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Dynamic Response of overhung Pelton Turbine Unit for Free Vibration

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Abstract

The study of dynamic behavior of rotating parts have significant role to the operating mechanism and failure associated with vibration. Shaft and runner are the main component of water turbines which are subjected to highly difficult working conditions. When rotors are operating at high speeds centrifugal actions cause to increase intensity of vibration. During the manufacturing and installation dynamics analysis is necessary.
This research work was focused on modeling of Pelton turbine unit; which cover dynamic behavior of overhung rigid runner on circular flexible shaft, which was supported by rigid bearings on one end and enable to determine the natural frequency of the system by using different models. The unit was modeled as discrete and continuous systems. Simplified Jeffcott rotor model and Rayleigh's energy method: effective mass models were used for the discrete system models. The model for continuous system was developed by calculating the kinetic and potential energy of the runner –bucket and shaft. The governing equations were formulated by using Langrange's equation and solved analytically by using Rayleigh-Ritz method.
The critical frequencies were determined for developed mathematical model based on the real Pelton turbine unit installed in Fluid Mechanics Laboratory of Department of Automobile and Mechanical Engineering , Thapathali campus. The critical frequencies for continuous system model were found to be 1470, 97rad/sec and 2139.45rad/sec for backward and forward whirl.It values using simplified Jeffcott rotor model was 1905.08rad/sec.Similarly, the values of critical frequencies by Rayleigh's Energy Method: Effective mass models were calculated to be 1875.65rad/sec and 1864.31rad/sec by considering effective mass of shaft and spring respectively. The continuous shaft-runner –bucket system can be model as discrete system with effective mass with fixed support at end of shaft to determine the critical frequencies of unit with reasonable accuracy. The model is simulated in ansys the critical frequencies for backward whirl and forward whirl is found 1396rad/sec and at 1875rad/sec at first mode of vibration which is close to analytical result.

Keywords

ROTODYNAMIC, DISCRETE, CONTINUOUS, MODEL, NATURAL FREQUENCY

1. Introduction

Rotor dynamic study the transverse/lateral(bending) longitudinal(axial), and torsional vibration of the rotating shafts and minimizing the probality of failure due to vibration.The possible forces responsible for the vibration increase in hydro turbine may be mechanical, hydraulic and electrical.study of dynamic response of pelton turbine unit, one of the widely used water turbines worldwide, for the improvement in performance as well as reliability, stability and the life span of the component of the hydraulic power system.if the frequency of external excitation concide with one of the natural frequencies of the system, resonance occurs leading to the dangerously large oscillations and may cause excessive deflection and failure.The failure of structures like [1] buildings, bridges, turbines, and airplane wings etc are associated with the occurrence of resonance. Hence, the calculation of the natural frequencies of the system is one of the important part of vibration study and analysis The model for the study of dynamic behavior of rotor and rotating part was first developed by german engineer august fopple in 1895 and American Henry Homan Jeffcot in 1919.This model is commonly known as the Fopple/Jeffcot rotor or fixed jeffcot rotor, which consisted of a single rigid disc centrally located on the flexible shaft of constant circular cross section supported by bearings placed at each end of shafts.the mass of the shafts for the model assume negligible.Fopple considered the system without damping but jeffcot considered the effect of damping in his research.Their analysis demonstrated that the supercritical operation was possible and stable.This model was oversimplication of the real world rotor systems, but it gave good understanding of the different characteristics of the real world rotors behavior like critical speeds, gyroscopic action and internal damping.

2. Mathematical Model Development and Analytical Solution

solution of most engineering problem require mathematical modeling physical system. The Basic component considered for the model development were runner-bukker assembly and flexible shaft supported by rigid bearings at on ends. The models were development considering the system as discrete and continuous systems and focused for the output of natural frequency.

Assumptions

- The model is assumed to be linear and discrete system for most of the models.
- Pelton turbine unit is the combination of shaft-runner-buckets-felxible bearings system but flexible shaft-rigid disk-rigid bearing are considered for model development to reduce the level of complexities.

2.1 Discrete System Models

Jeffcot rotor model and Rayleigh's energy method: Effective mass model were used for the discrete system models.

2.1.1 Jeffcot Rotor Models

For the shaft with fixed at one end, the stiffness of the shaft is given by the expression as (Rao, 2013).

$k_s = \frac{3EI}{L^3}$
Similarly,

$$\omega_n = \sqrt{\frac{K_s}{M_D}} = \sqrt{\frac{3EI}{L^3 M_D}} \qquad I = \frac{\pi D^4}{64} \tag{1}$$

2.1.2 Rayleigh's Energy Method: Effective Mass Models

The principle of conservation of energy (kinetic and strain energy) is used to evaluate the effective mass of the system. This method is employed for the lower degrees of freedom systems, mostly single degree of freedom systems.

A mass MD of the rotor disk (runner-buckets assembly) is mounted at the axial end of the shaft of stiffness Ks. The mass of the shaft is MS and length L. For the fixed supported uniform shaft, also considering the effective mass of the shaft, the undamped natural frequency of the system is expressed as (Thomson and Dahlen, 2005)

$$\omega_n = \sqrt{\frac{K_s}{(M_D + 0.235 m_s)}} = \sqrt{\frac{3EI}{(M_D + 0.235 m_s) L^3}} \tag{2}$$

A mass MD of the rotor disk is assumed as resting on a heavy spring (shaft) of stiffness Ks. The mass of the spring (shaft) is MS and L is the length of un-stretched spring. The natural frequency of the equivalent spring-mass system, considering the effective mass.of spring and assuming the shaft to be rotating in simple harmonic motion, is defined as (Kelly, 2012).

$$\omega_n = \sqrt{\frac{K_s}{\left(M_D + \frac{m_s}{3}\right)}} = \sqrt{\frac{3EI}{\left(M_D + \frac{m_s}{3}\right) L^3}} \tag{3}$$

2.2 Continuous System Model

Any rotation can be described by three successive rotations about linearly independent axes and these rotations are Euler angles. The positions, angular velocities and angular accelerations of a body that rotates about a fixed point, such as a gyroscope, and body that rotates about its center of mass (an aircraft, shaft of turbine etc.) can be described by Euler's angles (Ardakani and Bridges, 2010).

X, Y and Z is fixed inertial frame and x, y and z is the body fixed axis. Firstly, the rotation is counter clockwise from an initial XYZ system about the Z, z_1 axis as shown in Figure 2.1(a) into x_1, y_1, z_1 system by an angle ϕ.

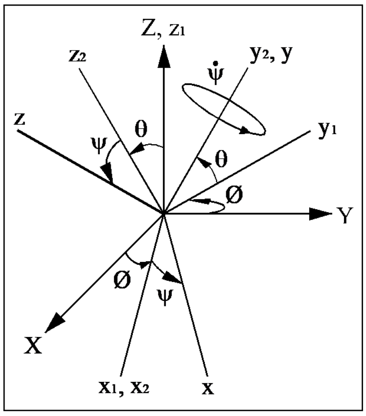

Figure 1: rotational matrix for 312 euler angles

2.2.1 Rotational Matrix for 3-1-2 Euler Angle

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi\cos\psi - \sin\phi\sin\theta\sin\psi & \sin\phi\cos\psi & -\cos\theta\sin\phi \\ -\sin\phi\cos\theta & \cos\phi\cos\theta & \sin\theta \\ \cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi - \cos\phi\sin\theta\cos\phi & \cos\phi \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

2.2.2 Angular Velocity of xyz Frame

Angular velocity for x , y, z frame

$$\therefore \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -\phi\cos\theta\sin\psi + \theta\cos\psi \\ \phi\sin\theta + \psi \\ \phi\cos\theta\cos\psi + \theta\sin\psi \end{bmatrix}$$

2.2.3 Disc

Thus, the kinetic energy of the disk is given by,

$T_D = \frac{1}{2}M_D\left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}\left[I_{Dxx}\omega_x^2 + I_{Dyy}\omega_y^2 + I_{Dzz}\omega_z^2\right]$

Where,

T_D – the kinetic energy of disk.

M_D –the mass of the disk.

I_{Dxx}, I_{Dyy}, and I_{Dzz} are the moment of inertia about the principal axes X, Y and Z respectively.

As the disk is assumed to be symmetrical, $I_{Dxx} = I_{Dyy}$.

2.2.4 shaft

The K.E. of the shaft is defined for an element and integrated over the length of the shaft 'L'. The K.E. of the shaft is given by the expression,

$$T_s = \frac{\rho A}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right)dy + \frac{\rho l_{xx}}{2}\int_0^1 \omega_x^2 dy + \frac{\rho l_{yy}}{2}\int_0^1 \omega_y^2 dy + \frac{\rho l_{zz}}{2}\int_0^1 \omega_z^2 dy \quad (4)$$

Where,

T_S – kinetic energy of the shaft,

ρ – the mass per unit volume,

A – the cross-sectional area of shaft and it is assumed to be constant,

I – the area moment of inertia of the shaft cross-section about the neutral axis and it is also supposed to be constant. Total kinetic energy of shaft and disc

$$T = \frac{1}{2}M_D\left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}l_{Dxx}\left(\dot{\theta}^2 + \dot{\phi}^2\right) + \frac{1}{2}I_{Dyy}\left(\Omega_2 + 2\dot{\phi}\theta\Omega\right) + \frac{\rho A}{2}\int_0^1 \left(\dot{u}^2 + \dot{w}^2\right)dy + \frac{\rho l}{2}\int_0^1 \left(\dot{\theta}^2 + \dot{\phi}^2\right)dy + \rho IL\Omega^2 + 2\rho l\Omega\int_0^1 \theta\dot{\phi}dy \quad (5)$$

Total potential energy of system is

Total potential energy of the system is,

$$U = \frac{EI}{2}\int_0^1 \left[\left(\frac{\partial^2 u}{\partial y^2}\right) + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] dy$$

The bearings at the support are considered as rigid and isotropic with negligible damping

2.2.5 Lagrange's Equation

Using Lagrange's equation for the system of rigid bodies in the form;

$$\frac{d}{dl}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q_1} + \frac{\partial U}{\partial q_1} = F\dot{q}_1$$

Where,

N is the number of degrees of freedom $(l \le i \le N)$.

qi are the system's generalized independent coordinates.

Fqi are the generalized forces.

denotes differentiation with respect to time t.

2.2.6 Rayleigh-Ritz Method of Analytical Solution

Rayleigh-Ritz method is also known as assumed modes method. For proper description of the lateral vibration behavior of the rotor, it is necessary to write the displacement u and w of the rotor in terms of shape function f(y), before applying the expressions obtained in the Lagrange equation.

$$f(y) = \{\cosh\beta y - \cos\beta y\} - \frac{\cosh\beta l + \cos\beta l}{\sinh\beta l + \sin\beta l}\left(\sinh\beta y - \sin\beta y\right)$$

Where,

βl= 1.875

2.2.7 Total kinetic energy of shaft disc assembly

Therefore, the kinetic energy of the disk-shaft assembly is,

$T = T_D + T_s$

$= \left[\frac{1}{2}\left(4M_D + 450.3I_{Dxx} + 0.101\rho A + 35.7\rho I\right)\left(\dot{q_1}^2 + \dot{q_2}^2\right) - \left(450.3I_{Dyy} + 71.5\rho I\right)\Omega\dot{q}_1\dot{q}_2\right.$

$T = \frac{1}{2}m\left(\dot{q_1}^2 + \dot{q_2}^2\right) - a\Omega\dot{q}_1\dot{q}_2$

Meq. = ($4M_D$ + 450.3 I_{Dxx} + 0.101 ρ A +35.76ρ I)
a = 450.3 I_{Dyy}+ 71.52 ρ I

2.2.8 Total strain energy of shafts –disc assembly

$U = \frac{EI}{2}\left[\int_0^L h^2(y)\, q_1^2 dy + \int_0^L h^2(y)\, q_2^2 dy\right]$
= $\frac{EI}{2} \times 5623.26\left(q_1^2 + q_2^2\right)$
Where,
k eq =5623.26 EI
Free vibration equation of motion by using langrange'equation
$\begin{bmatrix} m & o \\ o & m \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \Omega\begin{bmatrix} 0 & -a \\ a & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

3. Analytical solution of equation of motion

The Newton's equations of motion
$m\ddot{q}_1 - a\Omega\dot{q}_2 + Kq_1 = 0$
$m\ddot{q}_2 + a\Omega\dot{q}_1 + Kq_2 = 0$ Equation takes the form i.e. q_1 and q_2 to be a harmonic function,
then q_1 and $q_2$are expressed as $(Baruh, 1999)$,
$q_1 = Q_1 e^{\lambda t}$
$q_2 = Q_2 e^{\lambda t}$
characterstics equation by using above eq
$m^2\lambda^4 + \left(2mK + a^2\Omega^2\right)\lambda^2 + K^2 = 0$
Solution of Equation which is quadratic in Ω^2 resulted as
$\Omega_1 = \sqrt{\frac{K}{s(sm+a)}}$
$\Omega_2 = \sqrt{\frac{K}{s(sm-a)}}$
$\omega = s\Omega$
Critical speed of the system is defined as,
s=1
$\Omega_{cr} = \omega_n$
$\Omega_{1cr} = \omega_{1n} = \sqrt{\frac{K}{(m+a)}}$
$\Omega_{2cr} = \omega_{2n} = \sqrt{\frac{K}{(m-a)}}$
Where. $\Omega_{1cr}(\omega_{1n})$ and $\Omega_{2cr}(\omega_{2n})$ are the critical speeds or natural frequencies of the system for forward whirl and backward whirl.

4. Result and Analysis

The developed mathematical models were solved analytically to find the natural frequencies under undamped free vibration condition of the system. The results obtained from the analytical solutions by using the different developed mathematical models were then analyzed. The developed mathematical models were solved for the undamped natural frequencies of the Pelton turbine unit installed with following specifications

Table 1: Parameter used for calculation of critical frequency

Parameters	Values
Total mass of runner bucket assembly, M_D	9.5 Kg
Mass of shaft, m_s	1.28kg
Diameter of shaft, D	40mm
Length of shaft, L	130mm
Density of shaft material, ρ	7860 kg/m^3
Young's Modulus of Elasticity of the Shaft Material, E	202Gpa
Moment of inertia of shafts, I	= 1.25 $\times 10^{-7} m^4$
Mass moment of inertia of disc about X-axis	0.0192kgm^2
Mass moment of inertia of disc about Y-axis	0.038kgm^2
Equivalent mass of shaft disc assembly, Meq	48kg
Gyroscopic effect , a	17kg

4.1 Analytical results

The results from mathematical model were found by using above parameters

Table 2: results from discrete model

Natural frequency from Jeffcott rotor model	1905.08rad/sec
Natural frequency from considering effective mass of shaft	1875.69rad/sec
Natural frequency from considering effective mass of spring	1864.3rad/sec

Table 3: results from continuous model

Natural frequency when system is at rest	1714.19rad/sec
Critical frequency for forward whrilling	2139.45rad/sec
Critical frequency for backward whrilling	1470.97rad/sec

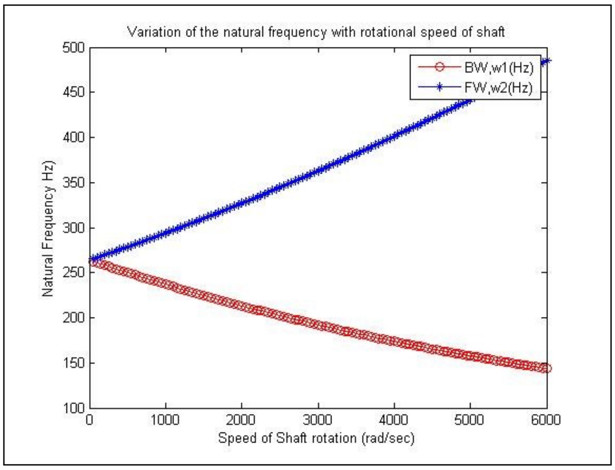

Figure 2: cambel diagram

4.2 results from simulation

By designing equivalent rotor disc model in solidwork and imported in ANSYS following results were found.

Table 4: natural frequency and corresponding mode shape from simulation

Frequency of first mode of vibration	283.06Hz
Frequency of second mode of vibration	286.29Hz
Frequency of third mode of vibration	365.91Hz
Frequency of fourth mode of vibration	1392.4Hz
Frequency of fifth mode of vibration	1399.1Hz
Frequency of six mode of vibration	1600.7Hz

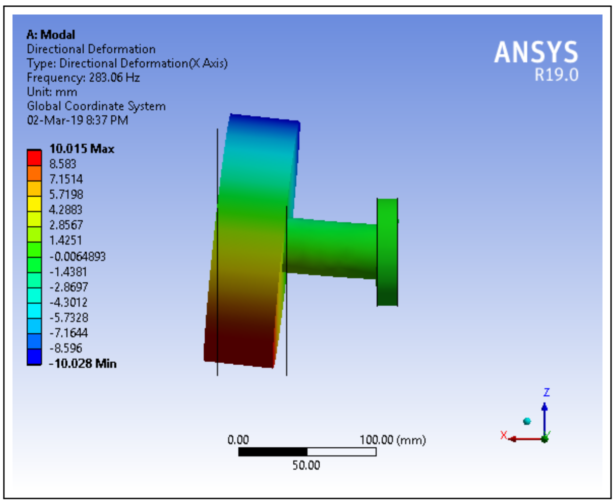

Figure 3: Directional deformation and stress distribution for First mode of vibration

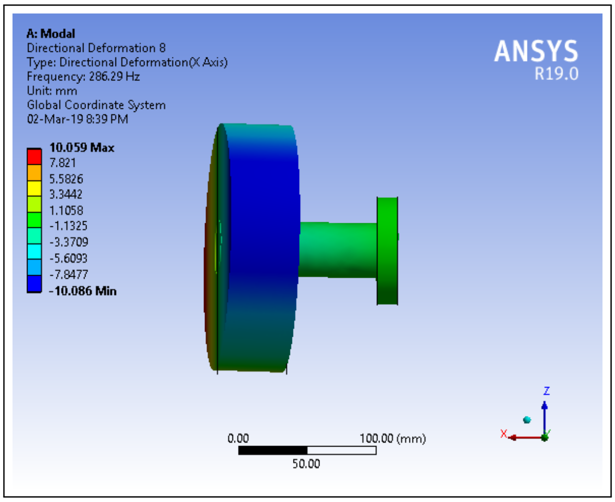

Figure 4: Directional deformation and stress distribution for Second mode of vibration

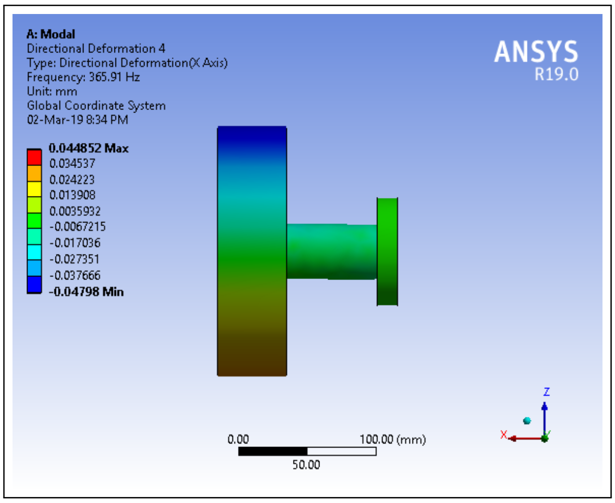

Figure 5: Directional deformation and stress distribution for Third mode of vibration

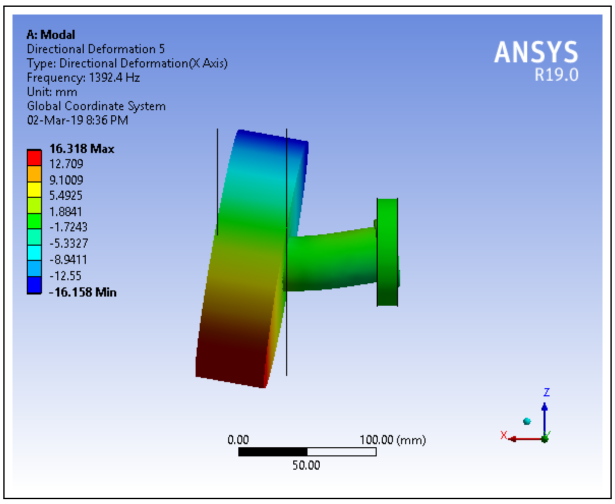

Figure 6: Directional deformation and stress distribution for Fourth mode of vibration

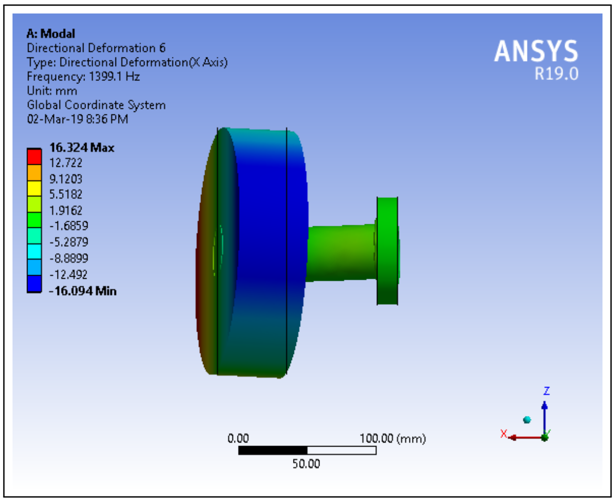

Figure 7: Directional deformation and stress distribution for Fifth mode of vibration

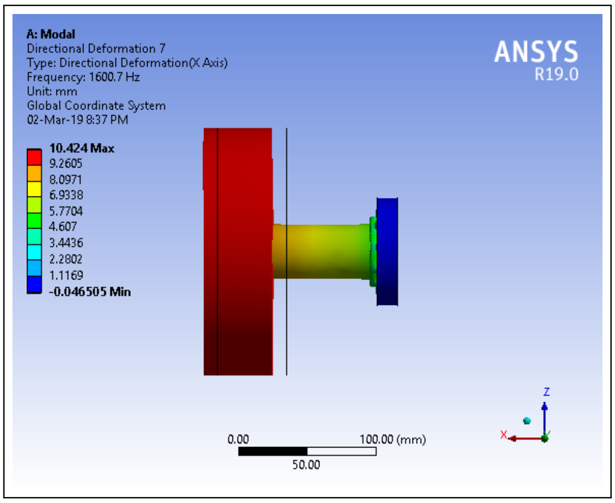

Figure 8: Directional deformation and stress distribution for Sixth mode of vibration

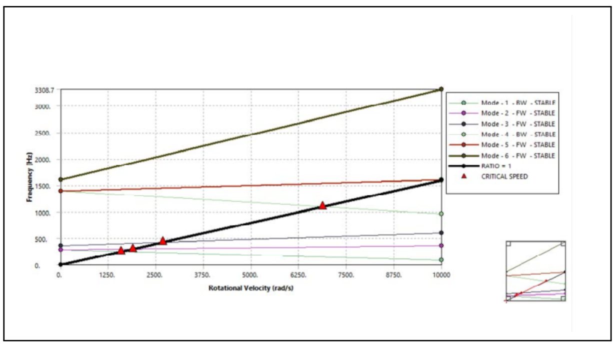

Figure 9: Cambel diagram from ANSYS simulation

5. Conclusion

The model fixed at one end and Rayleigh's energy method: Effective mass model for discrete system and continuous system model has been developed for the selected Pelton turbine unit.

- The values of critical frequency by using cantilever model fixed at one end was found to be 1905rad/sec. Similarly, critical frequencies by Rayleigh's energy method: Effective mass models were 1875.69 rad/sec and 1864.31 rad/sec for fixed supported shaft at end and for equivalent spring-mass configuration. For continuous system model, critical frequencies were found to be 1470.97 rad/sec and 2139.45 rad/sec for backward and forward whirl respectively.

- The natural frequency of the Pelton turbine unit by modeling it as a single degree of freedom discrete system by considering the effective mass of shaft which was fixed supported at end i.e.1875.69 rad/sec, provided close result to the millidegrees of freedom continuous system model of the unit i.e. 1470.97 rad/sec and 2139.45 rad/sec for backward and forward whirl respectively. The results from ansys simulation was 1875rad/sec and 1399rad/sec respectively which was closed to analytical solution with reasonable accuracy.

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