

# Analysis of Non-linear Semi-active Suspension Model for Lightweight Vehicles

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## Abstract

The role of vehicle suspension system is important in influencing the overall performance of a vehicle. All of the physical systems are inherently non-linear in nature but for simplicity in mathematical modelling linearity is considered. Suspension systems can be better understood through non-linear analysis. Performance of the suspension system is determined by the ride comfort and vehicle handling. It can be measured by the vehicle sprung mass displacement and acceleration. This paper aims to model passive and semi-active suspension systems in both linear and non-linear environment using MATLAB/SIMULINK. The study of a 2-DOF quarter car suspension model shows that linear semi-active suspension system performs better than a passive suspension system. By introducing non-linearity in the parameters of the suspension system, the actual response of the system can be realized. For the step input of one unit, it has been found that the maximum sprung mass displacement for linear passive suspension is 1.4 units and that for semi-active suspension is 1.03 units leading to the reduction in settling time from 2.7 sec to 1.82 sec. Introducing nonlinear parameters and taking step input of 0.1 units, the maximum sprung mass displacement becomes 0.08 units which is slightly higher than that of linear semi-active suspension sprung mass displacement value 0.064 units.

## Keywords

Non-linear vibration, Semi-active suspension, Passive suspension, MATLAB/SIMULINK

## 1. Introduction

A suspension system is the part of a vehicle which supports its weight and keeps the tire firmly in contact with road. A good automotive suspension system provides high level of ride comfort [1]. There are mainly three types of vehicle suspension system i.e., passive, semi active and active suspensions depending on mode of operation to improve vehicle ride comfort, safety, minimum damage of roads and overall performance [2]. Passive suspension system is a conventional suspension system consisting of spring and damper in which these two components do not add energy to the system [3]. In semi active suspension system the viscous damping co-efficient of shock absorber changes but no energy is added to the suspension system [4]. An active suspension system controls the vehicle movement of the wheels relative to chassis or vehicle body with an on board system rather than passive system where the movement is entirely determined by the road surface [5].

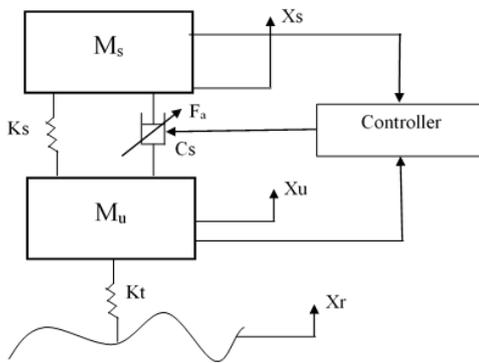
In consideration of the vehicle safety and passenger comfort, suspension system is very important as it carries weight of vehicle structure, driver, and passenger and also absorbs the vibration passing through the vehicle body. As per ISO 2631-1:1997 standards the effect of vibration on comfort of normal healthy person exposed to whole body periodic, random and transient vibration during travel, at work or during leisure activities are standardized. Practically there is existence of nonlinearity in automotive vehicle as it consists of suspension system, tires, and others components having nonlinear properties. So the movement of vehicle on road gives chaotic response [6]. The quarter car model of vehicle suspension, which represents 1/4th of the vehicle suspension model, can be developed for the efficient analysis of the overall suspension system of the vehicle. The ride quality and road handling performance of the semi-active suspension system is better as compared to the passive suspension system [7]. It is necessary to consider the nonlinearities in

suspension system for analysis of dynamic vehicle suspension system.

In this paper a passive and a semi active suspension system has been modelled using both linear and non-linear approach to the system. A mathematical model has been developed using laws of motion considering the nonlinearities in spring and damper of the suspension system. MATLAB/SIMULINK tool has been used for modeling, simulation and analysis of the system.

## 2. Mathematical Modeling

Mathematical equations for suspension system have been derived using basic laws of mechanics. Both linear and non-linear aspect of semi active suspension has been studied in this paper. It is considered that the tyre material has stiffness as well as damping property. Damping effect being minimal, has been neglected. Minor forces such as body force, backlash etc. have been neglected to reduce complexity of the system and also these forces have minimal effect on the overall performance of the system.



**Figure 1:** Semi active linear suspension system.

Equations for semi active linear suspension system:

$$M_s \ddot{X}_s - C_s(\dot{X}_u - \dot{X}_s) - K_s(X_u - X_s) = F_a \quad (1)$$

$$M_u \ddot{X}_u + C_s(\dot{X}_u - \dot{X}_s) + K_s(X_u - X_s) + K_t(X_u - X_r) = F_a \quad (2)$$

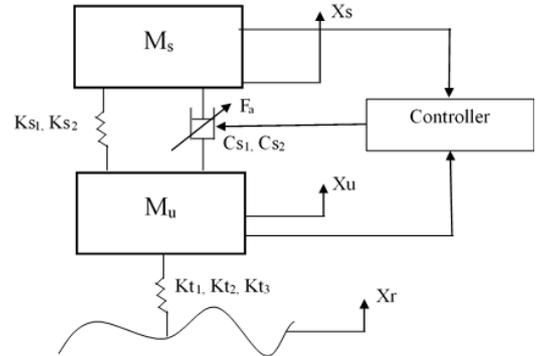
Equations 1 and 2 are the mathematical expressions representing the semi active suspension shown in Figure 1. These equations obtained using basic laws of mechanics are the basis for MATLAB/Simulink model development.

Tire force:

$$F_t = K_{t1}(X_u - X_r) + K_{t2}(X_u - X_r)^2 - K_{t3}(X_u - X_r)^3 \quad (3)$$

Spring force:

$$F_s = K_{s1}(X_s - X_u) + K_{s2}(X_s - X_u)^2 \quad (4)$$



**Figure 2:** Semi active non-linear suspension system.

Damping Force:

$$F_d = C_{s1}(\dot{X}_s - \dot{X}_u) + C_{s2}(\dot{X}_s - \dot{X}_u)^2 \quad (5)$$

Equations for semi active non-linear suspension system:

$$M_s \ddot{X}_s = -K_{s1}(X_u - X_s) - K_{s2}(X_u - X_s)^2 - C_{s1}(\dot{X}_u - \dot{X}_s) - C_{s2}(\dot{X}_u - \dot{X}_s)^2 + F_a \quad (6)$$

$$M_u \ddot{X}_u = K_{s1}(X_u - X_s) + K_{s2}(X_u - X_s)^2 + C_{s1}(\dot{X}_u - \dot{X}_s) + C_{s2}(\dot{X}_u - \dot{X}_s)^2 - K_{t1}(X_u - X_r) - K_{t2}(X_u - X_r)^2 + K_{t3}(X_u - X_r)^3 + F_a \quad (7)$$

Equations 3, 4, 5, 6 and 7 are the mathematical expressions representing the semi active suspension shown in Figure 2. The symbols used in mathematical modeling are listed below:

$M_s$  = Sprung mass

$M_u$  = Unsprung mass

$K_s$  = Suspension spring coefficient

$C_s$  = Suspension damping coefficient

$K_t$  = Tyre stiffness coefficient

$X_s$  = Sprung mass displacement

$X_u$  = Unsprung mass displacement

$X_r$  = Road disturbance

$F_a$  = Damping force

### 3. Simulink Model

Simulink model of a semi active suspension consists of a controllable damper. There are two types of damper which are most popular in suspension systems: ER (Electrorheological) damper and MR (Magnetorheological) damper. MR dampers are widely used in suspensions in recent times. The behavior of MR dampers can be modelled using a Bingham model [8]. Bingham viscoplastic model has been used for modeling of MR damper in this paper. In this model, the damping force is defined as

$$F_a = f_c \text{sgn} \dot{X}_s + c_o \dot{X}_s + f_o \quad (8)$$

where  $c_o$  is the damping coefficient,  $f_c$  is the frictional force directly related to the yield stress,  $X_s$  is the sprung mass displacement and  $\dot{X}_s$  its time derivative. Figure 3 is the representation of Bingham viscoplastic model of an MR damper.

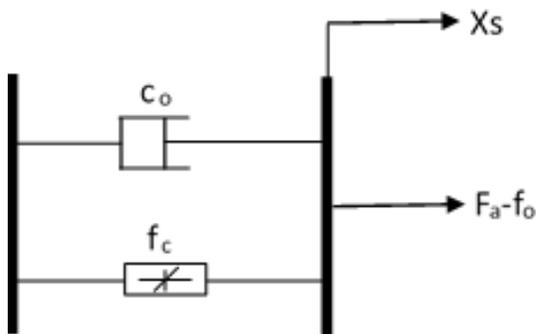


Figure 3: Bingham viscoplastic model of MR damper

Figure 4 shows Simulink model of semi active suspension. A subsystem has been created for ease of system design. Figure 5 and Figure 6 show subsystem design for linear and non-linear suspension respectively. Bingham model has been used for modeling the MR damper.

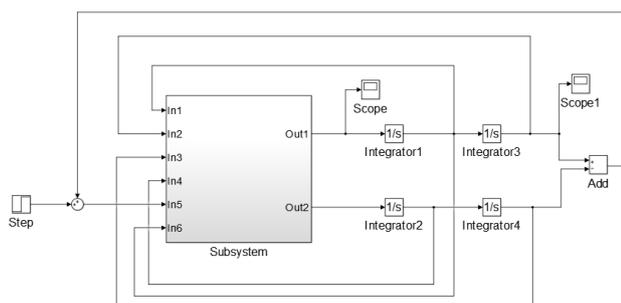


Figure 4: Semi active suspension SIMULINK model

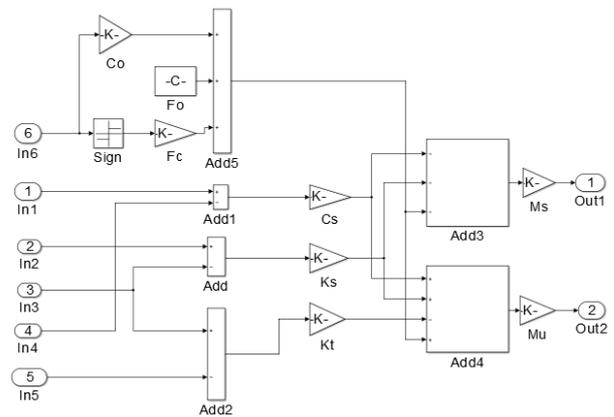


Figure 5: Subsystem for linear semi active suspension

The Simulink models have been designed on the basis of mathematical equations. Difference in sprung mass displacement ( $X_s$ ) and unsprung mass displacement ( $X_u$ ) has been fed back to the input as shown in the model. Major parameters for testing the performance of suspension systems are sprung mass acceleration and sprung mass displacement.

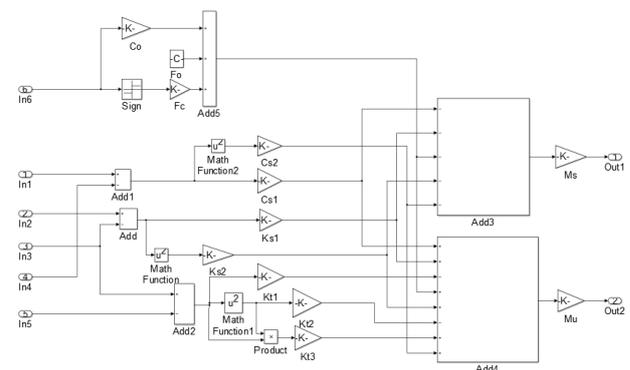


Figure 6: Subsystem for non-linear semi active suspension

### 4. Result and Discussion

Performance of linear semi active suspension has been compared with linear passive suspension and non-linear semi active suspension. Table 1 shows the parameters used for simulation of linear suspension and Table 2 shows the parameters used for simulation of non-linear suspension [9].

Using these parameters a linear passive and semi active suspension was simulated and the results were compared. Then, the non-linear semi active suspension model was simulated and the final results were compared. Graphs of sprung mass acceleration and

sprung mass displacement have been discussed in this section.

**Table 1:** Suspension parameters for linear system

Suspension Parameters	Symbol	Unit	Value
Sprung mass	$M_s$	kg	295
Unsprung mass	$M_u$	kg	39
Damping coefficient	$C_s$	Ns/m	2031
Spring stiffness coefficient	$K_s$	N/m	9015
Tyre stiffness coefficient	$K_t$	N/m	41815.66

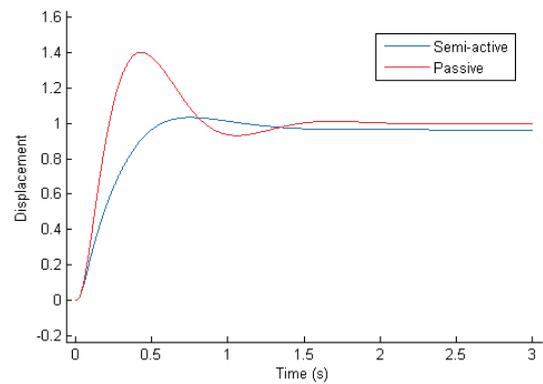
**Table 2:** Suspension parameters for linear system

Suspension Parameters	Symbol	Unit	Value
Sprung mass	$M_s$	kg	295
Unsprung mass	$M_u$	kg	39
Linear damping coefficient	$C_{s1}$	Ns/m	3482
Non-linear damping coefficient	$C_{s2}$	$Ns/m^2$	580
Linear spring stiffness coefficient	$K_{s1}$	N/m	15302
Non-linear square spring stiffness coefficient	$K_{s2}$	$N/m^2$	2728
Linear tyre stiffness coefficient	$K_{t1}$	N/m	60063
Non-linear square tyre stiffness coefficient	$K_{t2}$	$N/m^2$	42509
Non-linear cubic tyre stiffness coefficient	$K_{t3}$	$N/m^3$	22875

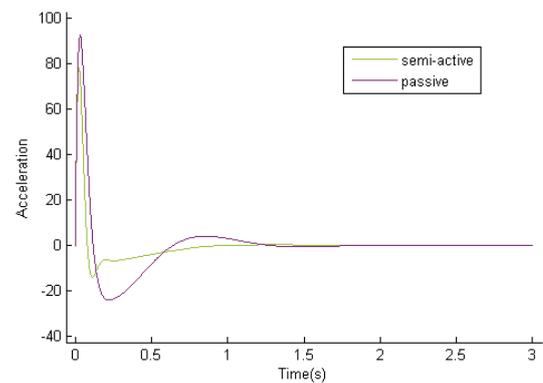
Figure 7 shows sprung mass displacement curve for passive suspension vs semi-active suspension. Maximum displacement for passive suspension is 1.4 units whereas maximum displacement for semi-active suspension is 1.03 units for step input of 1 unit. Settling time for passive suspension in 2.7 sec. and that for semi-active suspension is 1.82 sec. Figure 8 shows sprung mass acceleration curve for passive vs semi-active suspension. Maximum acceleration for passive suspension is 92.55 units and that for semi-active suspension is 77.91 units.

Figure 9 shows sprung mass displacement curve for semi-active linear vs semi-active non-linear suspension. Maximum displacement for semi-active linear suspension is 0.064 units and maximum displacement for non-linear suspension is 0.08 units for step input of 0.1 unit. Settling time for linear semi-active suspension is 0.67 sec and settling time of non-linear semi-active suspension is 0.61 sec.

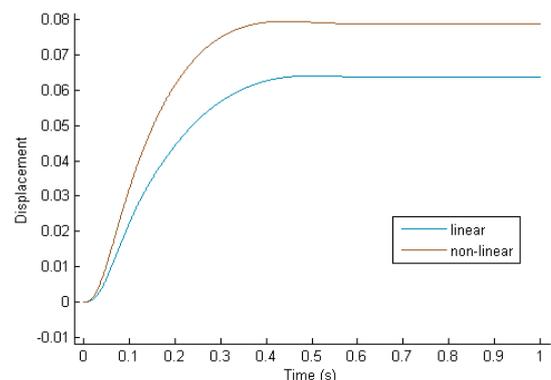
Figure 10 shows sprung mass acceleration comparison of linear and non-linear semi-active suspension. Maximum acceleration for linear semi-active suspension is 7.55 units whereas maximum acceleration for non-linear semi-active suspension is 11.03 units. Figure 11 shows the sprung mass displacement comparison between linear passive, linear semi-active and non-linear semi-active suspension.



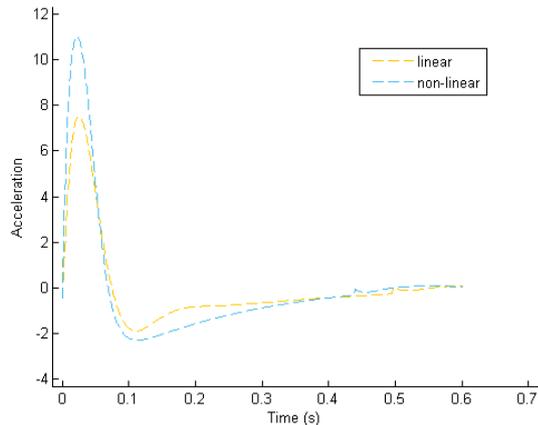
**Figure 7:** Displacement curve for passive vs semi-active suspension



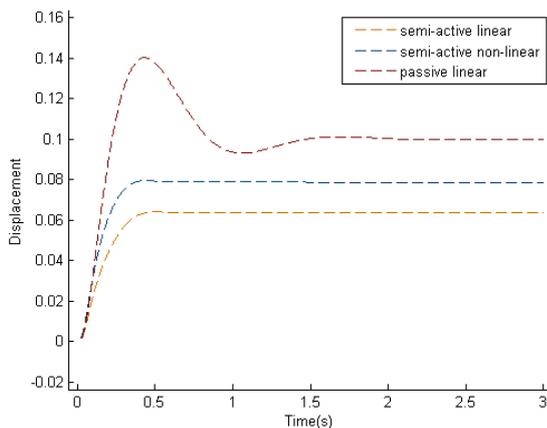
**Figure 8:** Acceleration curve for passive vs semi-active suspension



**Figure 9:** Displacement curve for semi-active linear vs non-linear model



**Figure 10:** Acceleration curve for semi-active linear vs non-linear model



**Figure 11:** Displacement curve for passive linear vs semi-active linear vs semi-active non-linear model

## 5. Conclusion

A linear passive suspension, linear semi-active suspension and a non-linear semi-active suspension model was developed and their parameters (sprung mass displacement and sprung mass acceleration) were compared. The graphs suggest that semi-active suspension performs better than passive suspension as the maximum displacement has reduced from 1.4 units to 1.03 units and the settling time has improved from 2.7 sec. to 1.82 sec. The sprung mass acceleration comparison also indicates the same.

For non-linear model development, quadratic non-linearity has been considered for suspension stiffness whereas cubic non-linearity has been considered for tyre stiffness. Sprung mass displacement comparison between linear semi-active

and non-linear semi active suspension suggests that maximum amplitude of the real suspension system (0.08 units), which is non-linear, is slightly greater than that of a linear model (0.064 units). Sprung mass acceleration of non-linear model is greater than the linear model. A non-linear semi-active model provides a better realization of the actual response of a real suspension system.

## 6. Future Enhancements

For future enhancements, this model can be simulated using inputs other than step input such as sine input, trapezoidal input, random input etc. Furthermore, the simulated results can be validated using state space equations or through experiment. Active suspension systems can be modeled and its parameter can be compared with the parameters of semi-active suspension.

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