Proceedings of IOE Graduate Conference, 2019-Summer
Peer Reviewed
Year: 2019 Month: May Volume: 6
ISSN: 2350-8914 (Online), 2350-8906 (Print)

Transverse Vibration Modal Analysis on offset Rotor Shaft of large Centrifugal Fan

Shailendra Shah [a], Mahesh Chandra Luintel [b]

[a, b] *Department of Mechanical Engineering, Pulchowk Campus, IOE, Tribhuvan University, Nepal*
Corresponding Email: [a] shailendrashah7100@gmail.com, [b] mcluintel@ioe.edu.np,

Abstract

In this paper, transfer matrix method is used as a tool for modal analysis of offset rotor of centrifugal fan to obtain the natural frequencies of the rotating systems. Mathematical model that govern the transverse vibration mode of rotor shaft is developed by transfer matrix method as the rotor of the centrifugal fan is offset from the centre, and the weight of shaft and impeller is of considerable amount. The model is solved by using standard matrix method in MATLAB to obtain the natural frequencies and the corresponding mode shape of rotor system. The parameters obtained after the solution includes displacement, shear force, bending moment and slope at each station point. Numerical simulation of modal analysis of the offset rotor shaft is done in ANSYS workbench. The CAD model of impeller is done in SolidWorks and imported in ANSYS for modal analysis. The accuracy of the model and the solution technique has been demonstrated by the comparison with results from the mathematical model and numerical simulation from ANSYS.

Keywords

Transfer matrix method, Field matrix, Point matrix, ANSYS, critical speed, unbalance, offset

1. Introduction

Vibration of the rotating components in turbo machine plays a great role in the performance in industrial plants. Turbo machines also play important role in aviation, heavy industries, oil refining industries and other important areas[1]. Huge economic losses are encountered because of its failure in heavy industries. Fatigue failure of shaft and impeller are the fundamental problem in the industry. Moreover, the study of the vibration analysis of rotor shaft is beneficial to guarantee the safe operation of turbo machines and gives support to intense research on relevant fundamental scientific problems. Fan vibrations may lead to operational problems, shutdowns, and curtailed operations[2]. Proper understanding of vibration dynamics are required for design and control of rotating equipment.

Various methods have been used in analyzing the rotary systems. The Jeffcott model for rotors assumes a massless shaft having a rigid rotor on it [3]. Standard transfer matrix method is given in many handbooks. Prohl in 1945, one of the pioneers, derived it for critical speed analysis of rotor system [4]. Lund and Orcutt in 1967 developed Shaft's transfer matrix in a continuous fashion but neglected the effect of both rotary inertia and gyroscopic effect [5]. Analytical results are being generated to demonstrate the need for and the advantage of transfer matrix method in modelling procedures [6]. Their research work describes the analytical basis and the method of application for direct representation of conical sections and trunnions for a transfer matrix analysis. Rotor unbalance and shaft misalignment are the two major domain of study in rotating machinery. Xu and Marangoni in 1993 developed a theoretical model of a complete motor-flexible coupling rotor system capable of describing the mechanical vibration resulting misalignment and unbalance [7]. A quantitative comparison is made between the Finite Element method and four variants of the transfer matrix method performed the quantitative comparison of Transfer Matrix Method, as applied to free vibration analysis of rotor systems [8]. The dynamics of a rotor-bearing system considering the gyroscopic effect has been analyzed [9]. The rotor response due to imbalances and offsets are studied by deriving the general transfer matrix method (TMM) for rotors containing global and local coupler offset [10]. The unbalance that exists in any rotor due to eccentricity

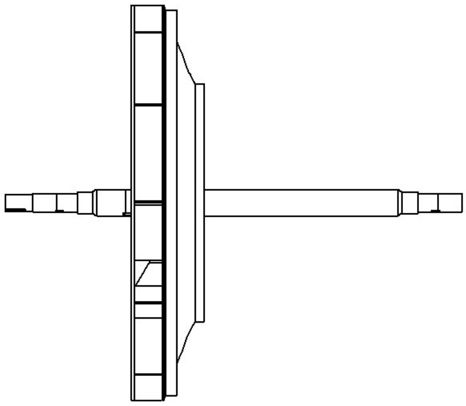

Figure 2.1: CAD model of offset rotor shaft of centrifugal fan

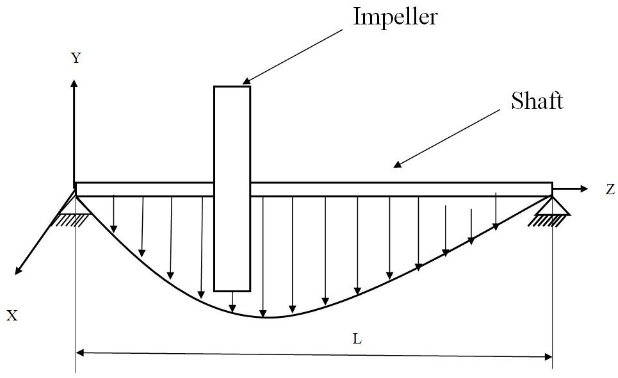

Figure 2.2: Schematic diagram of rotor system

has been used as excitation to perform harmonic analysis using ANSYS [11]. ANSYS parametric design language has been been implemented to achieve the results.

Transverse modal analysis of rotor of centrifugal fan with offset disk under free vibration is performed both by mathematical model development and numerical simulation. A mathematical model governing the transverse vibration of rotor shaft is determined by transfer matrix method. From the mathematical model, natural frequencies and mode shape of the rotor system is determined. Further, Numerical simulation is performed in ANSYS for modal analysis for such system. Unbalance effect is incorporated numerically by performing harmonic analysis of the rotor system in ANSYS.

2. Mathematical Model Development

The commonly used tools for rotor dynamic analysis employ D'Alembert principle, the Lagrange's equations, the transfer matrix method and the finite element method of multi-dof system. Since the rotor of the centrifugal fan is offset from the centre, and the weight of shaft and impeller is of considerable amount, Transfer Matrix Method (TMM) is used for the modal analysis. The advantage of TMM is high processing speed and low memory requirement [8]. The main focus of the analysis is to estimate the rotor system natural frequencies, mode shape and forced responses.

2.1 Modeling Assumption

In this analysis, the vibration amplitude is assumed to be small and linear. The shaft is considered to be isotropic and of having uniform cross section. Simply supported system is considered while deriving the mathematical model for the transverse vibration.
The transfer matrix of the shaft and the disk are obtained assuming that there is no gyroscopic effect, whirling effect and cross coupled terms are ignored. Bearing is assumed to be rigid and have low damping effect which is modeled by giving high stiffness coefficient to the bearing at both ends. Transfer matrix method obtained for one plane of motion is analogous to the perpendicular plane because of symmetry. The simple schematic diagram under consideration for rotor dynamic analysis is as shown in Figure 2.2.

2.2 Standard Transfer Matrix Method

Transfer matrix method (TMM), also called Mysklestand and Prohl method [4], the shaft is divided into a number of imaginary smaller beam elements and the governing equations are derived for each of these elements in order to determine the overall system behavior. This method is relatively simple and straightforward in application. For the rotor system under analysis, rotor is considered to have considerable mass and shaft is considered to be mass less. When the mass of the shaft is appreciable then it is divided up into a number of smaller masses concentrated (or lumped) at junctions or stations of beam segments so that concentrated masses and the shaft is modeled [12]. The station number is assigned whenever there is change in the state vector which includes the translational and rotational (tilting) displacement, shear forces and the bending moments.

2.2.1 Shaft transfer matrix or field matrix

The free body diagram of general element of shaft is shown in Figure 2.4. Translational displacement,

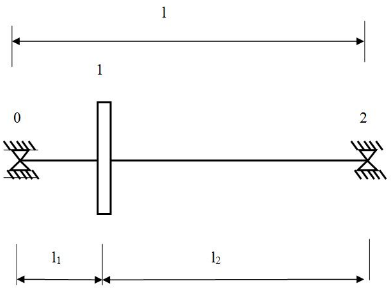

Figure 2.3: Schematic model for rotor model

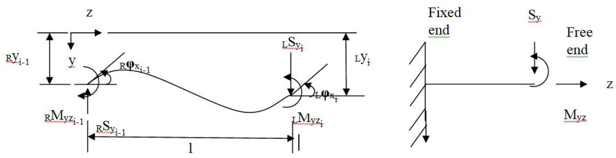

Figure 2.4: Free body diagram of general shaft element

rotational displacement, shear force and bending moment relation is obtained between $(i-1)^{th}$ and i^{th} station points. The displacement and the slope at the free end are related to the applied moment and the shear force at free end by considering the beam as though it were a cantilever and then the displacement and the slope of the fixed end is considered by considering the beam as a rigid. On assumption of small displacements, the two steps could be superimposed to get the total displacement and slope of the i^{th} shaft segment at the left of i^{th} station. The standard notation for the sign convention of the equation is used as followed in [13] and [14].

$$_Ly_i = {_Ry_{i-1}} - {_R\varphi_{i-1}}l - \frac{({_LM_{yz_{i-1}}}l^2 + {_LS_{y_{i-1}}})}{2EI} + \frac{{_LS_{y_{i-1}}}l^3}{3EI} \tag{1}$$

$$_L\varphi_i = {_R\varphi_{i-1}} - \frac{({_LM_{yz_{i-1}}}l + {_LS_{y_{i-1}}})}{EI} + \frac{{_LS_{y_{i-1}}}l^2}{2EI} \tag{2}$$

$$_LS_i = {_RS_{i-1}} \tag{3}$$

$$_LM_{yz_i} = {_LM_{yz_{i-1}}} + {_RS_{i-1}} \tag{4}$$

These equations can be rearranged and expressed in matrix form as $_L\{S\}_i = [F]_i \; _R\{S\}_{i-1}$ with

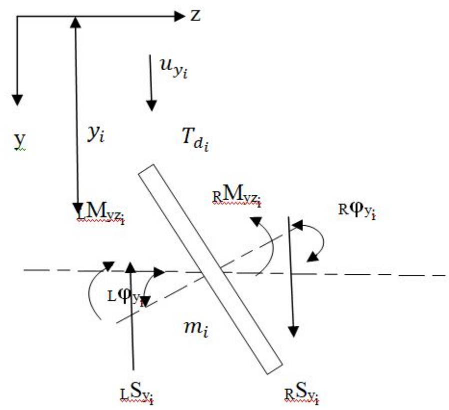

Figure 2.5: Free body diagram of disk element

$$_L\{S\}_i = {_L\begin{Bmatrix} -y \\ \varphi_x \\ M_{yz} \\ S_y \end{Bmatrix}_i}; \quad [F]_i = \begin{bmatrix} 1 & l & \frac{l^2}{2EI} & \frac{l^3}{6EI} \\ 0 & 1 & \frac{l}{EI} & \frac{l^2}{EI} \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix}_i;$$

$$_R\{S\}_{i-1} = {_R\begin{Bmatrix} -y \\ \varphi_x \\ M_{yz} \\ S_y \end{Bmatrix}_{i-1}}$$

2.2.2 Disk transfer matrix or point matrix

The free body diagram of the point mass m^i is shown in Figure 2.5 at i^{th} location. The relationship between forces and displacements at thin disc is given by its equation of motion

$$_Ry_i = {_Ly_i} \tag{5}$$

$$_R\varphi_i = {_L\varphi_i} \tag{6}$$

$$_RM_{yz_i} = -I_{d_i}\omega^2 {_L\varphi_i} + {_LM_{yz_i}} \tag{7}$$

$$_RS_{y_i} = -m_i\omega^2 {_Ry_i} + {_LS_{y_i}} - u_{y_i} \tag{8}$$

On combining these equations in a matrix form as $_R\{S\}_i = [P]_i \, _L\{S\}_i + \{u\}_i$ with

$$_R\{S\}_i = {_R\begin{Bmatrix} -y \\ \varphi_x \\ M_{yz} \\ S_y \end{Bmatrix}_i}; \; [P]_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\omega^2 I_d & 1 & 0 \\ m\omega^2 & 0 & 0 & 1 \end{bmatrix}_i;$$

$$ {}_L\{S\}_i = \begin{Bmatrix} -y \\ \varphi_x \\ M_{yz} \\ S_y \end{Bmatrix}_i^{\;L} ; \{u\}_i = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -u_y \end{Bmatrix}_i $$

2.2.3 Overall transfer matrix

The point and field matrices can be used to form the overall transfer matrix to relate the state vector at one extreme end station (i.e., the left) to the other extreme end (i.e., the right). When there is no coupling between the vertical and horizontal planes (gyroscopic effects neglected) and no damping in the system is considered, then the overall transfer matrix [T] takes the size of 5x5. The equation can be written as

$$ {}_R\{S\}_i = [T]_i \, {}_R\{S\}_0 \tag{9} $$

$$ \begin{Bmatrix} -y \\ \varphi_x \\ M_{yz} \\ S_y \\ 1 \end{Bmatrix}_n = \begin{bmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} & t_{1,5} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} & t_{2,5} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} & t_{3,5} \\ t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4} & t_{4,5} \\ t_{5,1} & t_{5,2} & t_{5,3} & t_{5,4} & t_{5,5} \end{bmatrix} \begin{Bmatrix} -y \\ \varphi_x \\ M_{yz} \\ S_y \\ 1 \end{Bmatrix}_0 \tag{10} $$

On the application of boundary condition of simply supported system the Equation11,

$$ \begin{Bmatrix} 0 \\ \varphi_x \\ 0 \\ S_y \\ 1 \end{Bmatrix}_n = \begin{bmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} & t_{1,5} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} & t_{2,5} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} & t_{3,5} \\ t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4} & t_{4,5} \\ t_{5,1} & t_{5,2} & t_{5,3} & t_{5,4} & t_{5,5} \end{bmatrix} \begin{Bmatrix} 0 \\ \varphi_x \\ 0 \\ S_y \\ 1 \end{Bmatrix}_0 \tag{11} $$

2.3 Governing mathematical model

The Governing equations for the Simply supported system (Pinned roller supports) is given by 12and 13 with the application of boundary condition in the given system

$$ \begin{bmatrix} t_{1,2} & t_{1,4} \\ t_{3,2} & t_{3,4} \end{bmatrix} \begin{Bmatrix} \varphi_x \\ S_y \end{Bmatrix} = \begin{Bmatrix} -t_{1,5} \\ -t_{3,5} \end{Bmatrix} \tag{12} $$

$$ \begin{Bmatrix} \varphi_x \\ S_y \end{Bmatrix}_R = \begin{bmatrix} t_{1,2} & t_{1,4} \\ t_{3,2} & t_{3,4} \end{bmatrix} \begin{Bmatrix} \varphi_x \\ S_y \end{Bmatrix} + \begin{Bmatrix} t_{2,5} \\ t_{4,5} \end{Bmatrix} \tag{13} $$

3. Rotor dynamic analysis in ANSYS

ANSYS softwares are extensively used finite element simulation tool to solve different varieties of problem in many engineering industries [11].In recent years, the rotor dynamic capabilities of ANSYS program has been improved much subjected to the analysis need, feasible method and computational time [15].

Table 1: Parameters for modal analysis

Characteristic	Value
Rotational Speed (rpm)	990
Material	EN8
Density (kg/m^3)	7850
Elastic Modulus (Gpa)	2.1
Poisson's Ratio	0.3

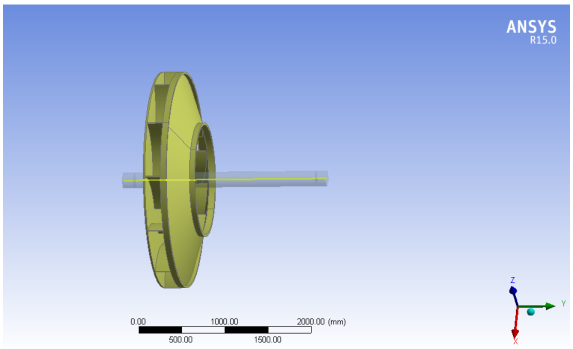

Figure 3.1: Geometry model used in ANSYS

3.1 Geometry

The modal analysis theory as described is used to obtain the various modes of vibration of the rotor system. The geometry of rotor system is the simplified by assuming the shaft of rotor to be of uniform cross-section. The impeller is designed in solidworks and is imported in ANSYS Design modeler. One dimensional line sketch is done where a circular cross-section is attached. This decreases the complexity in the analysis. Rotor Geometry model is shown in Figure 3.1.

3.2 Model development

Before Building up the modal analysis of the rotor of the preheater fan, various constraints were imposed as to determine the critical speed and modal frequency. Boundary Conditions:

- Bearing support:
  Ground to Line Body connection is established at the extreme end of the shaft for modeling the bearing support in the rotor system. The connection type is body-ground. Stiffness of the both bearing K11 and K22 is given a value

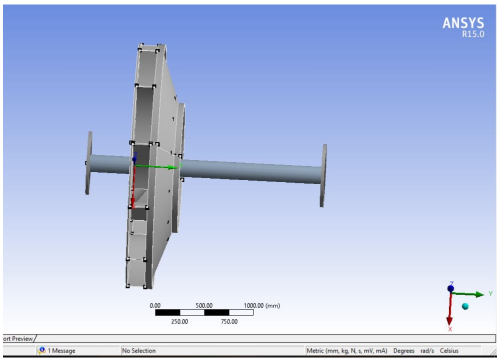

Figure 3.2: Model development for rotor dynamic simulation

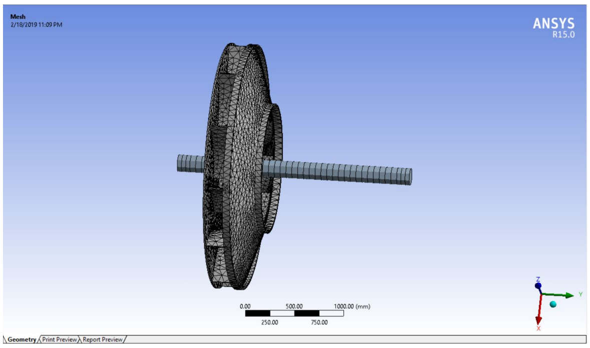

Figure 3.3: Mesh development for rotor dynamic shaft

of 1.e+005N/mm for the rigid body motion. Rotation plane of the bearing support is of X-Z plane.

- Remote displacement:
  The rotor system is supposed to be simply supported such that the bearing axis end points have zero displacement and zero moment.
- Rotational velocity:
  In order to obtain Campbell diagram, various rotational speed has to be specified for the various mode.

3.3 Mesh

Automatic mesh is generated in ANSYS 15. Because of the dynamic loads, the three-dimensional solid element is needed to analysis the response of the structural with large dynamic loads in order to improve the accuracy of analysis.

3.4 Analysis setting

Solution and Post processing: The solution of the rotor dynamic system is done as per the programmed

Table 2: Rotor dynamic analysis setting

Object name	Analysis setting
Max no of modes	8
Solver controls	
Damped	yes
Solver type	Program controlled
Rotor dynamic controls	
Coriolis Effect	On
Campbell diagram	On
Number of points	4

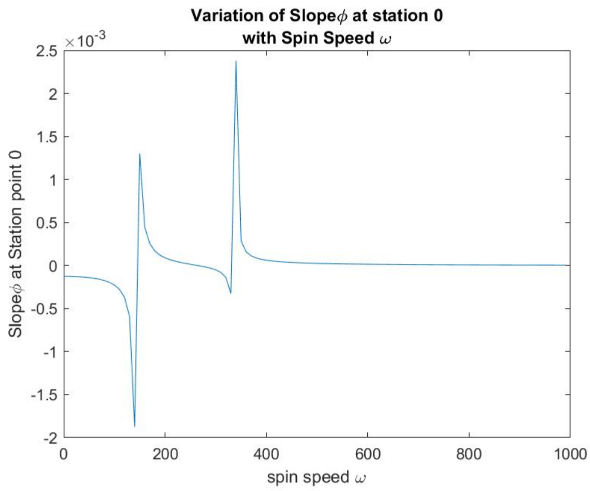

Figure 4.1: Variation of slope at station 0 with spin speed

controlled shown in the analysis Table 2. Since, only the transverse displacement is considered for analysis in this thesis, the solution of mode 2 gives the desired result.

4. Results

4.1 Mathematical model solution

A mathematical model that governs the transverse mode of vibration of offset rotor is obtained by transfer matrix method. The obtained governing equation is solved by the standard matrix method for its solution. The standard matrix method is coded in MATLAB and the result is generated as shown in Figures 4.1 – 4.8.

A plot of the state variable and spin speed of the rotor at station point 0, 1 and 2 is shown in Figures. The resonant condition can be seen as large amplitudes of vibration and it indicate critical speed.

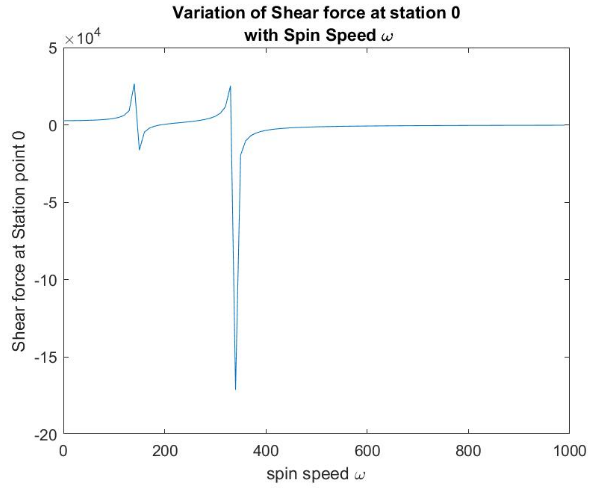

Figure 4.2: Variation of Shear force at station 0 with spin speed

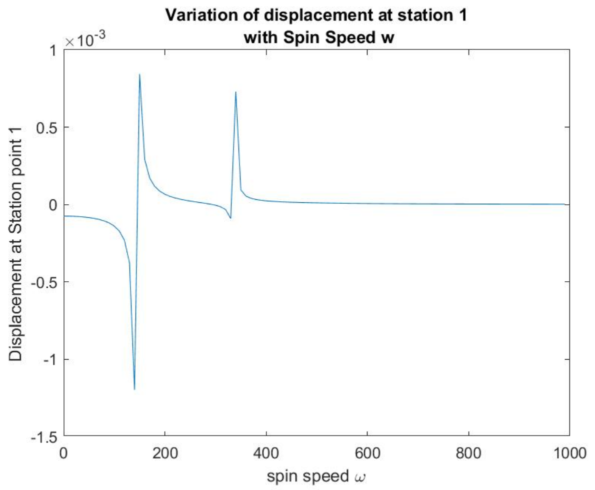

Figure 4.3: Variation of displacement at station 1 with spin speed

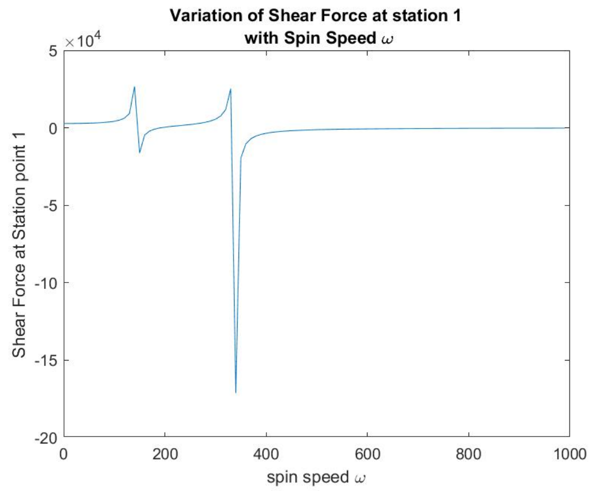

Figure 4.4: Variation of Shear force at station 1 with spin speed

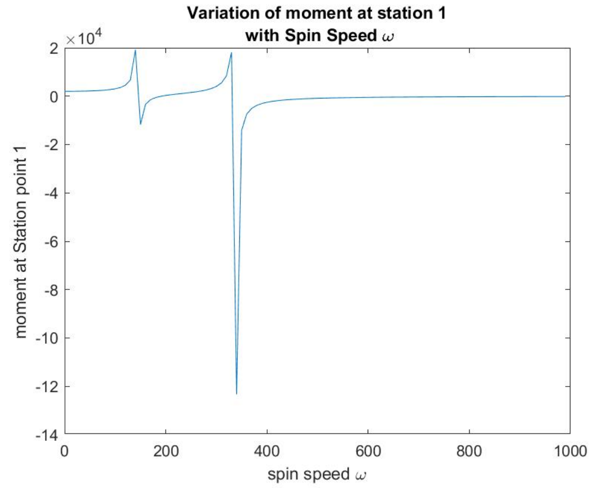

Figure 4.5: Variation of moment at station 1 with spin speed

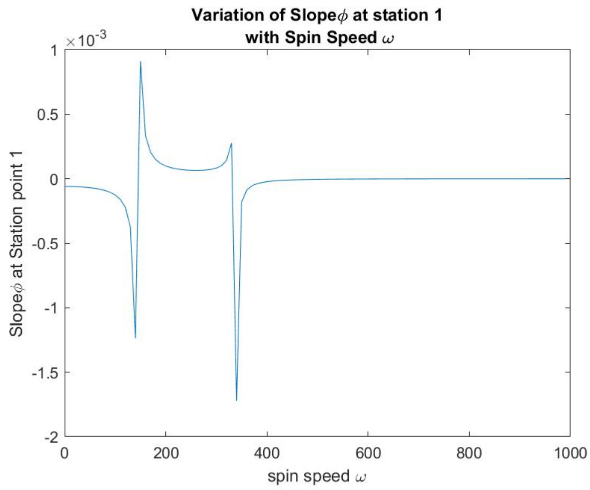

Figure 4.6: Variation of slope at station 1 with spin speed

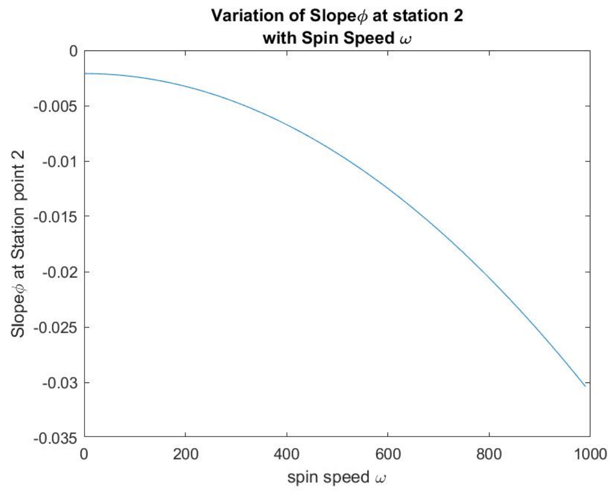

Figure 4.7: Variation of slope at station 2 with spin speed

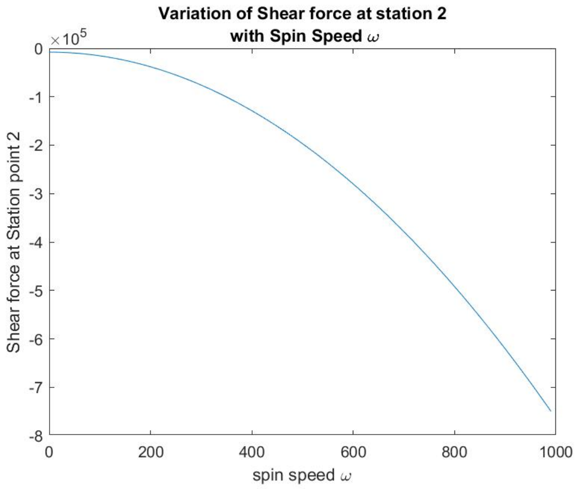

Figure 4.8: Variation of Shear force at station 2 with spin speed

4.2 Rotor dynamic solution in ANSYS

4.2.1 Campbell diagram

Determination of eigen frequencies of rotating system for different operating speed is carried out in this analysis. In this, modal analysis is performed starting from 0 rad/s to 1000 rad/s.

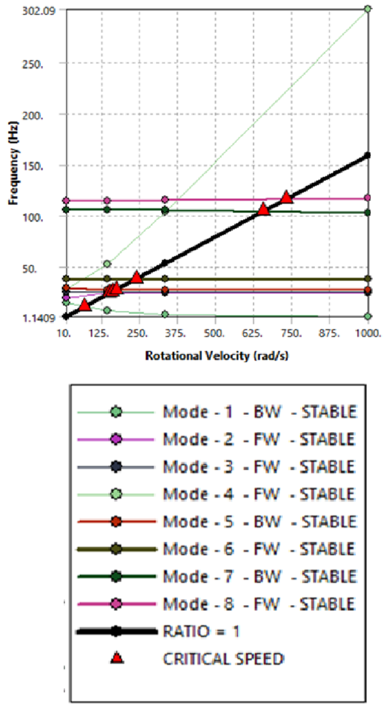

Figure 4.9: Campbell diagram

4.2.2 Deflection results

Mode shape 2 is the transverse vibration of the shape as shown in the Figure 4.10 and Figure 4.11.

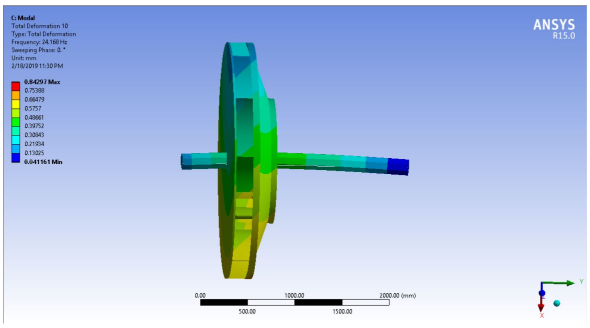

Figure 4.10: Displacement at 144.2495 rad/s

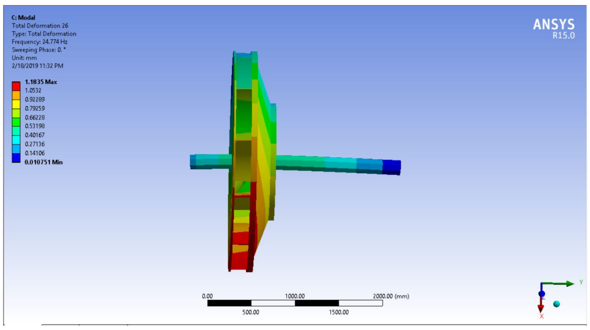

Figure 4.11: Displacement at 335.0 rad/s

5. Discussion

From the mathematical model of offset rotor shaft by transfer matrix method, only transverse mode of vibration is obtained. Other modes of vibration is not considered in analysis. The governing mathematical equation is given by 12 and 13. A MATLAB code is developed for the solution of governing equation by standard matrix method. From the solution, two natural frequencies 144.2495 rad/s and 338.7165 rad/s is obtained and the corresponding mode of vibration is also obtained. The displacement, slope, shear force and moment forces are obtained at various station point. The resonant condition can be seen as large amplitudes of vibration and indicate critical speed.
For the numerical solution ANSYS 15 is used. The geometry required for analysis is obtained from SolidWorks and imported to ANSYS workbench. The modal analysis is done with the setting as shown in the Table 2. The results obtained from modal analysis

showns transverse vibration, axial vibration and torsional vibration and its higher modes. But comparison is done only for the transverse mode of vibration with the mathematical model solution. From the numerical analysis, the campbell diagram shows the critical speed of 151.98 rad/s for the mode 2. This mode shows the transverse mode which is used for comparison.

The obtained natural frequencies of transverse vibration from mathematical model by transfer matrix method is 144.2495 rad/s and critical speed from campbell diagram from numerical simulation in ANSYS is 151.98 rad/s. The obtained results is of comparable range. The error in both the results is 5.08 %. This may be due to the various assumption done while deriving the mathematical model of transverse vibration which includes the neglecting the effect of gyroscopic effect, tilting of rotor and ignoring the cross coupled terms by transfer matrix method. But these effects are taken in consideration in solution while numerical solution in ANSYS. Thus, obtained mathematical model for offset rotor shaft of centrifugal fan by transfer matrix method can be considered consistent with the result.

6. Conclusion

A mathematical model that governs the vibration of offset rotor shaft of centrifugal fan is developed by transfer matrix method. The obtained mathematical model is solved by standard matrix method to obtain natural frequencies and corresponding modes of vibration. The natural frequencies obtained are 144.2495 rad/s and 338.7165 rad/s. Modal analysis is performed in ANSYS 15.0 and simulated numerically. The geometry required for simulation is generated in SolidWorks and imported in ANSYS. From the modal analysis, campbell diagram is obtained which determines various critical speed. Various modes of vibration is also obtained from numerical analysis and from which mode 2 shows the transverse mode of vibration. This mode is in our consideration and from the campbell diagram, critical speed obtained is 151.98 rad/s. The mathematical model results and simulation results are compared and result is obtained.

7. Recommendation

Only transverse mode of vibration is obtained while obtaining mathematical model development of offset rotor shaft by transfer matrix method. So other modes of vibration can also be considered for the further study. Similarly more generalized analysis of offset rotor can be analyzed by considering the gyroscopic effect and tilting of the rotor while whirling motion. Moreover, the effect of unbalance force can also be incorporated with the increase in spin speed of shaft. For further generalized study the effect of thermal load and aerodynamic load can also be considered.

8. Acknowledgments

The authors would like to acknowlege Institute of Engineering [IOE] and Department of Mechanical Engineering, Pulchowk Campus for their support and opportunity.

References

[1] Ming Zhang, Yan Liu, Weiqiang Wang, Pengfei Wang, and Jianfeng Li. The fatigue of impellers and blades. *Engineering Failure Analysis*, 62:208–231, 2016.

[2] Hamid Eskandari, Morteza Gharouni Nik, and Amir Pakzad. Foundation analyzing of centrifugal id fans in cement plants. *Alexandria Engineering Journal*, 55(2):1563–1572, 2016.

[3] JS Rao. *Rotor dynamics*. New Age International, 1996.

[4] MA Prohl. A general method for calculating critical speeds of flexible rotors. *Trans. ASME*, 67:A142, 1945.

[5] J Widebæk Lund and FK Orcutt. Calculations and experiments on the unbalance response of a flexible rotor. *Journal of Engineering for Industry*, 89(4):785–796, 1967.

[6] MS Darlow, BT Murphy, JA Elder, and GN Sandor. Extension of the transfer matrix method for rotordynamic analysis to include a direct representation of conical sections and trunnions. *Journal of Mechanical Design*, 102(1):122–129, 1980.

[7] M Xu and RD Marangoni. Vibration analysis of a motor-flexible coupling-rotor system subject to misalignment and unbalance, part i: theoretical model and analysis. *Journal of sound and vibration*, 176(5):663–679, 1994.

[8] Jorgen L Nikolajsen. Finite element and transfer matrix methods for rotordynamics: A comparison. In *ASME Turbo Expo 2001: Power for Land, Sea, and Air*, pages V004T03A002–V004T03A002. American Society of Mechanical Engineers, 2001.

[9] Mohammad Taghi Ahmadian, Hossein Sadeghi, Omid Ghasemalizadeh, and Mohammad Bonakdar. Analysis of rotor-bearing system using the transfer matrix method. In *CSC*, pages 265–272, 2007.

[10] Chao-Yang Tsai and Shyh-Chin Huang. Transfer matrix method to vibration analysis of rotors with

coupler offsets. *Shock and vibration*, 20(1):97–108, 2013.

[11] Ritesh Fegade, Vimal Patel, RS Nehete, and BM Bhandarkar. Unbalanced response of rotor using ansys parametric design for different bearings. *International Journal of Engineering Sciences & Emerging Technologies*, 7(1):506–515, 2014.

[12] Rajiv Tiwari. *Rotor systems: analysis and identification*. CRC Press, 2017.

[13] Singiresu S Rao and Fook Fah Yap. *Mechanical vibrations*, volume 4. Prentice hall Upper Saddle River, 2011.

[14] Arthur Peter Boresi, Richard Joseph Schmidt, and Omar M Sidebottom. *Advanced mechanics of materials*, volume 6. Wiley New York et al., 1985.

[15] Deepak Srikrishnanivas. Rotor dynamic analysis of rm12 jet engine rotor using ansys. 2012.