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Dynamic Response of Overhung Pelton Turbine Unit under Rotating Unbalance

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Abstract

This research work is carried out to model the excitation force imparted by water jet in the form of Fourier Series and determine the response of Overhang Pelton turbine unit located at Thapathali Campus lab premises analytically by developing mathematical model. Amplitude of forced vibration from analytical was compared with the simulation result from ANSYS Software. The mathematical model was developed by calculating the kinetic energy and the potential energy of both the disk and the shaft including the unbalanced condition of both. Lagrange's equation of motion was used to determine the equation of motion of unbalanced condition of the disk and the shaft. To validate the mathematical model ANSYS simulation was done. The simulation tool used was Harmonic Response of ANSYS Workbench. The data taken for modelling of runner and shaft for simulation purpose was from turbine located at lab of Thapathali Campus, IOE, TU. The runner was simply modelled as disk which was situated along the end of horizontal flexible shaft with fixed support at the other end. Fourier analysis was used to obtain the forcing function of water jet. The amplitude of vibration from analytical model was close to the amplitude obtained from ANSYS simulation in X – direction and Z- direction for unbalanced condition.

Keywords

Vibration, Overhung Pelton Turbine, Rotating Unbalance

1. Introduction

Pelton turbine is an impulse turbine and one of the widely used turbine in Nepal because of the availability high head and low flow rate in the country like Nepal. Vibrations are oscillations in the mechanical system. Vibrations are good as well as bad. However excessive vibrations are not good for any working system because it may lead to failure of the system and accidents in the working area. Vibration is undesirable, wasting energy and creating unwanted sound for most of the time. The vibrational motions of engines, electric motors, or any mechanical device in operation are typically unwanted. Such vibrations could be caused by imbalances in the rotating parts, uneven friction, or the meshing of gear teeth. Careful designs usually minimize unwanted vibrations.

Rotors are the rotating parts in different engineering applications such as turbines, compressors, fans, washing machines etc. Rotating machinery produces vibrations depending upon the structure of the mechanism involved in the process. The faults in the rotating machine can increase or excite the vibration. Vibration behavior of the machine due to imbalance is one of the main aspects of rotating machinery which must be studied in detail and considered while designing. Rotating Unbalance is the uneven distribution of mass around an axis of rotation. A rotating body is said to be out of balance when its center of mass is out of alignment with the center of rotation. This unbalance causes a moment which gives the rotors a wobbling movement characteristic. Thus, these dynamic behaviors of rotating machine due to unbalance is one of the most important topics to be studied.

There are numbers of hydropower using different kinds of turbines. In context of Nepal, Pelton turbine is used widely due to availability of high head. Study of dynamic behavior of those turbine enables us to get the information regarding operating mechanism and possible failures associated with the turbine. Thus, we can improve the performance, prevent emergency shutdown, increase life and reliability of the turbine with the information obtained from the study.

2. Methodology

2.1 Literature Review

Vibration deals with the oscillatory behavior of bodies. In recent times, many investigations have been motivated by the engineering applications of vibration, such as the design of machines, foundations, structures, engines, turbines, and control systems. Unbalance in rotating machinery is one of the main causes of vibration. A common source of forced harmonic force is imbalance in rotating machines such as turbines, centrifugal pumps, and turbogenerators. Imbalance in a rotating machine implies that the axis of rotation does not coincide with the center of mass of the whole system. Even a very small eccentricity can cause a large unbalanced force in high-speed machines such as turbines. The harmonic excitations are in the form of external force applied to the mass, base motion and force exerted on the mass of the system by a rotating unbalanced mass [1].

Pokharel et al. have presented modeling of Pelton turbine unit; which covers the dynamic behavior of an overhung rigid runner on circular flexible shaft, which was supported by rigid bearing on other end and enable to determine the natural frequency of the system by using different models. The unit was modeled as discrete and continuous systems. Overhung rotor model and Rayleigh's energy method: effective mass models were used for the discrete system models. The model for continuous system was developed by calculating the kinetic and potential energy of the runner –bucket and shaft. The governing equations were formulated by using Lagrange's equation and solved analytically by using Rayleigh-Ritz method.

Bhandari et al. developed the mathematical model by calculating the kinetic energy of the disk and both kinetic energy and potential energy of the shaft. Rayleigh-Ritz method was used to find natural mode of vibration and Lagrange's equation to derive the equation of motion for forced condition. Fourier analysis was done to obtain the function in its exact form. The developed methodologies were followed to find the analytical solution of dynamic response of selected Pelton turbine unit of 1 kW with single nozzle rated at 1500 RPM. A rigid disk (runner and buckets assembly) was situated along the end of flexible shaft with rigid and undamped simply supported bearings of the shaft[2].

Motra worked on mathematical modeling of the Pelton turbine unit by studying the dynamic behavior of the centrally located rigid runner on the circular flexible shaft, which was simply supported on the both ends by rigid bearings. This study was done to determine the natural frequency of the system by using different models. The unit was modeled as discrete and continuous systems. Föppl/Jeffcott rotor and Rayleigh's energy method: Effective mass models were used for the discrete system models. The model for the continuous system was developed by calculating the kinetic and potential energy of the runner-buckets and shaft assembly. The governing equations were formulated by using Lagrange's equation and solved analytically by using Rayleigh-Ritz method for the continuous system model [3]. Karki et al. studied the same turbine configuration under the conditions of forced vibrations. The excitation force imparted by the jet water was modelled in the form of Fourier series. Lagrange's equation, Rayleigh-Ritz method and Virtual work method was used to obtain the governing equation of motion under forced vibration condition. [4]

Bhatta et al. developed the method to study the forced vibration response of the Pelton turbine unit under rotating unbalance conditions. Rotating unbalance occurs when the center of mass of the rotating system does not coincide with its geometric center. The Pelton wheel is assumed as a rigid disk with lumped unbalance while the shaft, which is assumed as a Euler-Bernoulli beam, is flexible with continuous eccentricity distributions on orthogonal planes. The shaft is simply supported at the ends by rigid bearings. The equations of motion are derived by using Lagrange equation of motion from the energy expressions of the system with the help of assumed modes method. The excitation force imparted by the water jet is modelled in the form of Fourier series. The resulting equations are coupled non-homogeneous ordinary linear differential equations which are solved for the forced response of the system [5]. These works did not include the effects on mass unbalance on overhung Pelton turbine unit.

2.2 Development of Mathematical Model

Lagrange's equations are derived using energy methods. The basis for the derivation of Lagrange's equations is the principle of work and energy. It can be obtained from the kinetic energy and potential energy of the system.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{\mathrm{i}}}\right)-\frac{\partial T}{\partial q_{\mathrm{i}}}+\frac{\partial U}{\partial q_{\mathrm{i}}}=0 \quad (1)$$

Where,
q_{i}'s are the system's generalized independent coordinates.
$\dot{q}$ denotes differentiation with respect to time t.
T is the total Kinetic Energy of the system
U is the total Potential Energy of the system [6]

2.3 Mathematical Modelling

The shape function for the overhung Pelton turbine -[7]

$$f(y)=\cosh\beta y-\cos\beta y - \frac{\cosh\beta l+\cos\beta l}{\sinh\beta l+\sin\beta l}\left(\sinh\beta y-\sin\beta y\right) \quad (2)$$

The expressions for the displacements in x and z directions are expressed as:

$$u(y,t)=f(y)q_1(t) = q_1(t)\left[cosh\beta y-\cos\beta y-0.734\left(\sinh\beta y-\sin\beta y\right)\right] \quad (3)$$

$$w(y,t)=f(y)q_2(t) = q_2(t)\left[cosh\beta y-\cos\beta y-0.734\left(\sinh\beta y-\sin\beta y\right)\right] \quad (4)$$

The total kinetic energy of the system is (T):

$$T=T_{\mathrm{D}}+T_{ud}+T_S+T_{us}$$

$$\begin{aligned}T=\frac{1}{2}\Big[&(4)M_D+I_{Dxx}(488.4)+M_u(4)+ \rho A(0.13)+\rho I(35.74) +\mu(y)(0.13)\Big](\dot{q}_1^2+\dot{q}_2^2)+\Big[M_u d\Omega(2)\cos\Omega t\\ &+\mu(y)\Omega\int_0^L f(y)\Big(e_z(y)\cos\Omega t-e_x(y)\sin\Omega t\Big)dy\Big]\dot{q}_1-\Big[M_u d\Omega(2)\sin\Omega t\\ &+\mu(y)\Omega\int_0^L f(y)\Big(e_z(y)\cos\Omega t+e_x(y)\sin\Omega t\Big)dy\Big]\dot{q}_2 +\Big[I_{Dyy}(488.4)\Omega+2\rho I\Omega(35.74)\Big]\dot{q}_1\dot{q}_2\end{aligned} \quad (5)$$

Where,
T_{D} is the kinetic energy of the disk only
T_{ud} is the kinetic energy of the unbalanced mass in the disk
T_{S} is the kinetic energy of the shaft
T_{us} is the kinetic energy of the unbalanced shaft

The Equation (5) can be represented as -

$$T=\frac{1}{2}M(\dot{q}_1{}^2+\dot{q}_2{}^2)+A\dot{q}_1-B\dot{q}_2-C\dot{q}_1\dot{q}_2 \quad (6)$$

Where,
$M=(4)M_{\mathrm{D}}+I_{\mathrm{Dxx}}(488.4)+M_{\mathrm{u}}(4)+\rho A(0.13)+\rho I(35.74)+\mu(y)(0.13)$

$A=M_{\mathrm{u}}d\Omega(2)\cos\Omega t+\mu(y)\Omega\int_0^L f(y)\Big(e_{\mathrm{z}}(y)\cos\Omega t - e_{\mathrm{x}}(y)\sin\Omega t\Big)dy$

$B=M_{\mathrm{u}}d\Omega(2)\sin\Omega t+\mu(y)\Omega\int_0^L f(y)\Big(e_{\mathrm{z}}(y)\cos\Omega t + e_{\mathrm{x}}(y)\sin\Omega t\Big)dy$

$C=I_{\mathrm{Dyy}}(488.4)\Omega+2\rho I\Omega(35.74)$

The total strain energy of the system (U) :

$$U_{\mathrm{s}}=\frac{EI}{2}\left[\int_0^L h^2(y)q_1{}^2dy+\int_0^L h^2(y)q_2{}^2dy\right] \quad (7)$$

$$U_{\mathrm{s}}=\frac{1}{2}(5623.26EI)(q_1{}^2+q_2{}^2)$$

2.3.1 Lagrange equation of motion

Using Equation (1) for displacement variables q_1 and q_2, equations of motion can be obtained as -

$$\begin{aligned} \frac{d}{dt}\Big(\frac{\partial T}{\partial \dot{q}_1}\Big) - \frac{\partial T}{\partial q_1} + \frac{\partial U}{\partial q_1} &= 0 \\ \frac{d}{dt}\Big(\frac{\partial T}{\partial \dot{q}_1}\Big) = \frac{d}{dt}\Big(M\dot{q}_1 + A - C\Omega q_2\Big) \\ = \Big(M\ddot{q}_1 - C\Omega \dot{q}_2 + \frac{dA}{dt}\Big) \\ \frac{\partial T}{\partial q_1} &= 0 \\ \frac{\partial U}{\partial q_1} &= Kq_1 \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{d}{dt}\Big(\frac{\partial T}{\partial \dot{q}_2}\Big) - \frac{\partial T}{\partial q_2} + \frac{\partial U}{\partial q_2} &= 0 \\ \frac{d}{dt}\Big(\frac{\partial T}{\partial \dot{q}_2}\Big) = \frac{d}{dt}\Big(M\dot{q}_2 - B\Big) \\ = \Big(M\ddot{q}_2 - \frac{dB}{dt}\Big) \\ \frac{\partial T}{\partial q_2} &= -C\Omega \dot{q}_2 \\ \frac{\partial U}{\partial q_2} &= Kq_2 \end{aligned} \tag{9}$$

Hence, from equation (8) and equation (9) the general equation of motion for the system is in the form,

$$\begin{aligned} M\ddot{q}_1 - C\Omega \dot{q}_2 + Kq_1 &= -\frac{dA}{dt} \\ M\ddot{q}_2 + C\Omega \dot{q}_1 + Kq_2 &= \frac{dB}{dt} \end{aligned} \tag{10}$$

Solving,

$$\frac{dA}{dt} \qquad and \qquad \frac{dB}{dt}$$

We get,
Equations of motion due to mass unbalance present in disk and shaft as:

$$\begin{aligned} M\ddot{q}_1 - C\Omega \dot{q}_2 + Kq_1 &= (2M_{\mathrm{D}}e + I_1)\Omega^2 \sin\Omega t \\ &\quad + I_2\Omega^2 \cos\Omega t \\ M\ddot{q}_2 + C\Omega \dot{q}_1 + Kq_2 &= (2M_{\mathrm{D}}e + I_1)\Omega^2 \cos\Omega t \\ &\quad - I_2\Omega^2 \sin\Omega t \end{aligned} \tag{11}$$

2.3.2 External Force applied by jet and Fourier Series Representation

In Pelton turbine force exerted by water jet can be determine easily. In mechanical system often excitation forces are not harmonic function but periodic in nature. Hence excitation force is converted to periodic function by Fourier expansion. For calculation of forces the data taken was of the Pelton turbine located at the lab of Thapathali Campus is Force imparted by water jet in bucket can be given as;

$$F_{\mathrm{j}} = \rho_{\mathrm{w}} A_{\mathrm{j}} V_1 (V_{\mathrm{w1}} + V_{\mathrm{w2}}) \tag{12}$$

Where,
ρ_w is the density of water
A_j the area of water jet
V_1 is the velocity of water jet
V_{w1} is the component velocity in the direction of jet
V_{w2} is the component velocity in the direction of vane
$V_{w2} = k(V_1 - U)\cos\phi – U$, 'k' is the blade friction coefficient = 0.95
ϕ is the vane angle at outlet is 15 degree
U is the circumferential velocity of runner

$$U = \frac{\pi D_{\mathrm{w}} N}{60} \tag{13}$$

D_w is the diameter of Pelton wheel which is equal to 180 mm
N is the rated rpm of turbine which is 1500 rpm
$V_1 = C_v\sqrt{2gH}$
'C_v' is the velocity coefficient of turbine i.e., 0.98
'g' is the acceleration due to gravity
'H' is the net head of turbine

Putting in all value the force is calculated as :

F_j = 430.2842299 KN.

For Fourier series representation of the function can be written as -

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_{\mathrm{n}} cosn\Omega t + b_{\mathrm{n}} sinn\Omega t) \tag{14}$$

The coefficients of the Fourier series can be found as -

$$a_0 = \frac{2}{\tau}\int_{t_1}^{t_2} F(t)d(t)$$
$$a_n = \frac{2}{\tau}\int_{t_1}^{t_2} \Big[F(t)\cos(n\Omega t)\Big]d(t) \tag{15}$$
$$b_n = \frac{2}{\tau}\int_{t_1}^{t_2} \Big[F(t)\sin(n\Omega t)\Big]d(t)$$

We have,
$t_1 = 0$
$t_2 = \frac{61}{1000\Omega}$ s
$\tau = \frac{1}{16\Omega}$ s
where, t_1 and t_2 are the time of hitting and leaving the bucket.

Solving We get,

$$a_0 = 1.95F_j$$
$$a_n = \frac{0.3183}{n}F_j\sin(6.1324n) \tag{16}$$
$$b_n = \frac{0.3183}{n}F_j\big(1-\cos(6.1324n)\big)$$

Hence Fourier series can be expressed as:

$$F(t) = \frac{1.95}{2}F_j + \sum_{n=1}^{\infty}\Big[\frac{0.3183}{n}F_j\Big(sin(6.1324n)\,cos(n\Omega t)\Big) + \frac{0.3183}{n}F_j\Big(1-cos(6.1324n)\,sin(n\Omega t)\Big)\Big] \tag{17}$$

$$F(t) = 419.9574084 - 20.57330868\cos(\Omega t) + 1.554021834(\Omega t) - 20.33987196\cos((2\Omega t) + 3.090410828\sin(2\Omega t) - 19.95434235\cos(3\Omega t) + 4.591800915\sin(3\Omega t) - 19.42195718\cos(4\Omega t) + 6.041355004\sin(4\Omega t) - 18.74992956\cos(5\Omega t) + 7.423018231\sin(5\Omega t) \tag{18}$$

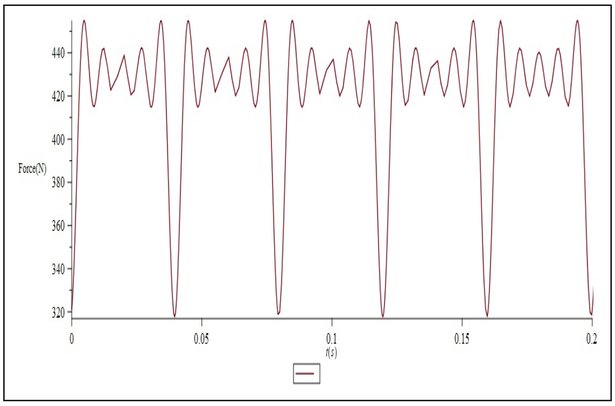

Figure 1: Force due to water jet in N

Equation of motions due to mass unbalance present in disk and shaft involving jet force is given by-

$$M\ddot{q}_1 - C\Omega\dot{q}_2 + Kq_1 = (2M_De + I_1)\Omega^2\sin\Omega t + I_2\Omega^2\cos\Omega t + F(t)$$
$$M\ddot{q}_2 + C\Omega\dot{q}_1 + Kq_2 = (2M_De + I_1)\Omega^2\cos\Omega t - I_2\Omega^2\sin\Omega t \tag{19}$$

3. Response of the System

3.1 Analytical solution for mathematical model

The solution to the equation of motion is in the form of -
$q_1(t) = A_0 + A_1$
$sin\Omega t + A_2 cos\Omega t + A_3\sin 2\Omega t + A_4\cos 2\Omega t + A_5\sin 3\Omega t + A_6\cos 3\Omega t + A_7\sin 4\Omega t + A_8\cos 4\Omega t + A_9\sin 5\Omega t + A_{10}\cos 5\Omega t$

$q_2(t) = B_0 + B_1\sin\Omega t + B_2\cos\Omega t + B_3\sin 2\Omega t + B_4\cos 2\Omega t + B_5\sin 3\Omega t + B_6\cos 3\Omega t + B_7\sin 4\Omega t + B_8\cos 4\Omega t + B_9\sin 5\Omega t + B_{10}\cos 5\Omega t$

Substituting these equations into equations of motion, coefficients of each harmonics are determined. Using the obtained coefficients, we get the expressions for forced response of the system as :

$q_1(t) = 294\times10^{-8} + 1.227\times10^{-8}\sin\Omega t - 20.05\times 10^{-8}\cos\Omega t - 8.64\times10^{-8}sin2\Omega t + 0.5755\times 10^{-8}cos2\Omega t - 1.6456\times10^{-8}sin3\Omega t + 7.06\times 10^{-8}cos3\Omega t - 0.91\times10^{-8}sin4\Omega t + 2.92\times 10^{-8}cos4\Omega t - 0.59\times10^{-8}sin5\Omega t + 1.49\times$

$10^{-8} cos5\Omega t$

$q_2(t) = -10.93 \times 10^{-8} \sin\Omega t + 0.162 \times 10^{-8} \cos\Omega t + 0.646 \times 10^{-8} sin2\Omega t + 9.699 \times 10^{-8} cos2\Omega t - 0.126 \times 10^{-8} sin3\Omega t + 2.915 \times 10^{-8} cos3\Omega t + 7.449 \times 10^{-8} sin4\Omega t + 2.317 \times 10^{-8} cos4\Omega t + 5.289 \times 10^{-8} sin5\Omega t + 2.093 \times 10^{-8} cos5\Omega t$

Putting value of $q_1(t)$ and $q_2(t)$ in the equation we get the displacement in X- direction and Z-direction as follows -

$$u(y,t) = f(y)q_1(t) = q_1(t)\left[cosh\beta y - \cos\beta y - 0.734\left(\sinh\beta y - \sin\beta y\right)\right] \quad (20)$$

$$w(y,t) = f(y)q_2(t) = q_2(t)\left[cosh\beta y - \cos\beta y - 0.734\left(\sinh\beta y - \sin\beta y\right)\right] \quad (21)$$

The maximum displacement in X- direction is $6.4 \times 10^{-6} m$
and the maximum displacement in Z- direction is $0.435 \times 10^{-6} m$

3.2 Simulation

The simulation was done in ANSYS 2016 workbench and the analysis system used is Harmonic Response. For simulation, the geometry was developed by using the data of overhung Pelton turbine located at the lab of Thapathali Campus, IOE, TU. The runner assembly is modelled as simple disk and the shaft is modelled as line body. Harmonic Response Analysis was done by adding masses of taken data on the disk and the shaft to create unbalance in the system. The model was created to have disk at one end and the rigid support of bearing at the other end of the shaft. The jet force was applied along the opposite direction of X – axis. The maximum displacement along X- direction, U(t) from the simulation was found to be $2.67 \times 10^{-6} m$ and the maximum displacement along Z – direction, W(t) from the simulation was found to be $3.3310 \times 10^{-6} m$.The specification used for construction of geometry is as shown in the table above.

Mass unbalance in the disk, M_u = 0.0001477 kg
Distance of mass unbalance in disk, d =0.05 m

Table 1: Parameters of Runner and Shaft.

Parameter	Value
Disk Parameters	
Output Power	1000 W
Rated RPM	1500 rpm
Thickness of Runner	50 mm
Dia. of Runner tip to tip	245 mm
Pitch Circle Dia. of Runner	180 mm
Inner Dia.	40 mm
Density of Runner Material	8300 kg/ m^3
Young's Modulus(Runner)	120 GPa
No. of Buckets	16
Thickness of Buckets	18 mm
Density of Bucket material	8300 kg/m^3
Total Mass of assembly	10.654 kg
Shaft Parameter	
Dia. of the shaft	40 mm
Material of the shaft	Mild steel
Density of shaft material	7860 kg/m^3
Young's Modulus (Shaft)	202 GPa
Nozzle Parameter	
Dia. of Nozzle opening	26 mm
Jet Area	5.04×10^{-6} m^2

Mass unbalance in the disk, $\mu(y)$= 1.292kg
Distance of mass unbalance in shaft, d_s = 0.020 m

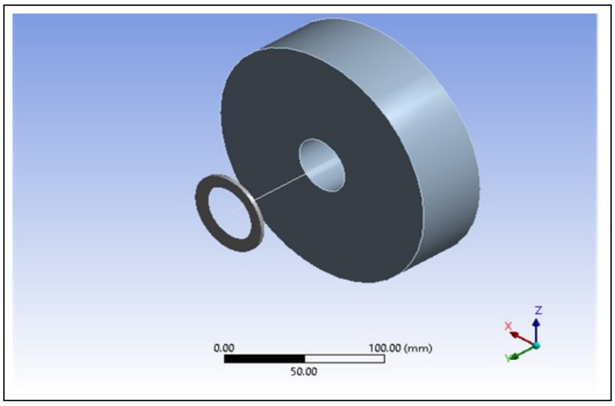

Figure 2: Geometric model

4. Result and Discussion

This research paper presents the methodology to study the dynamic response of Overhung Pelton Turbine unit under rotating unbalance system. The system was created by assuming runner blade assembly as disk and shaft as line body. The mathematical model was developed by using Lagrange's equation of motion. Fourier analysis was done to model water jet.

Summation of first five Fourier components was taken so as to represent the pressure pulse due to water jet.The equation of motion of unbalanced system consisting of forcing function was solved to get the amplitude of vibrations along x and z direction which were found to be 6.4 µm and 0.435 µm respectively.

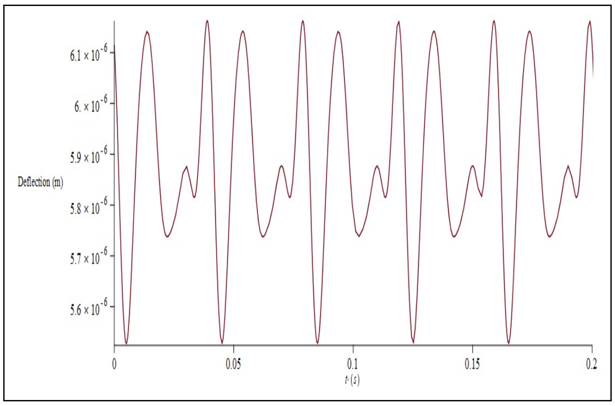

Figure 3: Deflection along X - direction in m

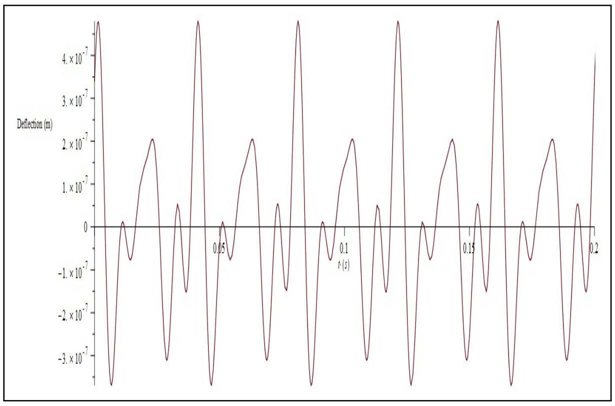

Figure 4: Deflection along z - direction in m

Simulation of geometric model was done with ANSYS 2016 workbench by applying Harmonic Response Analysis.The data for creating mathematical as well as geometric modelling was taken from the Thapathali Campus.The amplitude of vibration obtained from simulation along x and z direction which were found to be 2.66 µm and 3.33 µm respectively.

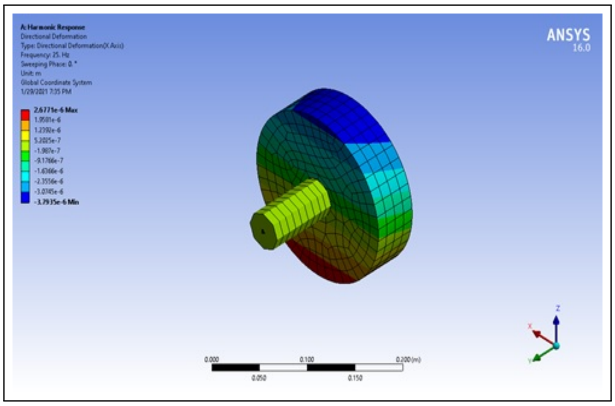

Figure 5: X - axis deflection

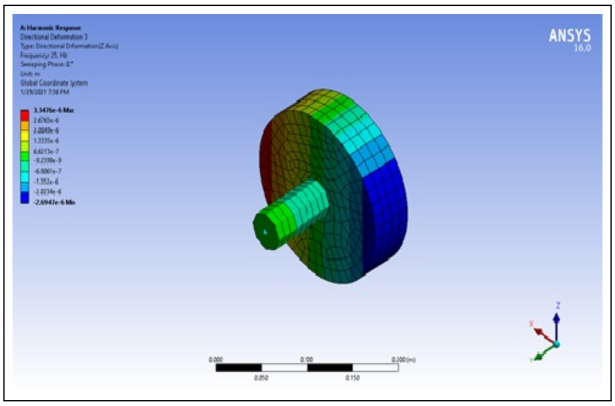

Figure 6: Z - axis deflection

The analytical result and the simulated result are close to each other. Hence this methodology can be applied to find the dynamic response of overhung Pelton turbine unit under rotating unbalance so as to limit the amplitude of vibration under acceptance level and avoid the failure of rotor system.

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