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Effects of Higher Modes on Capacity Curve and Hinge Formation

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Abstract

The effects of higher modes on the capacity curves and their effects on the formation of plastic hinges are studied herein for regular reinforced concrete structure with no any weak or soft story having time period typically less than 1 sec and designed through by prevailing Indian standard and modal pushover analysis procedure based on structural dynamics theory, which typically uses the concept of modes, as well as modal analysis of elastic system and further utilizes this concept to determine the pushover analysis for different accompanied modes of elastic system to determine the capacity curves for inelastic system. The theoretical validity of modal pushover analysis is explained with respect to elastic and inelastic systems. In this modal pushover analysis (MPA), the seismic demand due to individual terms in the modal expansion of the effective earthquake forces is determined by a pushover analysis using the inertia force distribution for each mode. The higher mode pushover curves show significant increase in base shear as well as significant decrease in roof displacement that are not detected by the first mode which is a part of FEMA-356 force distributions. To determine the difference of seismic response parameter at concerned point, local performance levels are used rather than global performance level for better accuracy. Fiber hinge plastic plastic hinges are used instead of conventional lumped plastic hinge modeling where stress-strain curves for confined concrete for inner core concrete and unconfined stress- strain for outer concrete is used. Although the formation of plastic hinges is found to differ from that of the fundamental mode formation Mechanism, the pattern is not studied here.

Keywords

Capacity curve, modal pushover analysis, fundamental mode, Base Shear, Displacement Demand

1. Introduction

The realistic behavior of new building and existing building is greatly realized during recent decades, for this non-linear analysis became inevitable to capture the actual ‘existing structures and of the proposed design of new structures is usually based on nonlinear static (pushover) analysis procedures, even if nonlinear response history analysis is more accurate, but it is onerous task, several times repetition required, depends on types of models itself, etc. During past decades, several methods for pushover analysis was made, among them FEMA 356 [1] includes-Equivalent Lateral force(ELF), Response spectrum Analysis (RSA), Uniform force distribution(UFD) and First mode force distribution(1st mode)-which gives results agreeable to that actual response history analysis (RHA) for linear elastic system, but their results differ for inelastic system, which most of structures really behaves during strong ground excitation [2]. The seismic demands are computed by non-linear static analysis of the structure subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a predetermined target displacement is reached. Both the force distribution and target displacement are based on the assumption that the response is controlled by the fundamental mode and that the mode shape remains unchanged after the structure yields. Obviously, after the structure yields, both assumptions are approximate, but investigations have led to good estimates of seismic demands. However, such satisfactory predictions of seismic demands are mostly restricted to low- and medium-rise structures provided the inelastic action is distributed throughout the height of the structure i.e. structure must be regular in every aspect like vertical, mass and stiffness regularity. In addition to estimating the seismic demands, such as story drifts and plastic hinge rotations, the nonlinear static procedures are also expected to provide insight into possible plastic hinge mechanisms of the building. Typically, however, the hinge formation due to dominant first mode force distribution is globally from bottom to top storey, local plastic hinge concentrated at particular storey was found after Northridge Earthquake,1945 [3]. It, intuitively, stated the effect of higher modes rather than only first mode effects. Thus, in pushover analysis, appropriate higher modes and its seismic response should be included in calculating seismic demands. Now the question arises, how higher modes and included and how its response is calculated? Also, their response must be better than unimodal (i.e. 1st mode) and closer to NLRHA. Since the analysis of inelastic system is further derived from elastic system, modal combination and its response is required and comparison with RHA and then subsequent inelastic system is presented.

1.1 Response History Analysis and Pushover analysis: Elastic System

The response of regular multi-story building can be uncoupled and can be determined for each horizontal direction,$\ddot{u}_g(t)$ independently.

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+[k]\{u\}=-[m]\{\iota\}\ddot{u}_g(t) \quad (1)$$

This dynamic equation can be solved directly to get $u(t)$ using numerical method technique which is RHA analysis. This equation may be solved independently or simultaneously depending on damping matrix[c]-either it is classically damped or not. Generally classical damping like Rayleigh damping is considered in terms of modal damping ratio ξ_n .If displacement vector u can be expand into modal contribution.

$$u(t)=\Sigma_{r=1}^{N}\phi_n q_r(t),\ \ u_n(t)=\phi_n q_n(t) \quad (2)$$

then, substituting this and pre-multiplying with ϕ_n^T,we get,

$$[M]_{n*n}\{\ddot{q}\}_{n*1}+[C]_{n*n}\{\dot{q}\}_{n*1}+[K]_{n*n}\{q\}_{n*1}=\{P(t)\}_{n*1} \quad (3)$$

It is uncoupled equation, can be solved independently. If excitation force is decomposed into modal excitation factor then,

$$\{p_{eff}(t)\}=[m]\{\iota\}\ddot{u}_g(t)=\Sigma_{n=1}^{N}\{S_n\}\ddot{u}_g(t) \quad (4)$$

$$\{S\}=\Sigma_{n=1}^{N}\{S_n\}=[m]\{\iota\}=\Sigma_{n=1}^{N},\Gamma_n[m]\{\phi_n\} \quad (5)$$

$$\Gamma_n=\frac{\{\phi_n^T\{S\}\}}{M_n}=\frac{L_n}{M_n} \quad (6)$$

where $L_n=\{\phi_n\}^T[m]\{\iota\}$ and $M_n=\{\phi_n\}^T[m]\{\phi_n\}$ are the mode participation ratio and generalized mass. Using all above expression in uncoupled equation above, we get,

$$\ddot{q_n}+2\xi_n\omega_n\dot{q_n}+\omega_n^2 q_n=-\Gamma_n\ddot{u}_g(t) \quad (7)$$

Where Γ_n is scalar only, it is just scale factor of response, so using D_n instead of q_n.we get,

$$\ddot{D}_n+2\xi_n\omega_n\dot{D}_n+\omega_n^2 D_n=-\ddot{u}_g(t) \quad (8)$$

which is SDOF system, its solution $D_n(t)$ can be obtained using integral solution or numerical techniques. Thus, dynamic response can be written as,

$$q_n(t)=\Gamma_n D_n(t) \quad (9)$$

The contribution of the nth mode to the nodal displacement $u_n(t)$ is:

$$\{u_n(t)\}=\{\phi_n\}q_n(t)=\Gamma_n\{\phi_n\}D_n(t) \quad (10)$$

The equivalent static force developed due to stiffness of structure is given as,

$$\{f_n(t)\}=[k]\{u_n(t)\}=[k]\Gamma_n\{\phi_n\}D_n(t)=\omega_2\Gamma_n\{\phi_n\}D_n(t)$$

$$\{f_n(t)\}=\{S_n\}(\omega^2 D_n(t))=\{S_n\}(A_n(t) \quad (11)$$

It clearly shows that equivalent force applied at each Dof depends on product of spatial distribution of force and pseudo acceleration response of structure of time period T_n, calculated from modal analysis- eigen value problem.

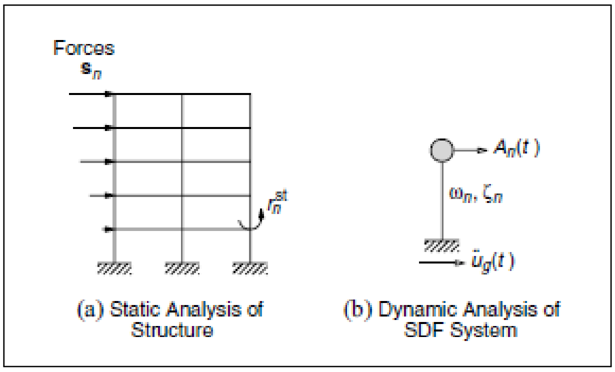

Figure 1: Conceptual explanation of modal RHA of elastic MDF systems [4]

The peak equivalent static force is given by,

$$\{f_{no}\} = \{Sn\}A_n = \Gamma_n[m]\{\phi_n\} \tag{12}$$

which gives peak response

$$r_{no} = r_n^{st}A_n$$

where, where An is the ordinate $A(Tn;\xi_n)$ of the pseudo-acceleration response (or design elastic response spectrum for the nth-mode SDF system, and $T_n = \frac{2\pi}{\omega_n}$ is the nth natural vibration period of the MDF system. Therefore, the response of the system to the total excitation $p_{eff}(t)$ is,

$$\{u(t)\} = \Sigma_{n=1}^{N}\{u_n(t)\} = \Sigma_{n=1}^{N}\{\phi_n\}\Gamma_n D_n(t) \tag{13}$$

$$\{r(t)\} = \Sigma_{n=1}^{N}\{r_n^{st}\}A_n(t) \tag{14}$$

This is Modal RHA analysis and if peak pseudo acceleration response A_n is placed instead of $A_n(t)$, then it will be modal RSA analysis. Clearly, if static force

$$\{f_{no}\} = \{S_n\}A_n = \Gamma_n[m]\{\phi_n\}A_n$$

is applied on structures gives displacement $\{u_{no}\}$ or vice-versa.

Clearly, To develop a pushover analysis procedure consistent with RSA, if structure is pushed with lateral force proportional to $\{f_{no}\}$, say $\{S_n\} = scalefactor * \{m\}\{\phi_n\}$ and displaced up to $\{u_{no}\}$,always gives $\{f_{no}\}$. Intuitively, elastic pushover analysis is simply RSA, therefore, elastic pushover analysis is seldom done, but required to understand modal analysis to apply in inelastic action

1.2 Dynamic and Pushover analysis: Inelastic System

Only difference between elastic and inelastic system is in stiffness matrix term, depends on material and geometric nonlinearity of each components and history of displacement -either increasing and decreasing, measured in terms of velocity. The dynamic equilibrium expression of inelastic system is,

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+\{f_s(u,\pm\dot{u})\} = -[m]\{\iota\}\ddot{u}_g(t) \tag{15}$$

If Force vs deformation(F-D) relation of each component is known, above equation can be solved directly using numerical techniques unaffected either it is classically or non-classically damped, which is called NLRHA. Solving above equation is complex, tedious and time demanding, any small instability can terminate process and result will be incomplete and erroneous. To understand the Modal pushover analysis (MPA), it becomes inevitable to understand NLRHA and UMRHA. Although modal analysis is not valid for inelastic system, its dynamic response is usually related to its corresponding linear system i.e. $\{\phi_n\}$ and ω_n is still valid. If $\{p_{eff}(t)\}$ is decomposed into modal contribution, as elastic system above, for the corresponding linear system of inelastic system as,

$$\{p_{eff}(t)\} = \Sigma_{n=1}^{N}\{S_n\}\ddot{u}_g(t) \tag{16}$$

then, the equation governing the n^th modal response is given as:

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+\{f_s(u,\pm\dot{u})\} = -\{S_n\}\ddot{u}_g(t) \tag{17}$$

But its solution for response is not defined by $\{u_n(t)\} = \{\phi_n\}q_n(t)$,because modes other than nth mode also contribute to the system response, i.e. vibration modes are coupled. Goel and Chopra(2002) [5] had shown that modes are weakly coupled and ,for inelastic system, can be written as:

$$\{u_n(t)\} = \Sigma_{r=1}^{N}\{\phi_r\}q_r(t) \approx \{\phi_n(t)\}q_n(t) \tag{18}$$

On solving above general equation by NLRHA and decomposing its response as modal cordiantes

$$q_n(t) = \{\phi_n\}[m]\{u_n(t)\}/M_n$$

at each instant of time,it can be shown that the effects of other modes on nth mode is minimal, so above approximation holds true.

Clearly, It is expressed above that equivalent force on inealstic system can also be expressed as product on static force $\{S_n\}$ and dynamic part $q_n(t)$ and effects of other modes on nth modes can be ignored, thus same concept as elastic system can be applied in inelastic system.Therfore, a static force

$$\{S_n^*\} = scalefactor * [m]\{\phi_n\}$$

for any mode can be applied on structure at defined Dof and pushed. Now the question may arise, how can we decide relative scale factor for different modes to be applied? If fundamental mode lateral force is only applied to structure, then any scale factor may be selected because in pushover analysis, load is applied form zero to full scale of load in small size of load for force-controlled and displacement for displacement controlled structure.But when multi modes are applied to structue, it will be faulty to apply scale factor randomly, must be relative to each other to account higher mode effect.

2. Description of Building selected

A fictitious 4-storey regular RC moment resisting building is selected with fairly uniform distribution of stiffness as well as strength throughout its height, without any obvious soft and/or weak story condition.The structure considered here is typically represents the majority of reinforced concrete building constructed. The time period of the building is deliberately chosen such that it is well below 1 sec.

Certain assumptions are made while performing analysis:

- Soil- Structure interaction has been neglected assuming rigid footings,
- Particularly good at working with mathematical symbols.
- Rigid floor diaphragm is assumed.
- Secondary elements resisting lateral forces are neglected- staircase and gravity columns.
- Secondary effects such as temperature, creep, shrinkage etc. are not considered.

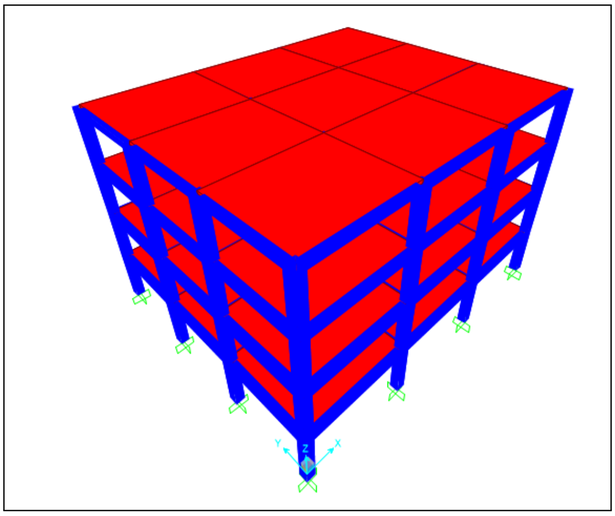

Figure 2: Sap2000 modelling of regular building

The building selected is modeled by using Sap2000 V21.20 and further designed by prevaling Indian standard Code IS456:2000 and seismic code IS 1893:2016 and also ensured that the change in storey stiffnes along the height of building is not such that it may makes the storey above or beneath as weak or soft storeys. Also, the torsional criteria is also checked as mentioned using the seismic code so that the response of structure should be predictable and lucid. During linear analysis, the storey drift limitation is also ensured.

To capture the pushover curve and further analysis, the designed model is exported to PERFORM 3D, which is sophisticated software to capture the significant portion of pushover curve without any unstabilities in numerical solution of non- linear equation at different incremental analysis.

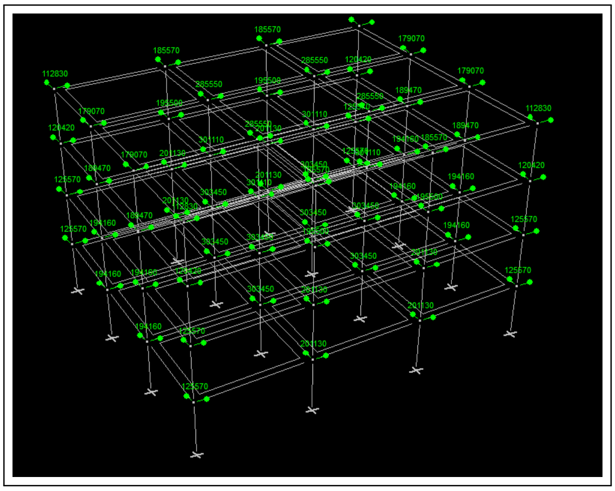

Figure 3: Exported to Perform 3D for analysis

Table 1: Design Parameters

Importance factor	1
Reduction factor	5
Seismic Zone	V
Concrete	Grade M20 Modulus of Elasticity $(E) = 22360KN/m^3$ Poison's ratio$(\upsilon) = 0.2$ Unit weight of rebar $(\gamma) = 25KN/m^3$
Reinforcement	Grade=HYSD500 TMT Grade=HYSD415 TMT Unit Weight$(\gamma) = 76.9kN/m^3$ Modulus of Elasticity $(E) = 200MPa$ Poisson's ratio$(\upsilon) = 0.3)$
Storey height	3m
No of bays	3 in X- and Y- direction
Design combination	IS 456:2000
Unit Wt of materials	IS 875 part 1
Live loads	IS 875 part 2
Seismic lateral load	IS 1893:2016 (part 1)

3. Methodology

The fictitious building was designed as per IS 456:2000 in SAP2000V21.2.0 and then again analyzed -members are detailed in section designer- to get exact rebar area and detailing. For nonlinear analysis, model was exported in PERFORM 3D, which is analysis by design software and check the limit ratios in terms of usage ration to calculate demand/capacity (D/C) ratios.

Fibre hinge was assigned to concentrated fixed length plastic hinge. Kent and Park (Mander curve) stress-strain curve for confined reinforced concrete member was assigned. Standard ASCE(41-13) uses plastic rotation value for plastic hinges which are ideal value, but in fibre hinge model limit ratios were used in terms of stress limit which was assigned directly in stress-strain curve- capture actual behavior of plastic hinge rotation. Local plastic hinges were assigned at sections which prior undergo inelastic action- edge of the beam and columns in MRF. Different limit usage ratios in terms of deformation, plastic rotation and drift were assigned to capture various performance levels in controlled way. To monitor the roof displacement, controlled points at each floor were assigned so that negative displacement of any storey could be recorded.

For multi-mode pushover analysis, equivalent static force was applied, from euation 6 as

$$\{S_n\} = \Gamma_n[m]\{\phi_n\}A_n$$

Modal Combination rule CQC was applied to combine modal response and finally plot between base shear and roof drift or roof displacement i.e. capacity curve was generated.

4. Result and Discussion

The result of pushover analysis considering fundamental mode and modes upto third modes, expressed in terms of Base shear with total lateral Drift in X direction and Roof displacement in same direction for selected building, is shown below.

To understand the complete failure of the structue, maximum target drift upto 10 % is considered, where,generally, 4% is taken. The graph shown below may seems steeper than usual found in other literature because of higher target drift/displacemtent.

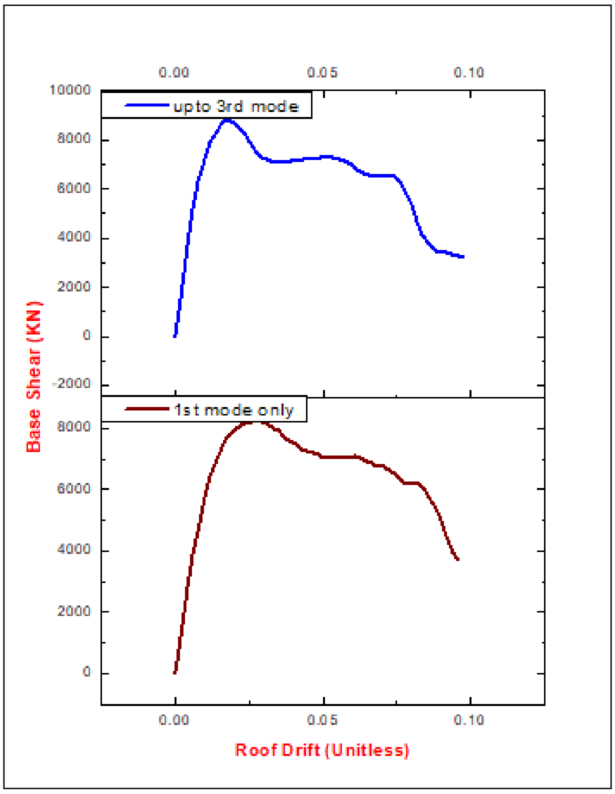

Figure 4: Capacity Curve for 1st mode and upto 3rd mode in terms of Base Shear Vs Roof Drift

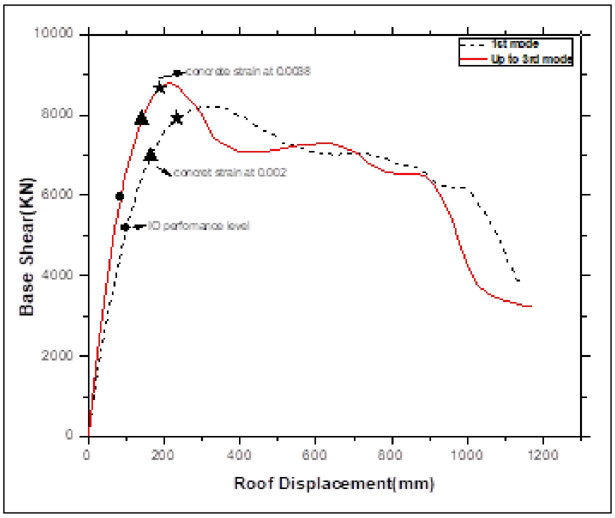

Figure 5: : Difference in Base Shear and Roof Displacement for Immediate Occupancy Performance Level

The capacity curve containing higher mode greater than 3rd mode is ,seemingly, found same as that of 3rd mode, so eradicated from the result.

Table 2: Numerical Results

Modes	**Base Shear** (KN)	**Roof Disp.** (mm)
IO perfomance level :		
1st mode only	5193	101.232
Up to 3rd mode	5947	85.188
comp.strain at 0.002:		
1st mode only	6965	162.96
Up to 3rd mode	7719	137.04
comp.strain at 0.0038:		
1st mode only	7912	239.52
Up to 3rd mode	8737	208.68

As Atc-40(1996) states that if the structure is regular and having time period up to 1 sec, only fundamental modes should be used for Pushover analysis [6]. Although, The building used has fundamental period of 0.67 sec(extracted from PERFORM 3D), which is typically less than 1 sec, also the structure is regular in all aspect i.e. stiffness and geometry, from table no 2, it is obvious to consider higher modes as the difference in seismic parameter is not trivial. From table2, it is found that base shear is increased by 14.52 % , 10.825 % and 10.42 % and displacement demand is decreased by 15.84 %, 15. 94 % and 12.86 % for Immediate Occupancy perfomance level, beam inelastic fibre component compressive stress at 0.002 and that at 0.0038 respectively. The base shear of capacity curve upto 3rd mode is increasing in decreasing rate while displacement demand is decreasing without any pattern with respect to that of first mode. Since the criteria for component performance level is ideal and generalized for all type of similar component, focus is given in actual parameter- strain ,rather than using standard rotation for component.

From figure 5, the ultimate deformation ductility in terms of displacement for capacity curve of first mode and upto third mode is 14.06% and 12.664 % respectively. Thus ultimate ductility in displacement term of capacity curve upto third mode decrease by 10 % with respect to that of first mode.

Table 3: Energy Equivalence of Capacity Curves

Modes	Energy Dissipated $(KN - mm)$
1st mode only	7473356.1413
Up to 3rd mode	7392109.4531

From table 3, it is clear that the energy released due to merger of three modes is 1.08 % lesser than that due to first mode only. Lesser energy stored means lesser potential energy, thus, structure may follow the path of 3rd mode curve during pushover analysis. Generally,it is not easy task to assure that the structure is regular in terms of stiffness, mass, even if the structure is regular in terms of primary element, secondary elements like stair case, gravity columns modify the response of structure. Similar will be the case with pent house, staircase void cover, etc. Therefore, relying completely on fundamental mode pushover curve,while in the midst of, the structure capacity curve may behave in higher modes,will be erroneous.

5. Conclusion

From the above results, following conclusion can be made.

1. Base Shear of Capacity Curve at IO performance level, inelastic beam component with compressive stress at 0.002 and 0.0038 is increased by 14.52 % , 10.825 % and 10.42 % respectively for up to third mode than that of fundamental mode.

2. Displacement Demand of Capacity Curve at IO performance level, inelastic beam component with compressive stress at 0.002 and 0.0038 is decreased by 15.84 %, 15. 94 % and 12.86 % respectively for up to third mode than that of fundamental mode.
3. The dissipated energy equivalence up to third mode is 1 % lesser than that of fundamental mode.
4. The ultimate ductility in terms of displacement of capacity curve upto third mode is decreases by 10 % that that of funadamental mode only.

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