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Effects of Change in Rise on the Structural Performance of Spherical Shell Roofs

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Abstract

This paper deals with the analysis of closed spherical shell roof, subjected to the self weight, with three different types of support conditions; fixed support , pinned support with restrained horizontal translation and spherical shells supported on edge beam. for this purpose, the three different spherical shells of fixed span 6 m, 20 m, and 45 m has been taken with the rises varying from span/3 to span/9 and thicknesses of 125 mm , 150 mm and 200 mm has been assigned to each dome respectively .The effects on the structural performance of the spherical dome with the change of rise has been analyzed applying the classical membrane theory and bending theories using Geckeler's theory and Hytenyi's modified approach [1].The program has been formulated using MAT-LAB.The influence of the moment is localized on the edge region and this influence length for each shells has also been determined as well as compared with length obtained from the relation given by Hoogenboom, 2006. Finite element analysis of each spherical shell has been performed on SAP 2000 and then comparison is done with the classical theories. Further the optimal rise for the spherical dome based on the structural performance has been suggested for practical application.

Keywords

Spherical Shells, classical shell theories, effects of rise, optimal rise, influence length

1. Introduction

Shells are the curved surfaces whose thickness is very small in comparison to other dimensions. Shells are capable of resisting imposed load by membrane stresses and moment stresses. Membrane stresses comprises of compressive as well as tensile stresses. Moment stresses arises when the membrane stresses are not fully compatible with the geometrical as well as loading conditions. The membrane stresses lie in the interior portion of the shell while the moment stresses lie in the place of openings, supports and at the junction of of two shells of different curvatures and stiffness.Thus, both the membrane analysis as well as moment analysis is required for the complete solution of the structural performance of the shell.

From the time immemorial the Spherical dome has been the major structures in any civilization across the world. The capability of dome to resist by the membrane stresses with its doubly curved surface makes it more stable . The symmetrical geometry enables the spherical dome where, internal stresses cancel each other making it less deform-able. The conversion of the normal stresses to the in-plane stresses is the mechanism of resisting the loads in spherical dome thus, unlike flat plates, it requires smaller thickness as larger bending stiffness is not required. Dome shell consists of different geometrical shapes. When segment of a circular curve revolves about vertical diameter, a spherical dome is obtained. Similarly, elliptical dome is obtained by the revolution of an elliptical curve. In this paper, only spherical dome with different rise and span with different support condition are analyzed.

As we all know the concrete is good in resisting compressive stress and major portion of the shell structures is resisted by the compressive membrane stresses unlike other flat slabs which are dominated by the bending. Thus, this signifies the efficient and optimum utilization of the material in shell structures. Beside this the shell structures are thin and can withstand large spans without any support, makes them light in weight which play a great role in minimizing the base shear of the structures for lateral loads like earthquake, thus it can be predicted that the shell structures functions satisfactorily in seismic zones like Nepal. Also, for the country like Nepal where there is a great possibility of tourism, the aesthetic of the city is very important and shell structures are very aesthetic in appearance. Thus, to build the structures, various codes and literature needs to be promulgated.

In this paper spherical is taken for the analysis and the effect on the stresses for different rise to span ratio is investigated as it is a prominent factor in optimizing and increasing the efficiency of the spherical dome design and construction. From a flat roof to the dome roof the main thing changing is the rise of the structure, as we increase the rise the membrane stresses become more and more prominent and bending stresses decreases. This behavior makes the structure utilize both the strength of the dome making it more efficient. Beside this the dome can be constructed for various edge restraints and the behavior of dome at the edges changes significantly with the change in the restraints. This change need to be addressed in analysis and design as at the edges, the bending stress becomes prominent and the influence of the bending is present in some part in the edge this needs to be determined as influence length. Thus, the domes with different rise to span ratio with different edge condition has been studied. further finite element analysis has been performed in SAP 2000 and the stresses are compared with that of the stresses obtained from the theoretical solution.

2. Related works

(Prabhapati. P, Dr. S. B. Vankundre and Veeresh Varur (2014) performed parametric study of three domes of base diameter 6m, 20m and 45m and the rise of each dome is varied from on fifth to one tenth. The membrane theory is applied to determine the membrane stresses and then design is carried out using working stress method as per IS 800; 2007 using ETABS software. From the analytical stress, dynamics optimization technique is carried out to optimize the weight of steel and volume of concrete. Then considerable economy can be achieved using the optimization method with very low amount of steel and concrete.[2]

(Ameen, 2011) carried a finite element analysis of the spherical dome of large diameter (50m, 70m and 100 m) with different uniform thickness (5, 7 and 10) cm respectively using ANSYS. The rise to the span ratio has been changed and it was shown that the ANSYS results have discrepancy about 10 % from the theoretical result published in (Billington, 1990). As a result, found that the finite element method is more reliable to design a dome. [3]

(VIT University, 2015) conducted a parametric study of behavior of spherical dome of constant thickness of 0.12 m and span of 10m using ANSYS. The rise to span ratio has been changed from 0 to 4 for comparing the stresses in the dome and found that there is an exponential increase in the membrane stress in the mid-span in every stress plot comparison and there is a significant change in membrane stresses as it makes a transition from non-shallow shell to shallow shell. Also with the decrease in the rise the dome extension ability of dome decreases and a suitable rise to span ratio in the range of 0.13 to 0.16 was suggested for an optimum behavior of shell.[4]

3. Research Objective

The main objective is to the address the effects on the structural performance of spherical shell roofs due to change in rise to span ratio for different domes sizes with different support conditions. Further this thesis aims at comparing the results obtained from theoretical methods with that from finite element method, performed using SAP 2000.

- To determine the moment stress influence in the edge region at the supports.
- To determine the optimal rise based on the structural performance of spherical shells

4. Methods of analysis

4.1 Classical theories

4.1.1 Exact solution approach

Both the membrane theory as well as moment theory has been applied to the spherical shells and involves the following steps:

- Total stresses on a shell element are combined forming equilibrium equations.
- In membrane theory the number of equation equals the number of unknowns thus the equations are sufficient to solve the problem.
- In moment theory the kinematic equations are employed to satisfy the compatibility conditions

and then constitutive relations are built to solve the problem.

- The differential equations are solved to get the solutions and then boundary condition are satisfied to get the resultant stresses and bending moments.

4.1.2 Geckeler's and Hytenyi's approximation approach Procedures

- The particular solution is found by assuming that the load is resisted entirely by the membrane forces.
- The resulting forces and deformation at the boundary condition will generally not be compatible with the solution obtained form the particular solution.
- Thus the additional forces and displacement need to be introduced at the boundaries such that the total solution is compatible with the actual boundary conditions. The additional forces and displacement are called as edge effects
- The magnitude of the edge effect are introduced to eradicate the errors in the particular solution. The solution obtained is called homogeneous solution.

4.2 Finite element analysis

- SAP 2000 version is used for the modeling and analysis of spherical shell roofs with uniformly distributed self-weight. FEM includes geometrical modeling, assigning material properties and support conditions, analyzing and obtaining outputs.

4.3 Optimal rise determination algorithm

- Input the span, initial thickness and the range of rise for which the spherical shells are to be analyzed.
- Considering buckling and minimum thickness criteria the thickness for each rise get refined using iteration techniques.[5]
- For each span there are different rises and the corresponding thickness to analyze the structural performance of spherical dome.
- In case of edge beam the dimension of the edge beam are assigned in proportion to the thickness of the shell.
- Absolute maximum values of meridional moments, meridional stress, Hoop stresses and deflections are obtained for each rise for the spherical dome of fixed spans.
- The plot of rise /span vs log10 of maximum stresses is used to determine the optimal range of rises for the spherical dome with different support conditions.

5. Description of the model

5.1 Geometrical description

Spherical shell roofs of various spans; 6 m, 20 m, and 45 m, has been taken with different thickness and the rise has been varied from 1/3 to 1/9 of its span. The geometrical features consists of radius of curvature of spherical shells and the angle subtended at the center, which are determined form the span and rise of spherical shell.

Table 1: Dimension of the spherical shell roof

SN	Span of the roof(m)	thickness (mm)
1	6	125
2	20	150
3	45	200

5.2 Material properties

Concrete of grade 25 MPa has been selected as construction material. The modulus of elasticity of material is taken as 25000 MPa and possion's ratio is taken as 0.2.

5.3 Loading details

The spherical shell have been analyzed for self-weight only and the weight of the shell is calculated considering the density of concrete as 25 KN/m3.

5.4 Boundary conditions

Following boundary conditions have been preferred for the analysis:

a) Edges clamped rigidly
b) Pinned support with restrained horizontal translation
c) Edges with ring beam

6. Observations

Applying both aforementioned theories to perform the analysis for the three different spans; 6 m, 20 m and 45 m with rise varying from span/3 to span/9 which are supported on three different types of support conditions, observations were obtained:

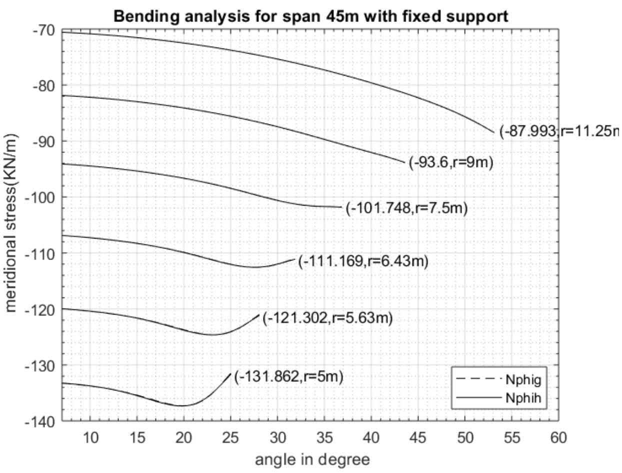

Figure 1: meridional Stresses for clamped support spherical shells with fixed span 45m and different rises

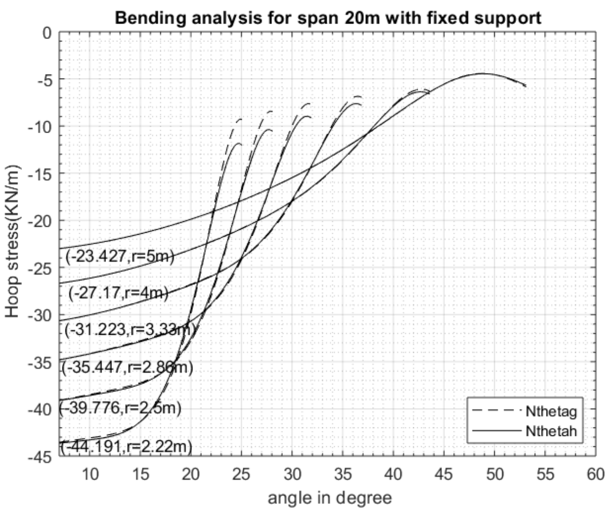

Figure 2: hoop Stresses for clamped support spherical shells with fixed span 45m and different rises

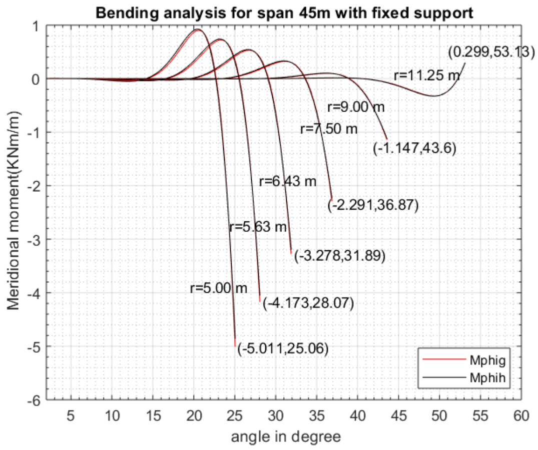

Figure 3: Meridional moment for clamped support with fixed span 45 m and different rises

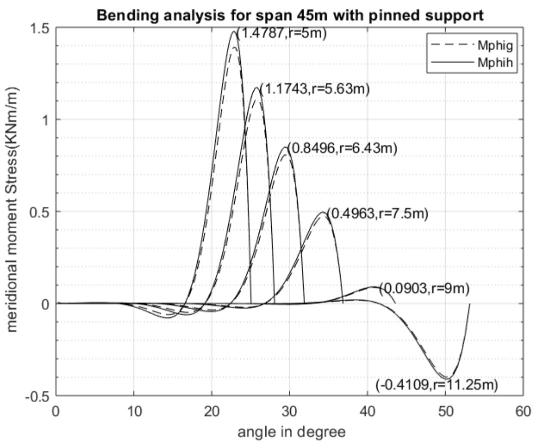

Figure 4: Meridional moment for pinned support spherical shells with fixed span 45m and different rises

Similarly observations for 45 m span shells with edge beam 700 * 900 mm cross-section are:

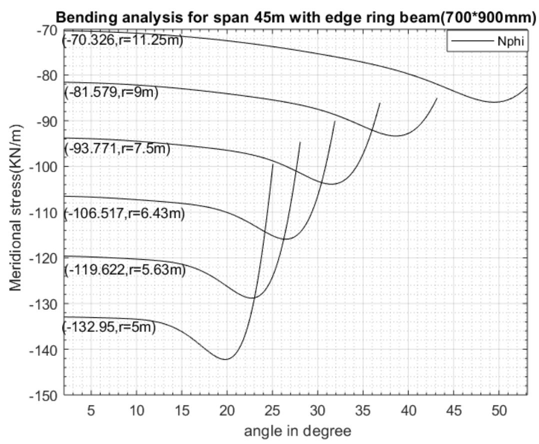

Figure 5: meridional stresses for the spherical shells with span 45 m supported on edge ring beam

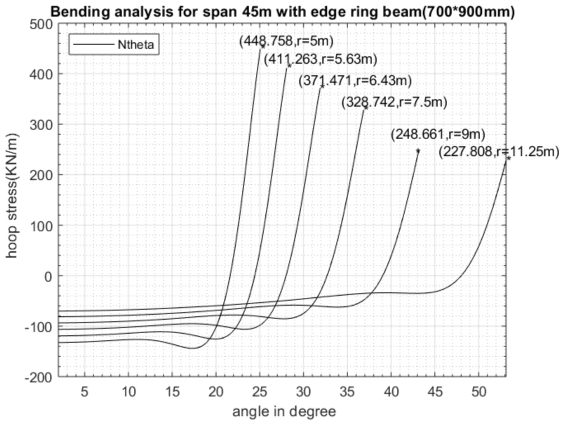

Figure 6: meridional stresses for the spherical shells with span 45 m supported on edge ring beam

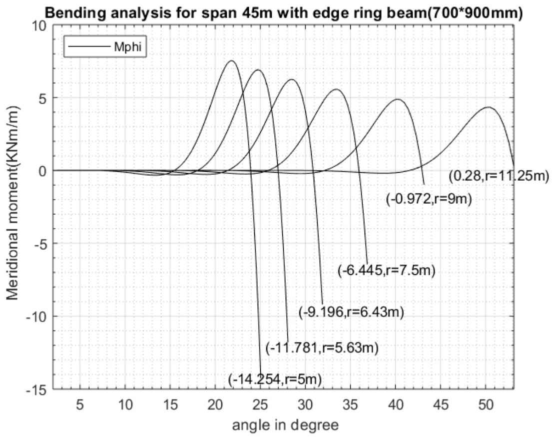

Figure 7: Hoop stresses for the spherical shells with span 45 m supported on edge ring beam

After observing the bending stress seems to be localized in the edge region and the length to which the bending is localized is referred as influence lenght and is obtained by Hoogenboom 2006, l=2.4sqrt(a*t) and after comparing it with the results obtained from actual analysis we get the results as shown in Table 2.

Table 2: Influence length of 45 m span spherical shells with fixed support

rise(m)	observed length(m)	By Hoogenboom(m)
11.25	6.45	6.972
9	7.35	7.5
7.5	7.68	8.05
6.43	7.83	8.578
5.625	8.05	9.089
5	9.27	9.582

Each of the spherical shells has been modeled and FEM analysis has been performed in SAP-2000 and the results were compared with that of the results form theoretical methods and following are some of the results obtained after the comparison for 20 m span and 6 m span shells

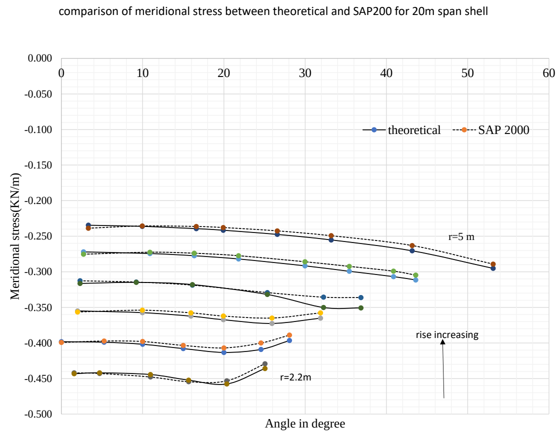

Figure 8: Meridional stress of 20 m span shell from SAP 2000 compared with theroretical method

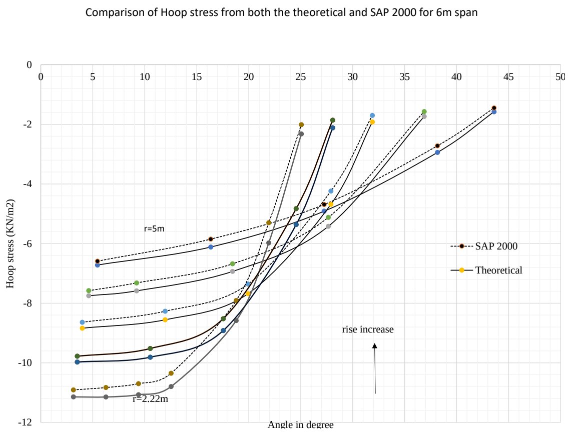

Figure 9: hoop stress of 6 m span shell from SAP 2000 compared with therоretical method

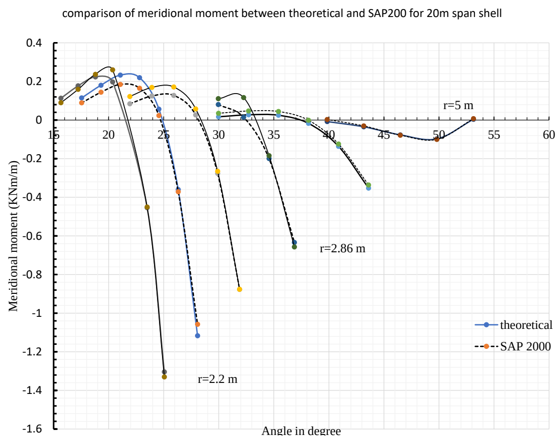

Figure 10: Meridional moment of 45 m span shell from SAP 2000 compared with theroretical method

To obtain optimal rise for a shell, the following results were obtained for 45 m span:

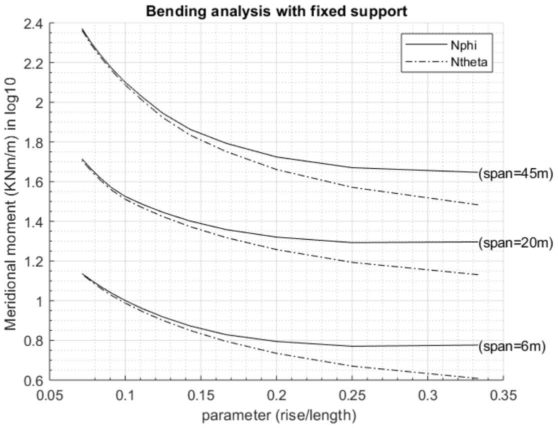

Figure 11: membrane stresses for fixed span spherical shells with different rise/span ratio for fixed support

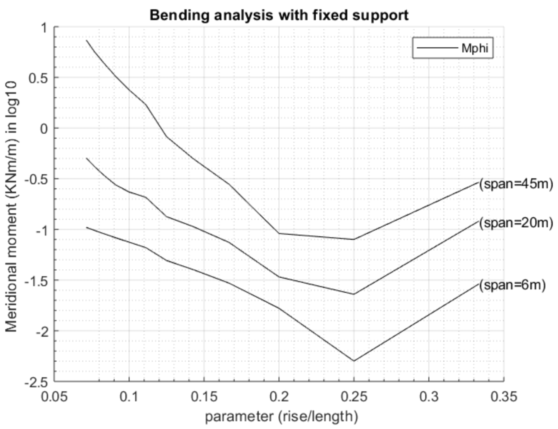

Figure 12: Moment stresses for fixed span spherical shells with different rise/span ratio for pinned support

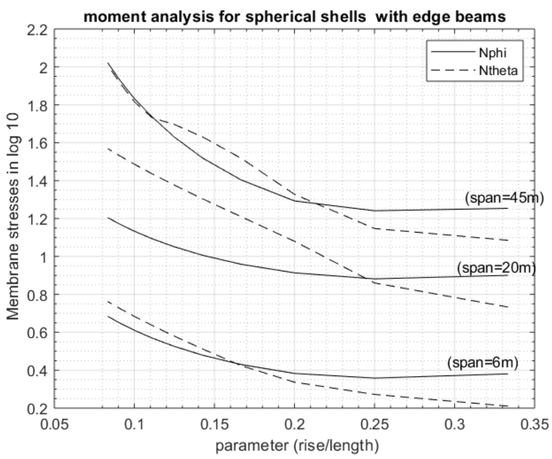

Figure 13: membrane stresses for spherical shells with different rise/span ratio for edge beam with gradually varying thickness

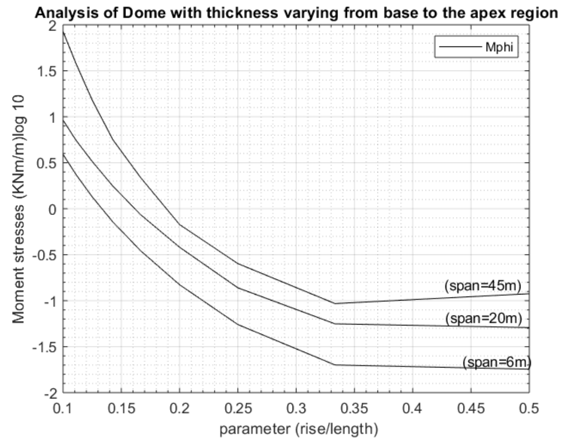

Figure 14: moment stresses for spherical shells with different rise/span ratio for edge beam with gradually varying thickness

7. Discussion

After analyzing all the sets of spherical domes of spans 6 m and 20 m with different rises gives the similar variation as shown in the above observations for 45 m span shell. The following observation can be depicted after completely analyzing all the spherical shells.

1. The membrane stresses; meridional stresses and hoop stress tends to decrease as the rise increases and the maximum value of meridional stress lies in between edge to apex while maximum compression hoop stress lies in apex
2. The rate of change of hoop stress from apex to edge is significant in which the edge is moment dominant as in Figure 2
3. The moment stresses are confined to the edge region only and can be referred as influence length
4. In the case of clamped support, while moving from the apex to the edge the value of bending stress changes its sign form negative moment to positive moment at the edges at a rise of about span /5.
5. Edge beam support condition tends to develop tensile hoop stress in the edge region as well as the moment also increases compared to other types of support.
6. Finite element modeling for clamped and pinned support is is obtained directly by assigning the respective restrained supports while for edge beam spring modeling, with finite stiffness assigned to

spring, was required as the stiffness of the beam influences the stresses in the shell.

7. The influence length as given by hoogenboon 2006 and observed length is observed for all the spherical shells with different support conditions and found that the it is maximum for edge beam support

8. In order to obtained maximum rise the variation of maximum maximum stresses with change in rise from span/2 to span/12 has been observed and the similar results were obtained for all the other spans of shells.

9. As the optimal rise for shells with edge beam ring tends to be more than other support condition, thus for more practical application the thickness has been gradually varied from edge to apex which lowers the economical rise for this case.

8. Results

Following results are deduced from the observations:

1. Both Hytenyi's and Geckeler's method can be employed for the analysis of spherical shells and gives a good approximation.

2. On comparing, the results of both theoretical methods and Finite element method [performed on SAP 2000 gives the results with in the discrepancy of upto 5% for membrane stresses and upto to 10% for moment stresses.

3. The moment stress influence can be well determined from the relation given Hoogenboom,2006.[6]

4. The optimal rise/span ratio based on the structural performance can be obtained at a range of 0.2-0.25 for clamped and pinned support condition while, 0.2- 0.25 for edge beam condition of constant thickness where edge beam dimensions varies in proportion to the thickness of the shell.

5. The optimal rise for the spherical shell with gradually varied thickness constrained with edge beam is found to be 0.17 -0.33. In this case also the dimension of the edge beam are varied in proportion to the shell thickness.

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