[9] Gustavo V Guinea, Jaime Planas, and Manuel Elices. Ki evaluation by the displacement extrapolation technique. *Engineering fracture mechanics*, 66(3):243–255, 2000.

[10] Roshdy S Barsoum. On the use of isoparametric finite elements in linear fracture mechanics. *International journal for numerical methods in engineering*, 10(1):25–37, 1976.

[11] Roshdy S Barsoum. Triangular quarter-point elements as elastic and perfectly-plastic crack tip elements. *International Journal for numerical Methods in engineering*, 11(1):85–98, 1977.

[12] ANSYS Mechanical. Ansys mechanical apdl theory reference; version 13.0. *ANSYS Inc, Canonsburg*, 2013.

[13] Andrea Carpinteri and Roberto Brighenti. Circumferential surface flaws in pipes under cyclic axial loading. *Engineering Fracture Mechanics*, 60(4):383–396, 1998.

[14] Dura Hari Bahadur, Wu Tieying, and Wu Zhigong. Damage tolerance and fatigue life estimation for pipe/rod bar structure. *Transaction of Nanjing University of Aeronautics and Astronautics*, pages 169–175, 2011.