

Optimal Location of Phasor Measurement Units in Transmission System: A case study of Integrated Nepal Power System

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Abstract

Power grid monitoring plays a vital role in stable and secure power transfer from generating stations to consumer load points. Comprehensive and precise monitoring and controlling is possible only when the measurement taken from the network is accurate and reliable. Incorporation of synchrophasors measurement units like Phasor Measurement Units (PMU) can enhance present power grid monitoring system. Due to high cost of PMU and its installation, it is not recommended to place PMU in every nodes of the system. Hence it is advisable to determine the optimal number and locations of PMU in the grid ensuring full system observability. This paper presents a procedure by which optimal location of PMU can be determined in order to render an observable system. The proposed algorithm calculates optimal number of PMUs along with their location by considering Zero Injection (ZI) bus which makes the system fully observable. In addition to that, System Observability Redundancy index has been calculated in order to select the set of solution that maximizes reliability of the network by installing minimum number of PMU. The simulation for proposed method is tested by using MATLAB for IEEE-14 bus, IEEE-30 bus, IEEE-57 bus and Integrated Nepal Power System (INPS)-74 bus systems ensuring full system observability. The results obtained in this paper proved the effectiveness of the proposed method since the number of PMUs obtained is comparable with other available techniques.

Keywords

Phasor Measurement Unit (PMU), Simulated Annealing (SA), Bus Observability Index (BOI), System Observability Redundancy Index (SORI), Integrated Nepal Power System (INPS)

1. Introduction

Electricity demand has always been increasing, penetration of renewables into the network is also increasing and power industry is going through an evolutionary shift towards smarter grid technology which results in complexity in power system network. Such complex networks require better monitoring and controlling in order to maintain reliability, economic efficiency and protection of the system to be able to survive from contingencies. State estimation is an efficient means in determining real time states of the network [1]. The classical state estimation of a power system is based upon measurements collected from Supervisory Control and Data Acquisition System (SCADA) where measurements might be unsynchronized and have low sampling rates which results in less accurate data[2]. The biggest blackout in North America, one of the factors that caused the incident was lack of real time data gathering during

the time to the incident happened, leading to catastrophic blackout. Fifty million people in eight US states and two Canadian provinces were affected by the incident. After that incident, Wide Area Monitoring system (WAMS) became important for protecting the system from such blackouts [3].

PMU is the major component of WAMS which can provide time stamped synchrophasors data of voltage and current, system frequency and rate of change of frequency. Power system laboratory located at Virginia Tech developed the first PMU prototype in middle of 1980s. PMU device is normally interfaced with conventional CTs and PTs, use accurate time reference signals provided by GPS so that accurate time synchronized measurements of voltages and currents from a wide geographic area can be made available to system monitoring, control and protections system. The experience from deployed PMUs in various countries have proven that synchronized measurement technology is required to accurately analyze and

control grid performance both in real time and offline, for post disturbance data analysis and early warning system, improving system models for faster system restoration, observing under-damped low frequency oscillations and performance of protection system during power system disturbances [4].

It is possible to capture the system data simply by placing PMU in each and every node of the network without running state estimation problem. But it is not economical to place PMUs at each bus since the cost of PMU (cost of device, communication channels and installation cost) is high. Moreover it will also create technical complexity by placing PMU in all nodes. So we need a placement technique which will provide a complete observable system.

Currently, there has been considerable research activity on the problem of finding optimal number of PMUs and their optimal locations under different system conditions which can be generally divided into two main groups: conventional techniques and heuristic algorithms.

Reference [5] proposed a procedure for multi staging of PMU placement problem using ILP considering the zero injection buses which further reduces the optimal number of PMU. Paper [6] presented Balas Additive Algorithm for solving binary linear programming problems to reduce the number of PMUs required and to identify optimal placement for complete observability of power system keeping state estimation in focus. The proposed algorithm resulted in different sets of solutions, with each solution resulting in the same number of PMUs but at different locations from which final solution is obtained by the combination of buses with maximum redundancy. Advanced heuristic algorithms, not only analyze the system observability, but also dominate some difficulties of conventional methods such as PMU failure or branch outage.

Sensitivity constraint [7], lack of communication in substation constraint [8], critical measurements [9], fault observability [10] and mean square error (MSE) [11] are other objects that have been considered by heuristic optimization algorithms.

In this paper, Simulated Annealing algorithm (SA) has been implemented to determine optimal number of PMU along with their corresponding locations considering ZI bus. SA is genetic probabilistic meta-heuristic technique which solves the problem of combination optimization by simulating the physical

anneal process of solid matter. The method is best over other search method since it has ability to approach global optimum in presence of large number of local optima. Moreover, SA can deal with highly non-linear models, chaotic and noisy data and many constraints problems required for large system.

2. Methodology

2.1 Power System Observability

A power system is said to be observable if measurements available in the system are sufficient in number and locations to allow state vector of whole system to be estimated. Power system observability is an essential concept for state estimation. Only when a power system is guaranteed to be observable, state estimation of the system is accurate and comprehensive. According to Kirchhoff's and Ohms law, three kind of virtual measurements for topological observability can be applied in power system [12].

1. If a PMU is placed at a bus, this bus and all of its neighbor buses can be observed (Figure 1(a)).
2. For a zero injection node, which is observed, if all of its connected nodes are observable except one, then the unobserved node can be observed (Figure 1(b)).
3. If all the nodes connected to a zero injection node are observable, then the zero injection node can be observed too (Figure 1(c)).

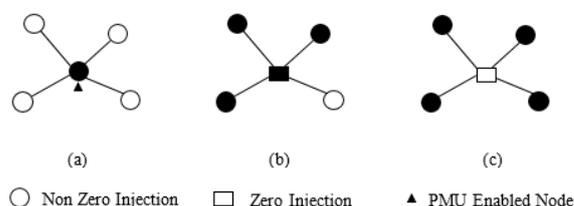


Figure 1: Topological observability rules based on PMU

2.2 OPP Problem Formulation

The main objective of optimal PMU placement is to determine the minimal number of PMUs to be installed at strategic locations so that the entire power system becomes completely observable for state estimation.

For n-bus system, the OPP problem can be formulated as:

$$\text{Minimize, } \sum_{i=1}^N w_i x_i \quad (1)$$

subject to $F(\bar{X}) = \bar{I}$

where

w_i is the cost of installation a PMU at bus i ,

$x_i = 1$ if a PMU is installed at bus i otherwise 0,

\bar{X} is a vector composed of x_i ,

$F(\bar{X})$ is a vector function that represents the observability constraint functions

\bar{I} is a column vector which has n entries and whose all elements must be 1 or greater than 1 to ensure full system observability.

According to three rules for system observability mentioned in [12], the constraint vector function can be obtained as

$$F(\bar{X}) = A\bar{X} \quad (2)$$

where A is a binary adjacency matrix. The element of matrix A is defined as

$$a_{mn} = \begin{cases} 1 & \text{if } m = n \\ 1 & \text{if bus m and n are connected} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

where a_{mn} represents the element located at m^{th} row and n^{th} column in A.

For the bus i in an n-bus system, the corresponding constraint function f_i can be represented as

$$f_i(\bar{X}) = \sum_{j=1}^n a_{ij} x_j \quad (4)$$

If $f_i(\bar{X}) \neq 0$, namely, if any $a_{ij} = 1$ and $x_j = 1 (j = 1, 2, \dots, n)$, the bus i is observable. If all buses are observable, i.e. if all $f_i(\bar{X})$ in $F(\bar{X})$ are non-zero, the power system is fully observable.

Above equations are for a system without consideration of ZI buses. When ZI buses are considered, the constraint function vector $f_i(\bar{X})$ is modified into a non linear constraint function vector. According to three rules for system observability considering the ZI bus mentioned in [12], for each bus in the group composed of ZI bus and its adjacent buses, a bus is observable if other buses in that group is observable. Therefore, we introduce an auxiliary binary variable y_{ij} in equation 4, to represent that bus i is observable supported by other buses bus not bus i

in the group of ZI bus j and all of its adjacent buses. Hence equation 4 can be modified as:

$$f_i = \sum_{j=1}^n a_{ij} x_j + \sum_{j=1}^n a_{ij} z_j y_{ij} \quad (5)$$

where, z_j is a binary parameter whose value is 1 if the bus j is a ZI bus or zero otherwise and y_{ij} is an auxiliary binary variable whose value is 1 if all of bus j and its adjacent buses but not bus i are observable or zero otherwise.

In the $a_{ij} z_j y_{ij}$ part of equation 5, a_{ij} ensures bus i and j must be connected and z_j ensures that bus j must be a ZI bus.

In practice, y_{ij} has to be calculated via logical operation. If, for example, there is a ZI bus m to which k normal buses composed of bus 1 to k where $m \notin [1, k]$, are connected, then the constraints function for bus 1 to k and bus m can be obtained as

$$\begin{aligned} f_1 &= \sum_{j=1}^n a_{1j} x_j + (f_2 \wedge f_3 \wedge \dots \wedge f_k \wedge f_m) \\ f_2 &= \sum_{j=1}^n a_{2j} x_j + (f_1 \wedge f_3 \wedge \dots \wedge f_k \wedge f_m) \\ &\dots \\ f_k &= \sum_{j=1}^n a_{kj} x_j + (f_1 \wedge f_2 \wedge \dots \wedge f_{k-1} \wedge f_m) \\ f_m &= \sum_{j=1}^n a_{mj} x_j + (f_1 \wedge f_2 \wedge f_3 \wedge \dots \wedge f_k) \end{aligned} \quad (6)$$

where, \wedge is logical AND operator.

From above equations, we can observe that the constraints equations, that compose the constraint function, are coupled. If we calculate the value f_1 to f_n , (where n is the total number of buses in a given system) only once in a certain order, the final result may be inaccurate. Therefore, we need to calculate f_1 to f_n repeatedly and take into account their values obtained in the last iteration, until the values of f_1 to f_n does not change after an iteration.

2.3 Bus Observability Index and System Observability Redundancy Index

The minimum number of PMU placement problem may have multiple solutions. Hence to decide the superiority among various solutions available, two performance indices are coined which are called as Bus Observability Index (BOI) and System Observability Redundancy Index (SORI). BOI for bus-k (α_k) is defined as the number of PMUs that can

provide us with bus-k data. Thus the maximum possible value of bus observability index is equal to the maximum bus connectivity m_k of a bus plus one.

$$\alpha_k \leq m_{k+1} \quad (7)$$

SORI is defined as the summation of the BOI's of all buses in the system. Thus

$$\beta = \sum_{k=1}^m \alpha_k \quad (8)$$

Where β represents SORI. The main advantage of maximizing SORI is that even if there is a PMU loss, a larger portion of the system will remain observable.

2.4 Simulated Annealing Algorithm

Simulated annealing is a kind of stochastic local search method which is put forward by Metropolis in 1953. The basic idea of local search is that the algorithm will keep searching if there is any adjacent solution, which is better than current solution and accepts new found solution if it is better than previously found solution. A worse variation is also accepted as the new found solution with the probability that decreases as the computation proceeds. The slower the cooling schedule or rate of decrease the more likely the algorithm is to find an optimal or near-optimal solution. SA is probabilistic technique for approximating global optimum of given function. For problems where finding an approximate global optimum is more important than finding a precise local optimum in a fixed amount of time, SA may be preferable. By analogy with the physical process of annealing process of solid matter, each step of the SA algorithm replaces current solution by a random “nearby” solution, chosen with probability that depends upon both the difference between corresponding function values and also on global parameter T that gradually decreases during the process. The energy function or the objective function in OPP problem is to minimize total number of unobservable buses and constraint is to make the system fully observable.

Pseudo code for simulated annealing algorithm

1. Initializing: Generate a randomly initial point X_0 setting initial temperature $T = T_{init}$ and iteration number $k=0$.
2. Calculation of annealing energy: Evaluate the value of objective function at X_k ; $E_k = F(X_k)$.

3. Generating randomly neighbor point: create a stochastically point x_{k+1} , near the previous one and calculate the energy at this point $E_2 = F(X_{k+1})$ and determine $\Delta E = E_2 - E_1$.
4. Inspection of first condition: If $\Delta E < 0$, current point is acceptable and go to step 6. Otherwise generate a random number α which belongs to (0 1) and calculate probability $P = e^{\Delta E/T}$
5. Inspection of second condition: if $P > \alpha$ then current point is acceptable and goto step 3. otherwise return to step 3.
6. Inspection of iteration condition: if $k < N$ (N is the maximum number of tried within one iteration), then $k=k+1$ and go to step 3. Otherwise goto step 7.
7. Stop condition: if $T < sT$ (sT is stop temperature), stop the algorithm otherwise set $k=1$, reduce the temperature $T=cT$ (c being the cooling rate) then goto step 3.

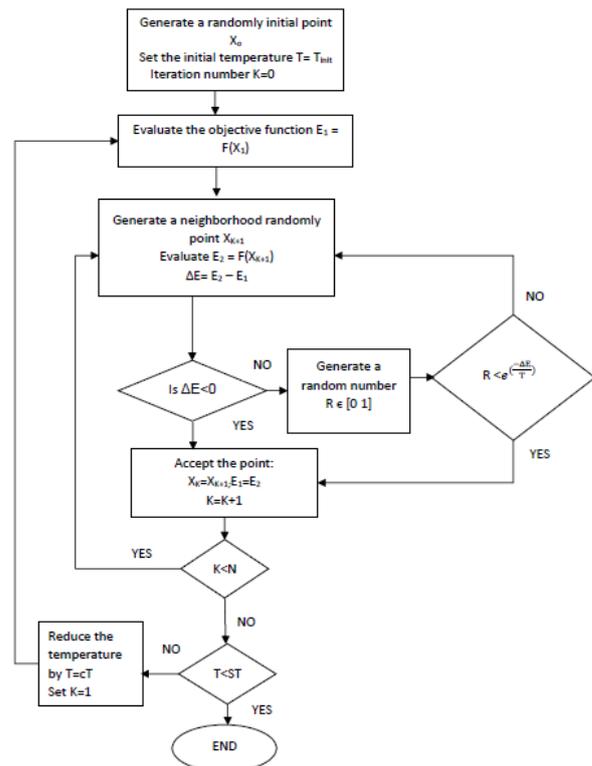


Figure 2: Flowchart for Simulated Annealing Algorithm

3. Simulation Results and Discussion

The formulation for optimal placement of PMUs and their locations for complete observability has been

simulated using simulated annealing algorithm. Developed algorithm has been tested for IEEE-14 bus, IEEE-30 bus and IEEE-57 bus system. The same algorithm is applied to find out optimal number of PMUs for existing INPS network.

The line data and bus data for IEEE 14-bus, 30-bus, 57-bus and Integrated Nepal Power System (INPS) were processed and fed as input data to the developed algorithm.

Table 1 provides the total number of ZI buses with their corresponding locations. ZI bus is determined by finding the bus with no load and no generators connected to it.

Table 1: Number of ZI buses and their locations for test systems

SN	Test Sytem	No. of ZI Bus	ZI Bus Location
1	14-Bus	1	7
2	30-Bus	5	6, 9, 22, 25, 27
3	57-Bus	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 40, 45, 46, 48
4	INPS		16 5, 6, 10, 11, 12, 14, 25, 26, 33, 39, 43, 44, 51, 67, 69, 71

Table 2: PMU Placement for IEEE-14 Test Bus System

SN	No. of PMUs	Location	SORI
1	5	1 3 9 10 13	18
2	5	3 5 8 11 13	17
3	4	2 4 11 13	18
4	3	2 6 9	15
5	4	4 5 10 13	18
6	4	2 8 11 13	14
7	4	1 4 10 13	16
8	6	2 3 4 10 12 14	23
9	3	2 6 9	15
10	5	4 5 9 10 12	22

Being a random search method, the simulations have been carried out for ten times to ensure the result and best solution is selected comparing SORI values (solution having the highest SORI value is the best solution) for maximizing redundancy. Table 2 shows the results for optimal location of PMU in IEEE-14 bus test system where minimum number of PMUs that can make the system fully observable is 3 and corresponding SORI value is 15.

In figure 3, after placement of PMU in bus no [2 6 9], whole network becomes observable. The full network observability can be ensured by evaluating values of constraint functions which must be non-zero values. In table 3, all values of constraint functions are non-zero value which means the test system of IEEE-14 bus is fully observable. Similarly, observability for other test system can also be proved by calculating the values of constraint functions accordingly.

Table 3: Values of constraint functions after Optimal PMUs Placement in IEEE- 14 bus system

Constraint Function	Value	Constraint Function	value
f(1)	1	f(8)	1
f(2)	1	f(9)	2
f(3)	1	f(10)	1
f(4)	1	f(11)	1
f(5)	2	f(12)	1
f(6)	1	f(13)	2
f(7)	2	f(14)	1

The same steps are carried out for other remaining test system and existing INPS network. Table 4 provides optimal number of PMUs required for making the system fully observable along with their corresponding locations while ensuring full network observability and total time required for the simulation for remaining test system.

Table 4: Optimum Number of PMUs required and their corresponding locations

Test Sytem	Optimal No.	Location [Bus No.]	Time Taken(Sec)
IEEE 30-Bus	7	[1 7 10 12 15 19 27]	38.204
IEEE 57-Bus	13	[2 6 12 13 20 24 29 30 32 38 51 54 56]	209.247
Existing INPS	19	[2 4 9 13 17 21 24 29 31 38 40 45 48 52 56 58 61 65 72]	433.874

Table 5: System Observability Redundancy Index (SORI) values

Test System	IEEE 14-bus	IEEE 30-bus	IEEE-57 bus	Existing INPS
SORI	15	32	57	72

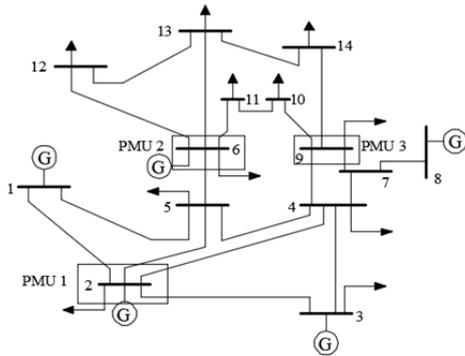


Figure 3: Optimal PMU Placement in IEEE-14 Bus Test System

Table 6: Comparisons between results obtained from BILP and SA

Test Sytem	BILP	SA
IEEE 14-Bus	4	3
IEEE 30-Bus	10	7
IEEE 57-Bus	23	13
Existing INPS	,	19

4. Conclusion

In this paper, PMU placement for power system observability for full network observability was presented. The problem was formulated considering zero injection bus for further reduction of optimal number of PMUs. The optimal placement problem was solved by using Simulated Annealing algorithm. The proposed algorithm being a random search method was simulated for ten times to ensure about the result. Ten solution sets sometimes generated same number of optimal PMUs located at different buses. So the best solution was selected having the highest value of SORI. Simulation results on IEEE-14 bus, IEEE-30 bus, IEEE-57 bus and INPS-74 bus test system indicated that the proposed placement method satisfactorily provides fully observable system with minimal number of PMUs.

The results obtained were quite compatible with those of other methods. Moreover, while comparing the

result with that of BILP method, simulated annealing algorithm resulted in reduced number of optimal PMUs.

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