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Fatigue Damage Model of Concrete Materials

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Abstract

Based on the continuum damage mechanics theory of concrete materials under fatigue loading; a phenomenological fatigue damage model due to nuclei-propagation of microdefects, microvoids and fractal of rendered concrete in conjunction with more compliant are developed by utilizing internal variable theory of thermodynamics. Experimental data from fatigue Test were employed to verify the Model and results shows that the model can describe the damage evolution of concrete materials under different fatigue loading by verifying the predicted fatigue life of concrete materials. Degradation of Young's Modulus, alteration in the mechanical behaviour, damage in elastic stiffness tensor, evolution in strain, damage development corresponding to the Normalized life, its accumulation subjected to constant amplitude fatigue loading including its verifications are well discussed.

Keywords

Concrete, Nuclei-propagation, continuum damage mechanics, fatigue loading, normalized life

1. Introduction

Fatigue is defined as the dominant failure characteristic of concrete structures under different cyclic loading which are the major causes for creation of damage and degradation of material property [1]. The foundation to predict abrupt material failure, fatigue strength of the plain cement concrete under serious service control was developed by author [2]. Based on the double bounding surface approach of damage mechanics, cyclic loaded prone structures such as bridges; highway pavements and dams which are subjected to uniaxial alternate tension-compression fatigue loading, a constitutive damage model was matured by Lu P. et al [3] and for gradual material degradation of longitudinal modulus along progressive creep strains, as brittle cracking and nonlinear creep which are two components for model development of cyclic loading, a fatigue model was proposed by author [4]. Fatigue Tests on concrete cylinders for residual strengths and secant fatigue modulus with an increment of fatigue loading cycles, a damage model for the estimation of failure cycles with secondary strain rates of S-N Curves that alters the capacity for predictions of strength and stiffness degradations during entire process of fatigue life, mainly not consigned with the recent theories of continuum damage mechanics, that was accepted as a brittle solids model tools described in the research [5]. Microdefects, microvoids and fractal which are the main causes for the degradation of Young's modulus of elasticity in fatigue process was reported by Gao et al [6]. Cyclic strain, the indicator of irreversible fatigue life, cyclic creep, and fatigue range that is higher than monotonic state strain and accumulated ultimate and plastic strains after each cycle before rupture, are dependent on the applied fatigue loading cycles of the concrete materials [7]. Thus, the mechanical behaviour of ordinary-reinforced concrete under cyclic loading is governed by microdefects like kinetics of the microstructure of the concrete materials and its life is dependent on the ranges of applied maximum stress and its ranges [8]. Nucleation, interaction and multi-microdefects growth that is the scenario of failure of concrete materials and cycles is inversely related to the applied stress. The model under such phenomenon is developed under bi-axial compression using bounding surface approach [9]. Anisotropic elastic degradation, strength reduction and increment of inelastic strain that are the main resources of the fatigue failures, a stress-based model having long computational time and large data storage even in a simple finite element analysis model was developed by author [10].

On the application of large number of loading cycles leading to sudden rupture or ”sudden deadth” of the concrete materials, residual strength, elastic stiffness, fatigue loading that influences its capacity, the strain level notably greater than failure strain occurred in the monotonic loading state was widely described [11]. More comprehensive and rational extension of works [11] had developed in the form of anisotropic fatigue model for concrete materials [12] and further extension as strain space formulation of continuum damage mechanics [13], along with widely extension in order to capture the stiffness degradation, strength reduction, a functional fatigue damage model regarding reduction and softening of the concrete material was well developed [14] that had attributed through complex degradation of mechanics for steel and concrete composites in fatigue damage process [15]. Fully accountable for fatigue damage mechanisms, concrete integrity deterioration, irreversible strain accumulation, and reinforcement crack growth incorporated into the constitutive models, compatibility equilibrium equations of the Disturbed Stress Field Model (DSFM), analysis algorithm to predict the fatigue residual capacity of the structural element regarding fatigue life are described by authors [16].

Fatigue model for all kinds of concrete like ordinary, lightweight, high strength, fiber reinforced in tension-compression, torsional, bending either uniaxial or multiaxial as a complex phenomenon, cubical polynomial curve fittings taking correlation coefficients above than 0.937 has well developed by Y. B. Chen et al [17]. Logarithmic of maximum strain rates with continual loading for second phase of the concrete in frazzle was proposed by author [18] with index formula connected to concrete fatigue strain fitting curve [19] with employment of verification tools through the experimental results [20]. Growth of Microcracks, inelastic flow with small amplitude of cyclic loading, that is unable to take guarantee of endurance limit like metal was well developed by author [21] in conjunction to damage model. Based on the mechanics of composite materials, the permanent damage failure for ordinary concrete in compression fatigue loading with cycle and time dependent, concept of dual nature model was well established by author [22]. Another, Cumulative fatigue damage analysis related to accelerated pavement testing was performed by author [23] along with experimental result for validation of model has taken from the research investigation of author [24] that illustrates, the occurance of fatigue damage in concrete is about 20% of its fatigue life. Thus, based on this fact, theoritical model was presented for the prediction of sap of concrete materials during tension-compression, the altered fatigue loading with employment of double bounding surface approach making on assumption of strain-energy released rate for optimum construction of damage-effective tensor [25] and researched theoritical and practical damage frazzle constitutive model, dominant microstructural patterns, inelastic flow, microcracking natures, nuecli-propagation of fractal, degradation of Young’s modulus of elasticity, the vital materials features for discovery of mechanical behavior of concrete under monotonic cyclic loading was well discussed by authors [26 – 39]. The locus of point on damage surface extended to failure surface for providing the strain at any cycles, a cumulative inelastic parameter for capturing the plastic strain, its accumulation to the normalized life, degradation of Young’s modulus, strength reductions and permanent deformation formulation in the frazzle environment as a damage model is proposed in this paper for new concrete material fatigue prescriptions.

Based on the continuum damage mechanics theory of concrete materials under fatigue loading; a phenomenological fatigue damage model due to nuclei-propagation of microdefects, microvoids and fractal of rendered concrete in conjunction with more compliant are developed by utilizing internal variable theory of thermodynamics. Experimental data from fatigue Test were employed to verify the Model and results shows that the model can describe the damage evolution of concrete materials under different fatigue loading by verifying the predicted fatigue life of concrete materials. Degradation of Young’s Modulus, alteration in the mechanical behaviour, damage in elastic stiffness tensor, evolution in strain, damage development corresponding to the Normalized life, its accumulation subjected to constant amplitude fatigue loading including its verifications are well discussed.

2. Damage Model

Degradation of elastic moduli and stiffness of the concrete at micro level field which creates non-inverse change under cyclic stress-strain with the increment of cyclic loading, are the vital causes of the fatigue failure. Based on the Continuum damage mechanics approach, degraded stiffness and elastic moduli that creates microstructural damage development, a versatile concept from internal variable theory of thermodynamics with isothermal assumptions from [12] and [13], small deformations and rate independent, the Helmholtz Free Energy (HFE) per unit volume can be expressed as Equation 1.

$$A(\varepsilon,K)=\frac{1}{2}\varepsilon:E(k):\varepsilon+A^i(k) \tag{1}$$

Where, $E,k,A^i(k),:$ strain tensor, current fourth-order stiffness tensor, cumulative scalar damage parameter, scalar function related to microcracks surface energy, tensor contraction operation. Following Equation 2, the constitutive relation for the concrete materials by using the Helmholtz Free energy is simply given by

$$\sigma=\frac{\partial A(\varepsilon,k)}{\partial\varepsilon}=E(k):\varepsilon \tag{2}$$

The differentiation of Equation 2 with respect to Normalized life (N), the frazzle cycle, yields,

$$\dot{\sigma}=E(k):\dot{\varepsilon}+\dot{E}(k):\varepsilon=\dot{\sigma}^e+\dot{\sigma}^i \tag{3}$$

Where in Equation 3, $\sigma,\dot{\sigma},\dot{\varepsilon},E,\dot{\sigma}^e,\dot{\sigma}^i$ are the stress tensor, rate of stress tensor, rate of strain tensor with respect to cyclic number (N), elastic stiffness tensor, elastic rate of stress tensor, rate of stress relaxation due to degradation of elastic properties of concrete materials respectively.

Damage alters the elastic properties of concrete, thus, decomposition should be added in the form of fourth-order elastic stiffness tensor which allows small deformations, the developed analytical form of Equation 4 can be written as,

$$E(k)=E^0+E^D \tag{4}$$

Where, E^0,E^D initial undamaged stiffness tensor and overall stiffness caused by cracks and microcracks damage. The constitutive relations for nonlinear behavior of the brittle materials describing stress-strain relation Equation 5 can be expressed as:

$$\dot{E}(k)=-\dot{E}^D(k)=-\dot{k}L(\varepsilon) \tag{5}$$

Where in Equation 5, $L(\varepsilon),k$ directional damage occurrence fourth order stiffness tensor, scalar measure of cumulative damage respectively. Internal dissipation for the isothermal and small deformations, the Helmholtz Free Energy can be written as Equation 6.

$$d_s=-\frac{\partial A(\varepsilon,\dot{k})}{\partial k}\dot{k}\geq 0 \tag{6}$$

Since, the Damage is assumed to be an irreversible phenomenon that leads to $k\geq 0$. Combined effect of Equations 1 and 6 in generalized form of damage surface can be written as,

$$\theta(\varepsilon,k)=\frac{1}{2}\varepsilon:L:\varepsilon-\frac{1}{2}p^2(\varepsilon,k)=0 \tag{7}$$

Where, in Equation (7), $P(\varepsilon,k),\theta(\varepsilon,k)$ critical strain damage function responsible for the damage growth, called critical strain as described by author [12] and [13], onset of damage in the materials that encloses the elastic domain $\theta(\varepsilon,k)\geq 0$ which is unattainable for rate independent material behaviour.

For capturing the compressive mode or mode II of cracking of concrete materials, a new specific versatile model is proposed with development of $L(\varepsilon)$, the response tensor, for the continuous existence and distribution of microcracks, on true line of continuum damage mechanics approach, the compressive mode or mode II is developed due to shear sliding of existing microcrack and opening of the crack slides as shown in Figure [1].

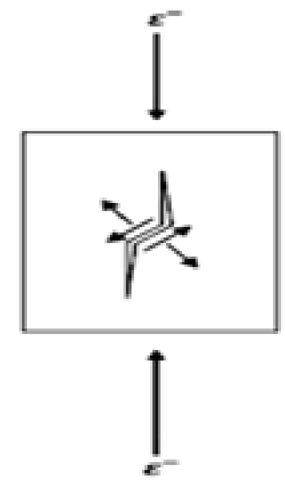


Figure 1: Fatigue crack opening in compressive mode (or mode II)

In compressive fatigue loading, the extended microcracks run in the direction that is parallel to the axis of loading resulted significant decrease in longitudinal stiffness which is due to development of tensile strains at the lateral directions and there are possibility for opening of cracks which is called ”cross effect”. Guided by experimental observations for concrete materials in compressive loading by authors [14, 40] and [41], the cross effect can be incorporated in above formulation by postulating the damage direction tensor L:

$$L = \frac{\varepsilon^- \otimes \varepsilon^-}{\varepsilon^- : \varepsilon^-} + \alpha \frac{\varepsilon^+ \otimes \varepsilon^+}{\varepsilon^+ : \varepsilon^+} \tag{8}$$

Where, $\otimes, \varepsilon^+, \varepsilon^-$ are the tensor product operator, the positive and negative cones of strain tensors, respectively. The coefficient α considers the extention of cross effect, which can be obtained through uniaxial compression monotonic loading test. The first part of Equation 8 refers the compressive loading directional damage, whereas the second part is the capability of the model for prediction of increment apparent Poisson's affected due to mode II damage. This means the response tensor presented in Equation 8 ensures that the prediction of damage is extended in all three principal directions of concrete during uniaxial compression test.

Guided by the experimental work of [42] and [43] and using constitutive relation for simple uniaxial compression test, the damage function can be expressed as Equations 9a, 9b, and 9c.

$$p(\varepsilon, k) = \left(\varepsilon_c \sqrt{1 + 2\alpha\upsilon^2}\right) \ln\left(\frac{E_0}{E_0 - k}\right)$$
$$= \varepsilon_0 \ln\left(\frac{E_0}{E_0 - k}\right) \tag{9a}$$

with

$$\upsilon = \frac{\upsilon_0}{1 - \frac{k_\alpha}{E_0}(1 + \upsilon_0)(1 - 2\upsilon_0)} \tag{9b}$$

$$\varepsilon_0 = \varepsilon_c \sqrt{1 + 2\alpha\upsilon^2} \tag{9c}$$

Where, $\varepsilon_c, E_0, \varepsilon_0, \upsilon_0, ln$ is ultimate strain corresponding to peak stress, initial Young's Modulus of elasticity, referential strain, initial Poisson's ratio under monotonic compressive loading of the concrete material and natural logarithm respectively.

Equation 9b implies the dependency of the Poisson's ratio υ at microcracking state of damage which is no longer constant. Authors [43, 44, 45] reported that increment of monotonic loading under uniaxial compression attains a certain stress level at which the considerably continuous increment of Poisson's ratio started.

3. Statistical Analysis

The damage surface from Equation 7 for the prediction of fatigue behaviour of concrete in strain space under fatigue loading at the prescribed stress level $\Delta\sigma = \sigma_{max} - \sigma_{min}$ remains constant; the damage surface expands successively to obtain the residual curve at failure which can be described in Figure [2].

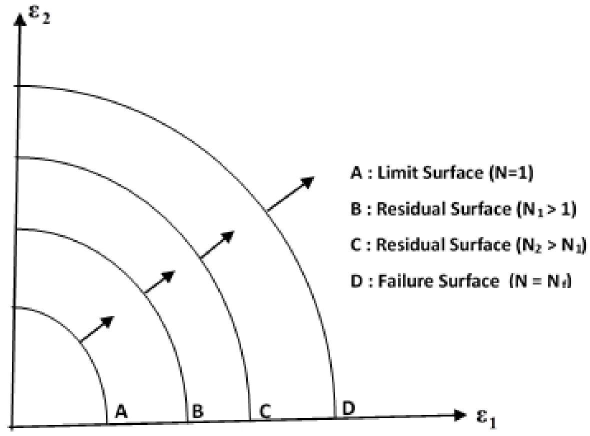


Figure 2: Residual Strain flow in biaxial compression Fatigue loading

Stimulation of cracks and microcracks (damage) in concrete material is mainly due to increase of frazzle strain. Increment of repeated loading from maximum to minimum stress levels, the damage and residual strain increased continuously resulting final decrease in residual strength of the material which is equal to magnitude of fatigue load. After that, concrete material cannot resist any additional fatigue cycle for failure. For development of the fatigue behaviour of concrete through Equation 7, an evolutionary modified equations are predicted and proposed for fatigue damage and residual strain by adopting the parameter 'k', Equation 10 can be expressed as,

$$k = A \int_0^N \left(\frac{\sqrt{\varepsilon^- : \varepsilon^-}}{\varepsilon_0}\right)^{n(1-r)} \mathrm{d}N \tag{10}$$

Where, $A, n, r, N, \varepsilon_0 A$ and n are the material constants; the stress ratio (ratio of minimum stress to maximum stress), negative cone of second order strain tensor, N is the number of fatigue cycles, referential strain in uniaxial compression respectively.

Researchers [8] and [9] reported that the number of cycles to the failure is directly proportional to the applied stress level duly fact that under higher value of compressive mean stress level, the formation of microcracks are inhibited due to crack closure effect. This result shows the increment of both ultimate and plastic strains, rendering concrete more flexible. The stress ratio 'r' adopted in Equation 10 is one of the modified model for reduction of such effects. Also, it is illustrated that the increment of maximum uniaxial compression strain when reaches to the referential strain, the role of cumulative damage parameter starts and increases linearly with the fatigue cycle at rate of constant A called damage cycle parameter.

By differentiating Equation 10 with respect to N, the increment of damage per cycle can be written as Equation 11.

$$\dot{k} = A\left(\frac{\sqrt{\overset{\cdot}{\varepsilon^- : \varepsilon^-}}}{\varepsilon_0}\right)^{n(1-r)} \tag{11}$$

Based on the stiffness degradation rule of concrete material under fatigue loading, the increment of ultimate strain and its associated inelastic strain at each cycle under monotonic loading state is obviously highlights the stiffness reduction addressed by Equations 10 and 11 clearly showing that the directly proportional relation between loading cycles and stress ratio up to accumulated plastic strain until fatigue failure. The related experimental results were also employed for such verification.

Followed by experimental observations and governing equations, the behavior of concrete under fatigue loading is captured by postulating an evolutionary proposed model's equation as described in Figure [3] to predict the failure surface in strain space of continuum damage mechanics theory with an assumption made in Equation 9, the product of two functions and modeled stress-strain diagram drawn in Figure [4] can be expressed as Equation 12.

$$p(\varepsilon,k,N,r) = G(N,r)\cdot p(\varepsilon,k) \tag{12}$$

Where, $G(N,r)$ is strain softening functions, dependent on N, number of cycles of loading and stress ratio. Further, for the settlement of minimum value of $G(N,r)$ equals to 1 as $G_{min}(N,r) = 1$. Substituting Equations 8 and 12 into Equation 7, the damage surface in fatigue environment can be written as Equation 13.

$$\theta(\varepsilon,k,N,r) = \frac{1}{2}\varepsilon : L : \varepsilon - \frac{1}{2}G^2(N,r)\cdot p^2(\varepsilon,k) = 0 \tag{13}$$

In uniaxial compression path of fatigue, the strain softening function is obtained as:

$$G(N,r) = \frac{\varepsilon}{\varepsilon_c} \tag{14}$$

Where, $\varepsilon = \varepsilon_{max}$ in Equation (13), if $G(N,r)$ equals to 1, the occurrence of failure of concrete becomes at one cycle at σ_{max} state. For maximum stress level, smaller than ultimate compressive strength of concrete material, the model function $G(N,r) > 1$. In order to obtain concrete evolutionary softening well fatigue damage model, depending upon the fatigue cycle and stress ratio, the function of $G(N,r)$ in Equation 14 is proposed as Equation 15.

$$G(N,r) = N^{\lambda(1-r)} \tag{15}$$

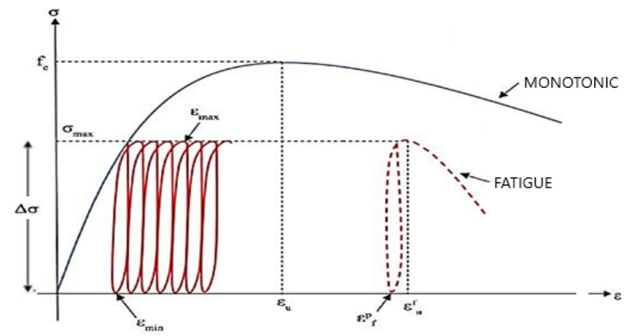


Figure 3: Idealized stress-strain of concrete material under monotonic fatigue loading

It is unable to predict the permanent strain of concrete materials after unloading, thus the damage occurred cannot be assumed as perfectly elastic for brittle materials. A complete closure of crack cannot be achieved due to development of sizeable crack tip process with misfits on the crack surface after reversal loading leads to develop the permanent strain during cyclic loading. To capture such phenomena, stress relaxation $\dot{\sigma}$ from Equation 3 containing two part like perfectly elastic damage and inelastic damage should be coaxial with each other can be modeled by the research of author [46] which can be adopted for fractional term of stress relaxation $\dot{\sigma}$, that can be further modified as $\mu\dot{\sigma}$, where, μ is cumulative fatigue inelastic parameter, value ranges from 0 to 1. The generalized Equations 3 and 5 for the latest extension for inelastic damage model can be obtained as Equations [16a], [16b] and [16c]

$$\dot{\sigma} = E(k) : \dot{\varepsilon} + \dot{\sigma}^i \tag{16a}$$
$$\dot{E} = -(1-\mu)\dot{k}L(\varepsilon) \tag{16b}$$
$$\dot{\sigma} = -k\dot{L} : \varepsilon \tag{16c}$$

Based on these findings, the versatile cumulative fatigue inelasticity parameter can be proposed and written in this paper as Equation 17.

$$\mu = \beta N^{-\gamma(1+r)} \tag{17}$$

Where, β ,and γ are constants, N is the cyclic number. Substituting both the strain softening function and the damage function into the damage surface, modified form of constitutive relation, stress-strain curves can be obtained after certain number of loading cycle. The new proposed fatigue damage model presented by Eqns [10] through [17] is capable for capturing the fatigue damage behavior of concrete such as degradation of Young's Modulus of elasticity, reduction in strength, and increase in ultimate as well as plastic strain during each fatigue cycle.

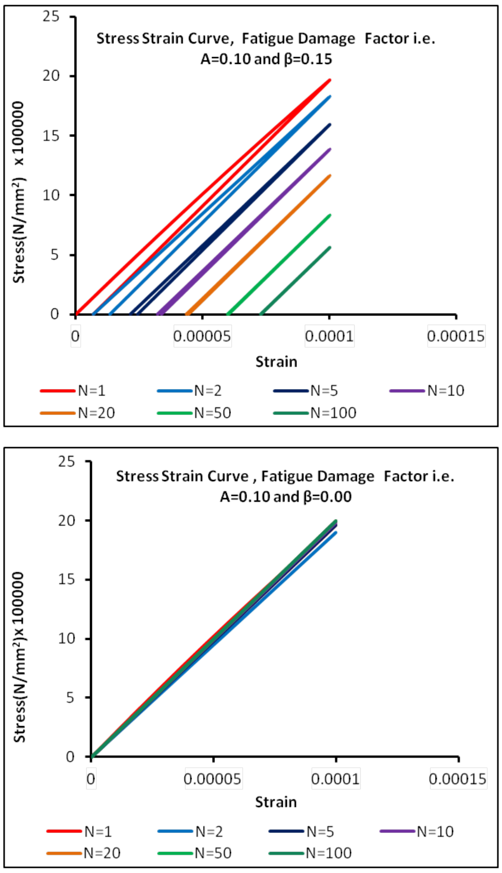


Figure 4: Concrete Stress-Strain diagram under fatigue loading

4. Damage Accumulation

When the constant amplitude of fatigue loading is subjected to the concrete materials, then the initiation of damage can be described by Equation 1. Whatever, on the application of variable amplitude monotonic compressive fatigue loading, the developed damage in the former stage first loading will affect the quantity of produced damage in the next stage of loading. The proposed model requires the six material parameters $(\alpha, A, n, \lambda, \beta$ and $\gamma)$, for the determination of sensitivity analysis based on the experimental results, that are well described in different literatures. The formulation of fatigue constitutive relation of concrete in compression must be endorsed by commonly used structural solution codes. The prescribed maximum stress $\dot{\sigma}$ and strain levels $\dot{\varepsilon}$ for the first cycle $(N = 1)$ of fatigue loading is attained from Equation 3 with updated value of k as $k_i = k_{i-1} + \dot{k}$ through Equation 11 along with updated E, υ from Equations 4 and 9b. For the known stress and strain at first cycle $(N = 1)$ of loading, the damage surface for fatigue loading at $N > 1$ can be obtained from Equation 13. To determine the fatigue strain, damage parameter, residual strength, inelastic strain level and for fatigue failure to the next loading cycles, the entire process shall be repeated. Emergence of Model in compression fatigue loading, the theoretical stress-strain relations, strength and stiffness reductions with accumulation of plastic strains and damage are well developed in Figure [1] with damage parameter from Figure [6] and Table [1]. Stimulated concrete fatigue material parameter as E^0 equals to 68869 MPa in Equation 4, υ equals to 0.20 in Equation 9b, α equals to 0.375 in Equation 9c, γ equals to 1.11111, β equals to 0.277728 in Equation 17, ε_c equals to 0.00201 in Equation 9c, ε equals to 0.00088 as stress level i.e. 0.74, f_c equals to 39.07 MPa, fatigue damage model as Figure [5] is developed.

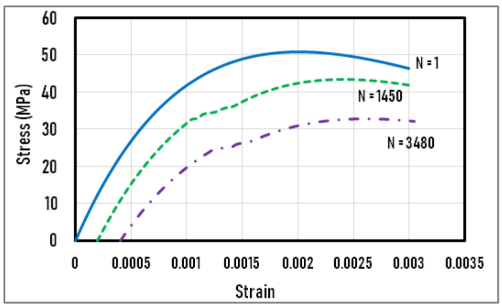
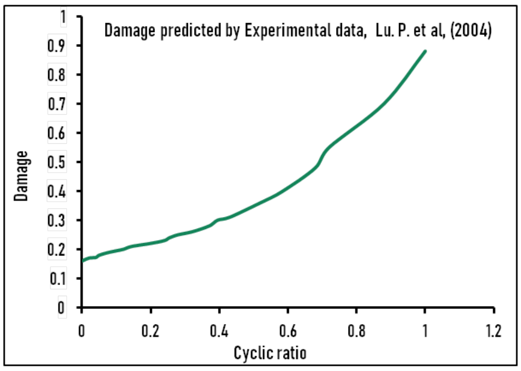


Figure 5: Concrete Stress-Strain curve in uniaxial Monotonic Fatigue loading

Table 1: Damage Data by [3] due to cyclic loading in concrete Materials

Stress Ratio	No. of Cycles	Stress Factor	Max. Stress	Damage	Cyclic Ratio	Damage	Cyclic Ratio
0.85	1479000	2.2	158.489319	0.16	0	0.23	0.24
0.84	1461600	2.3	199.526231	0.17	0.02	0.24	0.255
0.75	1305000	4.1	12589.2541	0.172	0.04	0.25	0.28
0.74	1287600	4.4	25118.8643	0.18	0.05	0.26	0.32
0.69	1200600	5.05	112201.845	0.19	0.08	0.28	0.37
0.685	1191900	5.2	158489.319	0.195	0.1	0.48	0.68
0.68	1183200	5.75	562341.325	0.2	0.12	0.55	0.72
0.65	1131000	6.2	1584893.19	0.21	0.145	0.7	0.88
0.63	1096200	6.3	1995262.31	0.22	0.198	0.88	1



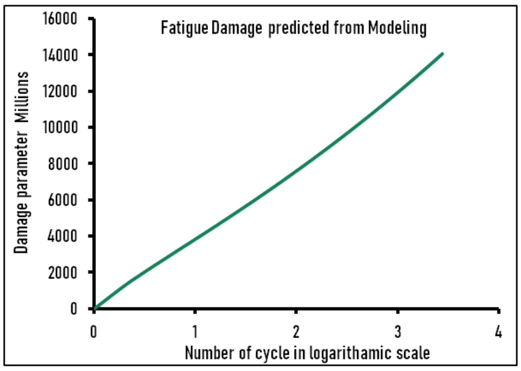


Figure 6: predicted as [3] and Modeling in uniaxial Monotonic Fatigue loading

5. Verification

According to the experimental data of author [3], it is a good regression that the proportional constant A equals 0.10, β equals to 0 and 0.15, changeable trend of the damage parameter in modeling and experimental is due to 100% and 85% of stress level in Figure [6]. The results show that the difference between the predicted residual fatigue life corresponding to normalized life and the experimental data are acceptable because of the big scatter of fatigue life and most points are within scatter ranges as shown in the Figure [4] and Figure [5]. Experimental fatigue life among concrete samples are subjected to the same fatigue loading are obviously very different due to bigger experimental data of author [3]. The predicted residual fatigue life by the proposed algorithm is in good agreement with the experiment, considering the bigger scatter of concrete as theory, Equations and model described above.

6. Conclusion

Based on the continuum damage mechanics theory of concrete materials under fatigue loading; a phenomenological fatigue damage model due to nuclei-propagation of microdefects, microvoids and fractal of rendered concrete in conjunction with more compliant are developed by utilizing internal variable theory of thermodynamics. Experimental data from fatigue Test were employed to verify the Model and results shows that the model can describe the damage evolution of concrete materials under different fatigue loading by verifying the predicted fatigue life of concrete materials. Degradation of Young's Modulus, alteration in the mechanical behaviour, damage in elastic stiffness tensor, evolution in strain, damage development corresponding to the Normalized life, its accumulation subjected to constant amplitude fatigue loading including its verifications are well discussed.

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