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# Theoretical Study of Twin-tube Damper

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**Abstract**
A mathematical model can be an important tool in the understanding and design of a damper. In this paper an attempt has been made to develop a mathematical model of a twin tube damper to represent the physical behavior of the damper. In contrast to the available model, this model has included the provision of explicitly defining the important characteristics of the damper system. The results obtained through the model have been compared with the experimental results found in literature.

**Keywords**
Damper, Shock-absorber, Mathematical Model, Viscosity

## 1. Introduction

The suspension system is an important element of a vehicle. It is essential to provide the required handling to the vehicle as well as to provide a comfortable ride quality to the passengers. The irregularities in the surface of the road induces vibration in the moving vehicle which reduces passenger comfort. Furthermore, the displacement of wheels during such vibrations may cause the wheels to loose contact with the road surface which may lead to a loss of control of the vehicle. The suspension system is made of components like tires, springs (stiffness element), shock absorbers or dampers (dissipation elements) and linkages and employs them to obtain its objectives. The dissipation element or the shock absorber dissipate the kinetic energy imparted to it due to the road surface as thermal energy of the fluid by forcing the fluid through different orifices.

The dampers lower the amplitude and/or the frequency of vibration and they can be classified as “hard” or “soft” and each has their own use. The hard dampers provide better traction and stability and are used when control is paramount like in vehicles which have to maneuver at high speeds or vehicles which need to carry very heavy loads. The soft dampers are used in vehicles which are designed for providing maximum comfort and their use results in reduced bumpiness of the vehicle. Their choice depends on the type of vehicle and their intended use [1]. Their use is chosen by conducting a vibration analysis of the vehicle by employing the stiffness and damping characteristics of the suspension system into the model of the vehicle. Much work has been done in the field of vibration analysis of the vehicle considering a half-car model or quarter-car models. However, the damping characteristics of the damper are either determined experimentally or by using equivalent models obtained from experimental data. This results in a rigid model which is applicable for a particular damper [2].

This paper is focused on developing a mathematical model for a hydraulic damper. The models strives to include all the physical phenomenon occurring within the damper. There are a few papers which present the mathematical model to describe the damper. A mathematical model of a mono-tube damper was developed. The author concludes the accuracy of the model with the experimentally obtained result [3]. A similar approach was taken to develop the mathematical model of a twin tube damper including the effects of compressibility and inertia [4]. However, both of these relied on experimental data provided by [5] for the values of different coefficients particularly the discharge coefficient used in the model. This arises the problem of requiring experimental analysis for even the theoretical study of the damper. A researcher may use the standard values obtained through experiment by other researchers, but the differences in the construction of the damper are not reflected in the coefficients. So, an effort has been made to propose a model with a reduced number of coefficients that are dependent on experimental data. This will not only provide a reliable theoretical model but also a model which is completely dependent on the characteristics of the damper rather than the values obtained from experiment for a certain damper by another researcher.

## 2. Methodology

This study focuses on developing a model which represents the basic working of a twin- tube damper and reduce the dependencies of earlier model on the experimentally obtained coefficients and values. A typical twin tube damper consists of two concentric cylinders, the internal cylinder and the external cylinder. The internal chamber and the external chamber are physically separated by the base valve at the base of the internal chamber. The internal chamber is filled with damping fluid and encloses the motion of the piston while the external chamber is partially filled with the damper fluid and partially filled with inert gas (nitrogen) at a pressure of around 5 bar. The piston divides the internal chamber into two chambers namely rebound chamber and the compression chamber. The rebound chamber is the volume of internal chamber above the piston while the compression chamber is the volume of internal chamber below the piston. The piston has a valve

The fluid flow occurs between the compression chamber and the rebound chamber. In case of compression stroke, the pressure of fluid in the compression chamber increase whereas the pressure in the rebound chamber decreases. This causes the fluid from the compression side of the piston to pass through different valves and orifices into the rebound side of the piston. As the fluid flows from one chamber to the other, it presents resistance to the motion of the piston.

During the compression stroke, as the piston moves into the internal chamber, the volume of fluid equal to that of the rod within the chamber needs to be displaced because the fluid is in-compressible and the mass needs to be conserved. So, the fluid moves to the external chamber. In case of rebound stroke, the external chamber compensates the volume of rod that leaves the rebound chamber. This flow takes place through the compression valve located at the base of the internal chamber.

When the fluid flows through the orifices, there exists a pressure difference across the piston which is the damping force provided by the damper.

### 2.1 Flow of Damper Fluid

The flow of damper fluid within the twin tube damper is illustrated in Figure 1. All the flow illustrated in the figure does not occur simultaneously or in the same stroke. The Valve flow from compression chamber to rebound chamber, represented by $Q_{vr}$ occurs only during compression stroke while the valve flow from compression chamber to reservoir chamber, $Q_{vc}$ occurs only during rebound stroke. This results in difference in mathematical model of the damper for different strokes.

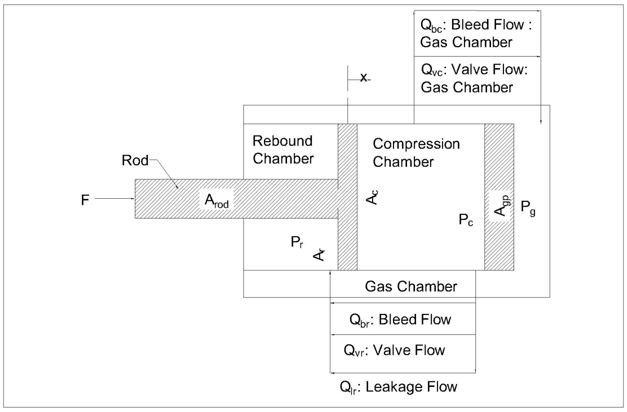


**Figure 1:** Flow of fluid in twin tube damper

#### 2.1.1 Flow through Rebound Piston

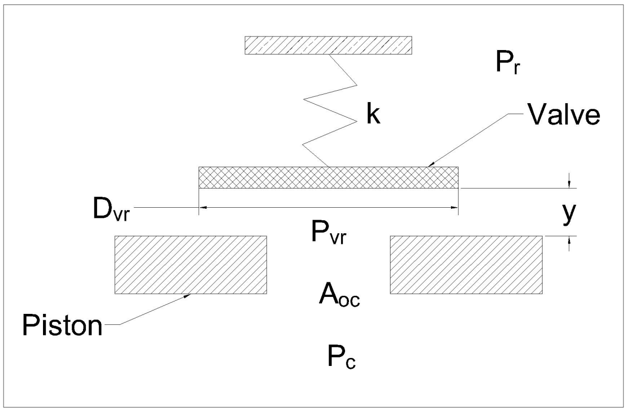


**Figure 2:** Flow through Rebound Piston

The flow through the rebound piston is related to the volume of fluid on the rebound side of the piston. To fill or empty the rebound side of the rebound piston flow must pass through the rebound piston and it may occur as valve flow, bleed flow and or leakage flow. So, the total flowrate for a piston moving with a velocity of ẋ can be represented as

$$A_r\dot{x} = Q_{vr} + Q_{br} + Q_{lr} \tag{1}$$

(a) **Flow through Piston Orifice**
The pressure loss during flow through rebound piston occurs in two stages. First, the pressure loss occurs as the flow moves from the compression chamber at pressure $P_c$ through the orifice to reach the pressure $P_{vr}$ just before the valve . Secondly, the pressure loss occurs when the flow is along the shim plate after which the fluid reaches the rebound chamber at pressure $P_r$. Since, the mass is conserved the flow entering the orifice passes through it , enters the valve and exits the valve i.e. the flow rate throughout the valve is constant, $Q_{vr}$. So, the flow through each piston orifice the can modeled as laminar flow in a pipe and hence the flowrate for each orifice is given as:

$$Q_{vr} = \frac{\pi D_o^4(P_c - P_{vr})}{128\mu L} \tag{2}$$

(b) **Flow through Rebound Valve**
In the valve the flow occurs due to the pressure difference ($P_{vr}$- $P_r$) and the flow can be represented as the flow between two stationary parallel plates. Mathematically,

$$Q_{vr} = \frac{a^3(P_{vr} - P_r)\pi}{6\mu ln\frac{r_o}{r_i}} \tag{3}$$

(c) **Flow through Bleed Orifice**
Bleed orifice behaves as a constant opening orifice where the flow occurs due to the pressure difference between compression chamber in an uninterrupted manner. The flow rate through the bleed orifice is:

$$Q_{br} = C_{Db}A_{br}\sqrt{\frac{2(P_c - P_r)}{\rho}} \tag{4}$$

(d) **Leakage Flow past Rebound Piston**
There is small leakage of fluid around the piston due to the pressure difference between compression chamber and rebound chamber i.e. ($P_c$-$P_r$). This flow can be modeled by assuming the laminar flow through parallel plates. This assumption is valid because the gap between the piston and the cylinder is very small compared to the circumference of the piston [6]. So, by applying Navier- Stokes equation we get

$$Q_{lr} = \frac{(P_c - P_r)b^3C_r}{12\mu l}$$

In the above equation, the velocity of piston has not been considered. Due to the velocity of the piston further leakage occurs past it which can be shown as:

$$Q_{lr} = \frac{\dot{x}b\pi D_p}{2}$$

So, combining the two equations, the total leakage flow past the piston is:

$$Q_{lr} = \frac{(P_c - P_r)b^3C_r}{12\mu l} + \frac{\dot{x}b\pi D_p}{2} \tag{5}$$

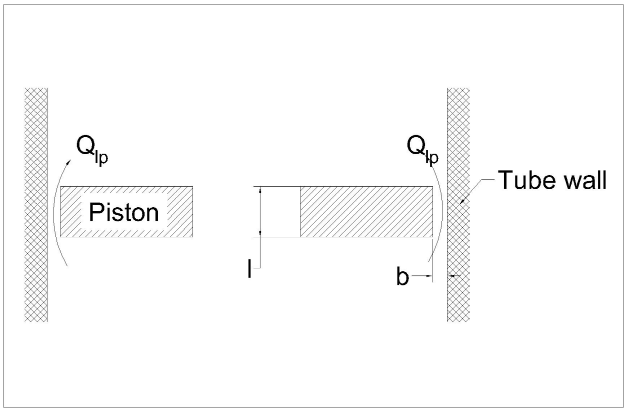


**Figure 3:** Leakage around Rebound Piston

#### 2.1.2 Flow through Compression Piston

The flow through Compression Valve occurs in order to accommodate the volume of piston rod that is either inserted or removed from the internal chamber. This flow is similar to the flow through the Rebound valve, however the leakage flow is not present. So a volume equal to the volume of the rod needs to flow through this valve either as bleed flow or valve flow, i.e

$$A_{rod}\dot{x} = Q_{bc} + Q_{vc} \tag{6}$$

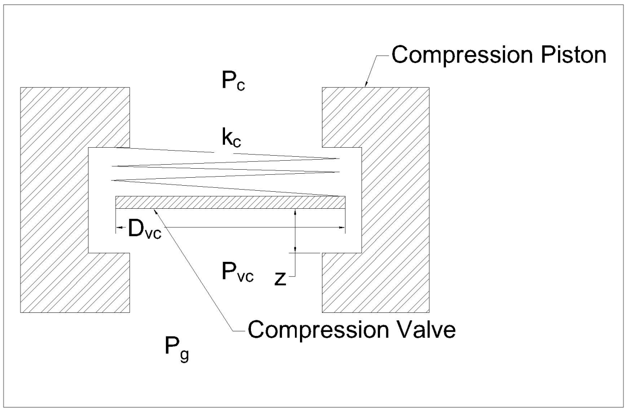


**Figure 4:** Flow through Compression Valve

(a) **Flow through piston orifice**
The flow passes through the orifice provided in the fixed compression piston. This flow is the result of the difference in pressure of reservoir chamber $P_g$ and the pressure at the valve opening, $P_{vc}$. $P_{vc}$ is obtained because of the pressure loss in the orifice. So, the flow is a constant area flow dependent on the above mentioned pressure difference and is given by

$$Q_{vc} = C_{Do} A_{oc} \sqrt{\frac{2(P_{vc} - P_c)}{\rho}} \quad (7)$$

(b) **Flow through Compression Valve**
The flow through the compression valve is a variable area flow dependent on pressure difference between the compression chamber and gas chamber (or reservoir). This pressure difference ($P_g$-$P_{vc}$) causes the valve to deflect by a value of z units which provides the area for the flow to occur. The flow can be represented as:

$$Q_{vc} = C_{Dv} \alpha \pi D_{vc} z \frac{2(P_g - P_{vc})}{\rho} \quad (8)$$

(c) **Flow through Bleed Orifice**
The bleed orifice is fixed orifice provided in between the orifice and the shim stack plates. This flow is independent of the shim-stack opening but depends on the pressure difference on either side of the bleed orifice. The resulting formulation of this type of flow is

$$Q_{vc} = C_{Db} A_{bc} \sqrt{\frac{2(P_g - P_c)}{\rho}} \quad (9)$$

## 2.2 Forces on Components

### 2.2.1 Static Force on Rebound Piston

When the damper is in static state as shown in Figure 5, there is no valve flow of damper fluid. So, the pressure in the whole damper is uniform. However, the area of the piston on the compression side is greater than the area on the rebound side. So a net force equal to $F=P_cA_c\text{-}P_rA_r$ which pushes the piston outwards.

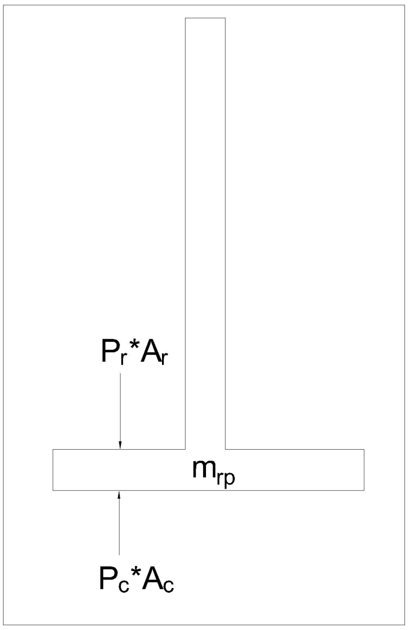


**Figure 5:** Static Force on Rebound Piston

### 2.2.2 Force on Rebound Piston

When the damper is moving, in addition to the static forces the forces as shown in free body diagram of the piston in Figure 6 during compression. The force (F) on the piston is the damping force generated by the damper. This force is the result of the pressure differences between the compression and rebound side of the piston as well as the friction forces. The friction force is the sum of all the friction forces that are present in the system.

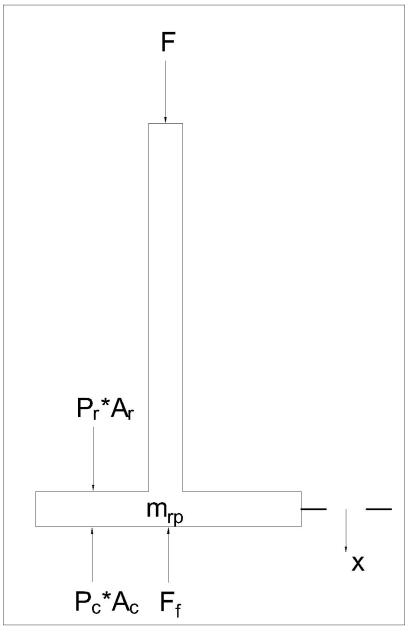


**Figure 6:** Force on Rebound Piston

By summation of forces in the vertical axis, Equation 10 is obtained:

$$m\ddot{x} = F - P_cA_c + P_rA_r - F_f \tag{10}$$

#### 2.2.3 Force on the Rebound Valve Shim plate

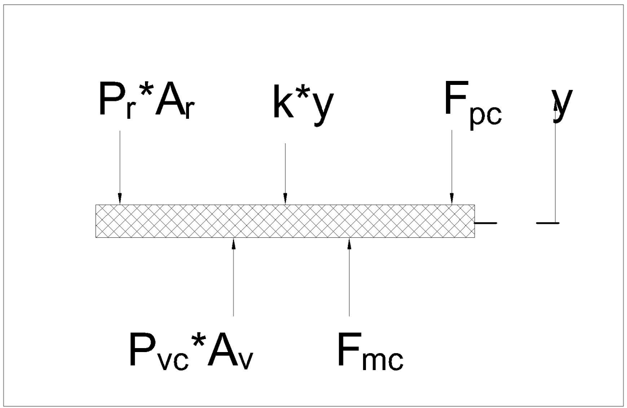


**Figure 7:** Force on Rebound Shim Plate

As shown in the free-body diagram in Figure 7 the forces on the plate can be summed up in the vertical direction to calculate the deflection as

$$k_ry = (P_{vr} - P_r)A_{vr} + F_m - F_{pr}$$

By applying the momentum balance equation to the fluid entering and exiting the valve, the momentum force can be represented as:

$$F_m = \frac{\rho C_f Q_{vr}^2}{A_or}$$

Hence, by combining the two equations we get

$$k_ry = (P_{vr} - P_r)A_{vr} + \frac{\rho C_f Q_{vr}^2}{A_or} - F_{pr} \tag{11}$$

#### 2.2.4 Force on Compression Shim Plate

The compression valve works similar to the rebound valve. So, by applying the same method to find the force balance equation for the compression valve shown in Figure 8, we obtain:

$$k_ry = (P_g - P_{vc})A_{vc} + \frac{\rho C_f Q_{vc}^2}{A_oc} - F_{pc} \tag{12}$$

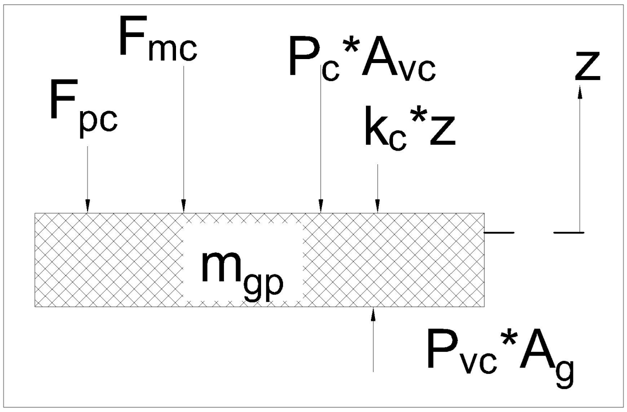


**Figure 8:** Force on Compression Shim Plate

### 2.3 Gas Compression

By assuming ideal gas, the gas in the gas chamber follows the ideal gas law. The gas volume changes to accommodate the volume of the piston rod. When the volume of the gas chamber changes its pressure changes and as per the ideal gas law we get:

$$P_g = \frac{P_{gi}A_{og}l_i}{A_{og}l_i - A_{rod}x} \tag{13}$$

## 3. Results and Discussions

A model based on the physical model of a commercially available damper was initially created. This provided the characteristics of a damper which is already optimized because it is commercially available.

The model represented through Equations 1 to 13 can be solved to obtain the Damping force. The model was solved for a frequency of 1 Hz and amplitude of 25 mm. In Figure 9, we can see than the maximum force attainable through the use of the damper is around 175N in the compression stroke and 100N in the rebound stroke. At 0m/s velocity i.e. when the piston is at the end of the stroke either at the top or bottom, the force it produces is nearly negligible. A small amount exists which is caused due to the hysteresis of the damper. As the damper begins to move from the bottom of the stroke, the damper accelerates, and the fluid is forced through the bleed valves in the rebound piston into the compression chamber. So, a force which opposes this motion of the piston is generated which is increasing until the middle of the stroke is reached. Once , the piston reaches the middle, it continues its motion towards the top of the stroke, however, the motion is decelerated. So, the damper still produces a force which opposes the upward motion of the piston but the force gets gradually lessened as the piston moves up, because the piston is itself decelerating. When the piston reaches the top of the stroke, it produces a negligible amount of force. Then, it reverses its direction and then starts to accelerate in the compression stroke. At the beginning of the stroke, the force produced by the damper has to oppose the accelerating motion of the damper, so the force also goes on increasing. It reaches the maximum force of 175N when the piston reaches the middle of the stroke after which the piston continues the compression stroke but in a decelerating manner. This decelerating motion of the piston means that the force produced by the damper to oppose it gradually decreases until it reaches the bottom of the stroke.

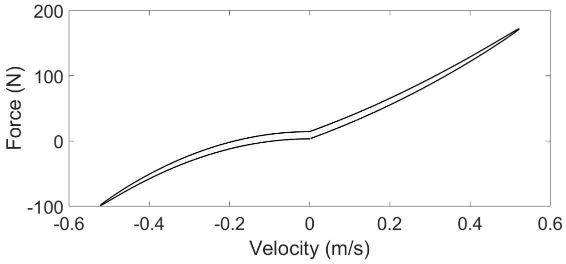


**Figure 9:** Velocity vs Damping Force

In Figure 9, the force generated by the damper is asymmetrical about the horizontal axis. This is because the fluid has to pass through different types of restrictions during the course of rebound stroke and compression stroke. Another way of representing the damper characteristics is the Force vs Displacement method as shown in Figure 10. The damping force is maximum when the piston is at the center of the stroke in both the rebound and compression stroke. However the force is maximum during the compression stroke.

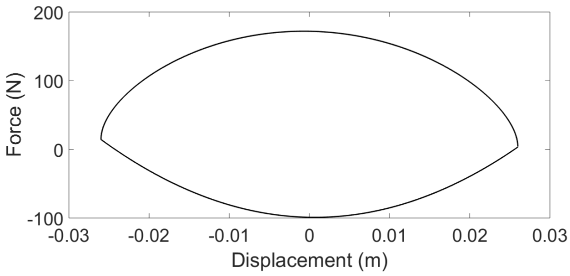


**Figure 10:** Displacement vs Force

Figure 11 is obtained when the viscosity of the damper fluid is neglected. So, when the viscosity of the damping fluid is neglected, the Force varies with the square of velocity of the fluid. If only the viscous flow is considered the force varies linearly with the velocity. This model considers both these flows, so the force-velocity relationship is as shown in Figure 9 which is nonlinear but the force increase at a greater rate with velocity.

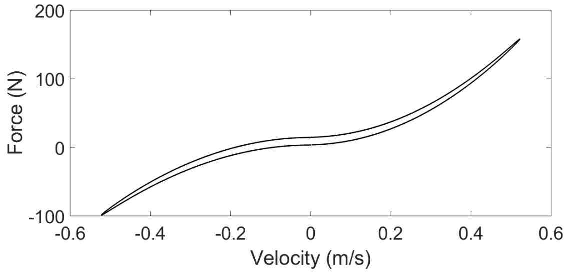


**Figure 11:** Velocity vs Force

This behavior has been presented by [7] in Figure 12. It can be seen that the force velocity relation obtained through this model is closer to the actual behavior of the damper than the relation obtained by neglecting viscosity.

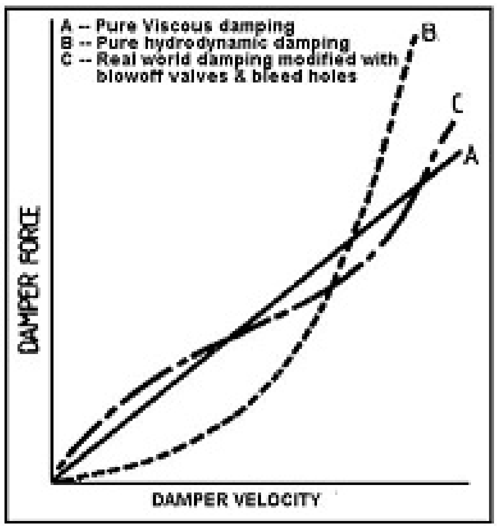


**Figure 12:** Velocity vs Force [7]

In an experimental study by [8] damping force was obtained as shown in Figure 13. It can be seen that by changing the excitation frequency, the magnitude of the force gets changed but the pattern of the variation remains the same. This pattern is the same as observed in the result of the mathematical modeling of this study.

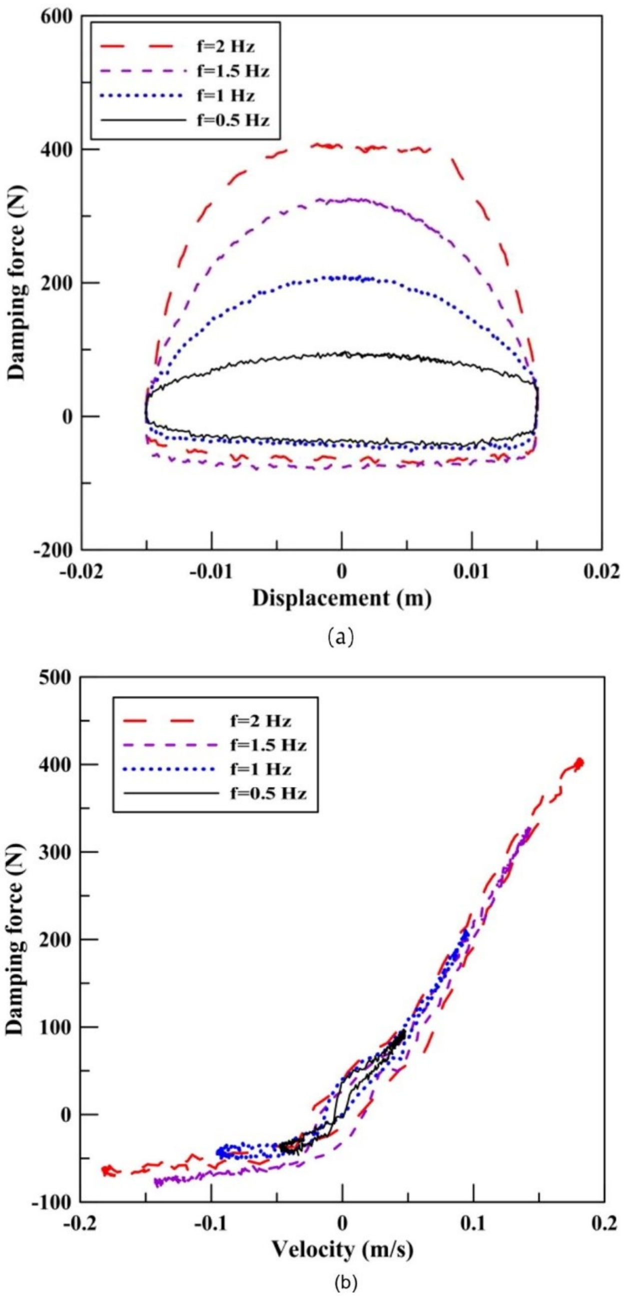


**Figure 13:** Experimental Results: (a) Displacement vs Force and (b) Velocity vs Force [8]

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