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Vibration Response of Pelton Turbine Unit under Rotating Unbalance

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Abstract

This study presents the method to study the forced vibration response of the Pelton turbine unit under rotating unbalance conditions. Rotating unbalance occurs when the center of mass of the rotating system does not coincide with it's geometric center. Due to the inherent imperfections in the manufacturing processes, some residual unbalance will always be present in the Pelton wheel and shaft. In addition to that, the loss of mass due to bucket erosion also contributes to the unbalance in the Pelton wheel. The Pelton wheel is assumed as a rigid disk with lumped unbalance while the shaft, which is assumed as a Euler-Bernaulli beam, is flexible with continuous eccentricity distributions on orthogonal planes. The shaft is simply supported at the ends by rigid bearings. The equations of motion are derived by using Lagrange equation of motion from the energy expressions of the system with the help of assumed modes method. The resulting equations are coupled non-homogeneous ordinary linear differential equations which are solved for the forced response of the system.

Keywords

Pelton turbine, Forced response, Rotating unbalance, Vibration

1. Introduction

Rotating machinery needs to be continuously monitored for faults. Faults may be developed in the system due to various operating conditions or due to the errors in manufacturing. Rotating unbalance is the uneven distribution of mass around an axis of rotation. A rotating body is said to be out of balance when its center of mass (inertial axis) is out of alignment with the center of rotation (geometric axis). This unbalance causes a moment which gives the rotor a wobbling movement characteristics. The combined effect of unequal mass distribution along with the radial acceleration due to rotation creates a centrifugal force which results in force on the bearings and vibrations within the system.

Silt erosion of hydro turbine components is one of the major problems for the efficient operation of hydropower plants. High content of unsettled silt particles passes through the turbines during rainy season resulting in erosion of turbine components. Buckets, nozzle and needle are the most affected parts of the impulse turbine [1]. Based upon 20 years of river discharge data *(1960s to 1980s)*, it was found that the correlation between water discharge and silt discharge was statistically highly significant in the river systems of Nepal [2]. The himalayan rivers contain excessive quantities of sediment in the form of hard abrasive particles. Turbines components exposed to sand-laden water are subjected to erosion causing reduction in efficiency and life of the turbine [3]. Thus, the problem of siltation is one of the major issues in the rivers of Nepal.

The silt particles present in the rivers may erode the buckets of Pelton turbine. Due to the imperfections in the manufacturing processes, it is not possible to have a perfectly balanced shaft and runner-bucket assembly. So, the combined effect, that results, is the uneven distribution of mass in the shaft and runner-bucket assembly. This uneven mass distribution will offset the center of rotation of both the shaft and runner-bucket assembly, giving rise to a problem of eccentric rotation. During eccentric rotation, various unbalanced forces and moments will act on the system.

Very recently, some researchers have been actively involved in the research works relating to the dynamic response of Pelton turbine unit [4, 5, 6]. Analyzing for the first mode of vibration, they have studied the free vibration [4] and forced vibration [5] response considering the shaft as a Euler-Bernaulli beam. The complete free and forced vibration analysis has also been done considering the shaft to be a Timoshenko shaft [6]. However, the researchers have not considered rotating unbalance in their mathematical models. This paper builds upon their works by adding the effect of unbalances in the mathematical model.

Rotating unbalance creates unnecessary vibrations, unwanted noise, excessive stress in the turbine parts and reduces the overall reliability of the machine. So, a clear understanding of the dynamics of the Pelton unit under these unbalanced conditions is necessary. Thus, this paper intends to provide a mathematical model of the Pelton unit for the first mode of vibration under the conditions of rotating unbalance.

2. Mathematical Modelling

2.1 Assumptions

The mathematical model is developed only for transverse vibrations. Axial and torsional vibrations are not considered in the model. The system is represented as a rigid disk with lumped unbalance which is fitted on a flexible shaft with continuous eccentricity distributions on orthogonal planes. The shaft is simply supported by rigid bearings at the ends. Non-linearity is not considered in the analysis.

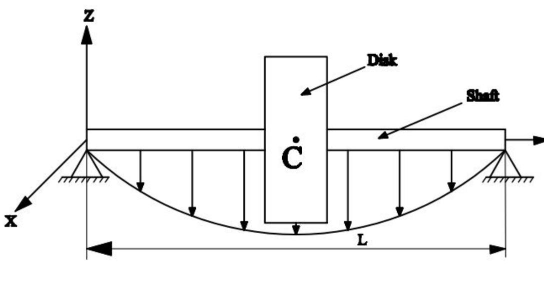

Figure 1: Shaft and Disk assembly with supports [4]

2.2 Energy expressions of the system

Let the coordinates of the center of rotating system be $u(y,t)$, v and $w(y,t)$ with respect to the fixed inertial frame. Since, the disk is considered rigid, only kinetic energy will be associated with it. On the contrary, both kinetic and strain energy will be associated with the shaft since it is considered flexible.

2.2.1 Angular velocity of the runner-buckets assembly

Using the rotation matrix to relate the inertial frame coordinates and body frame coordinates, the components of angular velocity of the runner-buckets assembly is found as:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \approx \begin{bmatrix} -\dot{\phi}\sin\psi + \dot{\theta}\cos\psi \\ \dot{\phi}\theta + \dot{\psi} \\ \dot{\phi}\cos\psi + \dot{\theta}\sin\psi \end{bmatrix} \tag{1}$$

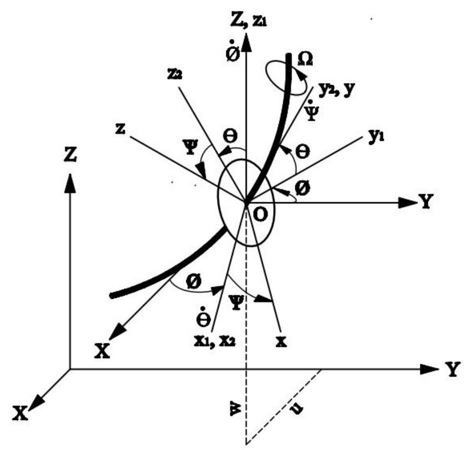

Figure 2: Disk rotating on flexible shafts with reference frames [5]

2.2.2 Kinetic energy of the disk

Let M_D be the mass of the disk, I_{Dxx}, I_{Dyy} and I_{Dzz} be the moment of inertia about X, Y and Z axis respectively. Let $\psi = \Omega$ be the angular velocity of the shaft. Then, the kinetic energy of the disk is given as [7]:

$$T_D = \frac{1}{2}M_D(\dot{u}^2+\dot{w}^2)+\frac{1}{2}[I_{Dxx}\omega_x^2+I_{Dyy}\omega_y^2+I_{Dzz}\omega_z^2] \tag{2}$$

The cross section of the disk is symmetrical which gives $I_{Dxx} = I_{Dzz}$. Then, using equation (1), we have,

$$T_D = \frac{1}{2}M_D(\dot{u}^2+\dot{w}^2)+\frac{1}{2}I_{Dxx}(\dot{\theta}^2+\dot{\phi}^2) + \frac{1}{2}I_{Dyy}(\Omega^2+2\dot{\phi}\theta\Omega) \tag{3}$$

2.2.3 Kinetic energy of the unbalanced mass in disk

Let M_u be the unbalanced mass situated at the distance d from the center of the disk. The expression for the kinetic energy of the unbalanced disk is given as:

$$T_{ud} = \frac{M_u}{2}(\dot{u}^2 + \dot{w}^2 + 2\Omega d\dot{u}\cos\Omega t - 2\Omega d\dot{w}\sin\Omega t + d^2\Omega^2) \quad (4)$$

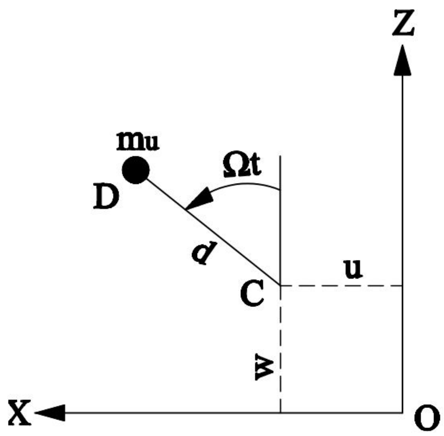

Figure 3: Mass unbalance in disk

2.2.4 Kinetic energy of the shaft

Let ρ be the density of the material of the shaft, A be the cross-sectional area of the shaft, I_1 and I_2 be the polar and diametrical area moment of inertia. The kinetic energy of the shaft is given by:

$$T_S = \frac{\rho A}{2}\int_0^L (\dot{u}^2 + \dot{w}^2)\,dy + \frac{\rho I_2}{2}\int_0^L \omega_x^2\,dy + \frac{\rho I_1}{2}\int_0^L \omega_y^2\,dy + \frac{\rho I_2}{2}\int_0^L \omega_z^2\,dy \quad (5)$$

where,

$$I_1 = \int_A (x^2 + z^2)\,dA$$
$$I_2 = \int_A x^2\,dA = \int_A z^2\,dA$$

Let $I_2 = I$, then $I_1 = 2I$. Then, using equation (1), the above expressions changes to,

$$T_S = \frac{\rho A}{2}\int_0^L (\dot{u}^2 + \dot{w}^2)\,dy + \frac{\rho I}{2}\int_0^L (\dot{\theta}^2 + \dot{\phi}^2)\,dy + \rho I L\Omega^2 + 2\rho I\Omega\int_0^L \theta\dot{\phi}\,dy \quad (6)$$

2.2.5 Kinetic energy of the unbalanced shaft

If $\mu(y)$ be the mass per unit length of the shaft, the kinetic energy resulting from the eccentricity distribution of the unbalanced shaft is:

$$T_{us} = \int_0^L \frac{\mu(y)}{2}[\dot{u}^2 + \dot{w}^2 + 2\,\dot{u}\,\Omega\,[e_z(y)\cos\Omega t - e_x(y)\sin\Omega t] - 2\,\dot{w}\,\Omega\,[e_z(y)\sin\Omega t + e_x(y)\cos\Omega t] + \Omega^2\,(e_x^2(y) + e_z^2(y))]\,dy$$

The mass per unit length is assumed constant. So, with that assumption, we can write,

$$T_{us} = \frac{\mu(y)}{2}\int_0^L [\dot{u}^2 + \dot{w}^2 + 2\,\dot{u}\,\Omega\,[e_z(y)\cos\Omega t - e_x(y)\sin\Omega t] - 2\,\dot{w}\,\Omega\,[e_z(y)\sin\Omega t + e_x(y)\cos\Omega t] + \Omega^2\,(e_x^2(y) + e_z^2(y))]\,dy \quad (7)$$

where,

$e_x(x)$ is the eccentricity distribution with respect to X-axis.
$e_z(x)$ is the eccentricity distribution with respect to Z-axis.

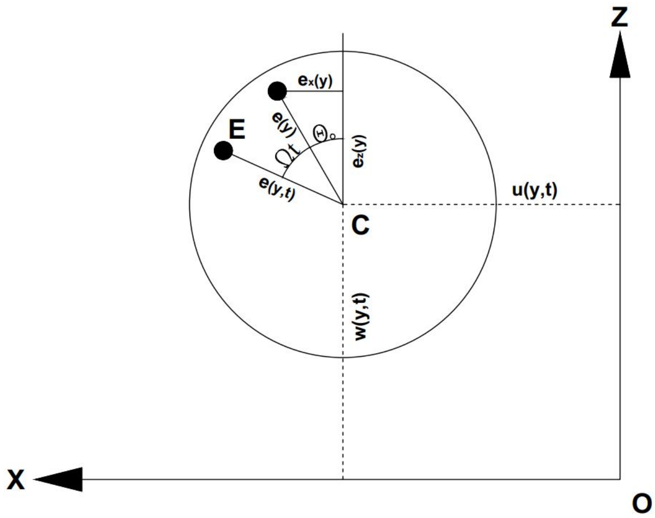

Figure 4: Mass unbalance in shaft

2.2.6 Strain energy of the shaft

If E be the modulus of elasticity of the shaft material, the expression for the strain energy of the shaft is given by:

$$U_S = \frac{EI}{2}\int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] dy \quad (8)$$

3. Equations of Motion

3.1 Assumed Modes Method

Assumed modes method is generally used to solve the forced vibration problem. In this method, the solution of the vibration problem of the continuous system is assumed in the form of series composed of linear admissible functions ϕ_i, which are functions of the spatial coordinates, multiplied by time-dependent generalized coordinates $q_i(t)$. ϕ_i are known trial functions that satisfy the geometry boundary conditions and $q_i(t)$ are unknown functions of time, also called generalized coordinates.

3.1.1 Displacement functions for the first mode of vibration

Using assumed modes method, let us assume the displacement functions as:

$$\begin{aligned} u(y,t) &= f(y)q_1(t) = f(y)q_1 \\ w(y,t) &= f(y)q_2(t) = f(y)q_2 \end{aligned} \tag{9}$$

where, q_1 and q_2 are generalized independent coordinates.

3.1.2 Choice of mode shape

The mode shape for simply supported beam is taken as:

$$f(y) = \sin\left(\frac{\pi y}{L}\right) \tag{10}$$

3.2 Lagrange equations of motion

Lagrange equations are used to derive the governing differential equations of motion for both linear and non-linear systems. It represents the equations of motion in terms of its generalized coordinates and can be solely obtained from the kinetic energy and potential energy of the system.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = 0 \qquad i = 1, 2, ..., n \tag{11}$$

The use of equation (11) leads for each value of i leads to n independent differential equations whose solution provides the time-dependent response of the system.

Using equations (9) and (10) in the energy expressions, the equations of motion in the following form is obtained from lagrange equations of motion:

$$\begin{aligned} M\ddot{q}_1 - C\,\Omega\,\dot{q}_2 + Kq_1 &= -\frac{dA}{dt} \\ M\ddot{q}_2 + C\,\Omega\,\dot{q}_1 + Kq_2 &= \frac{dB}{dt} \end{aligned} \tag{12}$$

where,

$$\begin{aligned} M &= \left[M_D + M_u + \frac{\rho AL}{2} + \frac{\rho I \pi^2}{2L} + \frac{\mu(y)L}{2}\right] \\ A &= M_u\, d\,\Omega \cos\Omega t + \mu(y)\,\Omega \int_0^L (e_z(y)\cos\Omega t \\ &\quad - e_x(y)\sin\Omega t) f(y)\,dy \\ B &= M_u\, d\,\Omega \sin\Omega t + \mu(y)\,\Omega \int_0^L (e_z(y)\sin\Omega t \\ &\quad + e_x(y)\cos\Omega t) f(y)\,dy \\ C &= \frac{\rho I \pi^2}{L} \end{aligned} \tag{13}$$

3.3 Equations of motion for different cases

The equations of motions considering disk only, shaft only and both the disk and shaft will be developed in this section.

3.3.1 Case I: Inherent mass unbalance in disk

For this case, equation (13) becomes:

$$\begin{aligned} M_1 &= \left[M_D + M_u + \frac{\rho AL}{2} + \frac{\rho I \pi^2}{2L}\right] \\ A_1 &= M_u\, d\,\Omega \cos\Omega t \\ B_1 &= M_u\, d\,\Omega \sin\Omega t \end{aligned} \tag{14}$$

So, the equations of motion in this case are:

$$\begin{aligned} M_1\,\ddot{q}_1 - C\,\Omega\,\dot{q}_2 + Kq_1 &= M_D\, e\,\Omega^2 \sin\Omega t \\ M_1\,\ddot{q}_2 + C\,\Omega\,\dot{q}_1 + Kq_2 &= M_D\, e\,\Omega^2 \cos\Omega t \end{aligned} \tag{15}$$

where, $e = \dfrac{M_u d}{M_D}$, is the eccentricity of the disk.

3.3.2 Case II: Mass unbalance present in the shaft

For this case, equation (13) becomes:

$$\begin{aligned} M_2 &= \left[M_D + \frac{\rho AL}{2} + \frac{\rho I \pi^2}{2L} + \frac{\mu(y)L}{2}\right] \\ A_2 &= \mu(y)\,\Omega \int_0^L (e_z(y)\cos\Omega t - e_x(y)\sin\Omega t) f(y)\,dy \\ B_2 &= \mu(y)\,\Omega \int_0^L (e_z(y)\sin\Omega t + e_x(y)\cos\Omega t) f(y)\,dy \end{aligned} \tag{16}$$

So, the equations of motion in this case are:

$$\begin{aligned} M_2\,\ddot{q}_1 - C\,\Omega\,\dot{q}_2 + Kq_1 &= I_1\,\Omega^2 \sin\Omega t + I_2\,\Omega^2 \cos\Omega t \\ M_2\,\ddot{q}_2 + C\,\Omega\,\dot{q}_1 + Kq_2 &= -I_2\,\Omega^2 \sin\Omega t + I_1\,\Omega^2 \cos\Omega t \end{aligned}$$

(17)

where,

$$I_1 = \mu(y) \int_0^L e_z(y) f(y)\, dy$$
$$I_2 = \mu(y) \int_0^L e_x(y) f(y)\, dy \tag{18}$$

3.3.3 Case III: Mass unbalance present in both disk and shaft

For this case, equation (13) becomes:

$$M_3 = \left[M_D + M_u + \frac{\rho AL}{2} + \frac{\rho I \pi^2}{2L} + \frac{\mu(y)L}{2} \right]$$
$$A_3 = M_u\, d\, \Omega \cos\Omega t + \mu(y)\, \Omega \int_0^L (e_z(y)\cos\Omega t - e_x(y)\sin\Omega t) f(y)\, dy$$
$$B_3 = M_u\, d\, \Omega \sin\Omega t + \mu(y)\, \Omega \int_0^L (e_z(y)\sin\Omega t + e_x(y)\cos\Omega t) f(y)\, dy \tag{19}$$

So, the equations of motion in this case are:

$$M_3\, \ddot{q}_1 - C\,\Omega\, \dot{q}_2 + K q_1 = (M_D\, e + I_1)\, \Omega^2 \sin\Omega t + I_2\, \Omega^2 \cos\Omega t$$
$$M_3\, \ddot{q}_2 + C\,\Omega\, \dot{q}_1 + K q_2 = (M_D\, e + I_1)\, \Omega^2 \cos\Omega t - I_2\, \Omega^2 \sin\Omega t \tag{20}$$

4. Solutions of Equations of Motion

4.1 Inherent mass unbalance in disk

The solutions to equations (15) are,

$$q_1 = \frac{M_D\, e\, \Omega^2}{K + (C - M_1)\Omega^2} \sin\Omega t$$
$$q_2 = \frac{M_D\, e\, \Omega^2}{K + (C - M_1)\Omega^2} \cos\Omega t \tag{21}$$

4.2 Mass unbalance present in shaft

The solutions to equation (17) are:

$$q_1 = \frac{I_1\, \Omega^2}{(C - M_2)\, \Omega^2 + K} \sin\Omega t + \frac{I_2\, \Omega^2}{(C - M_2)\, \Omega^2 + K} \cos\Omega t$$
$$q_2 = -\frac{I_2\, \Omega^2}{(C - M_2)\, \Omega^2 + K} \sin\Omega t + \frac{I_1\, \Omega^2}{(C - M_2)\, \Omega^2 + K} \cos\Omega t \tag{22}$$

4.3 Mass unbalance present in both disk and shaft

The solutions of equations (20) are:

$$q_1 = \frac{(M_D\, e + I_1)\, \Omega^2}{(C - M_3)\, \Omega^2 + K} \sin\Omega t + \frac{I_2\, \Omega^2}{(C - M_3)\, \Omega^2 + K} \cos\Omega t$$
$$q_2 = -\frac{I_2\, \Omega^2}{(C - M_3)\, \Omega^2 + K} \sin\Omega t + \frac{(M_D\, e + I_1)\, \Omega^2}{(C - M_3)\, \Omega^2 + K} \cos\Omega t \tag{23}$$

5. Results and Discussion

The calculations are done for the Pelton turbine unit rated at 1500 RPM. The basic data for disk and shaft is shown in the following table:

Table 1: Basic data for disk and shaft

Parameters	Values
Inner Diameter of disk, D_1	0.01 m
Outer Diameter of disk, D_2	0.155 m
Density of disk, ρ_d	7860 kg/m^3
Width of the disk, h	0.03 m
Diameter of the shaft, D	0.01 m
Density of shaft, ρ	7750 kg/m^3
Length of the shaft, L	0.4 m
Modulus of elasticity of shaft, E	207 GPa

Basic data for mass unbalance in disk:

Mass unbalance in disk, M_u = 0.0001477 kg

Distance of mass unbalance in disk, d = 0.03 m

Basic data for mass unbalance in shaft [8]:

$e_x(y) = 0.00533\, y^3$ - $0.0016\, y^2 + 0.000107\, y$ - 1×10^{-6}

$e_z(y) = 0.00533\, y^3$ - $0.0016\, y^2 + 0.000107\, y$ - 1×10^{-6}

5.1 Basic parameters for calculation

Using the values above, the parameters in the equations are:

$$A = 7.8539 \times 10^{-5}\, m^2 \qquad e = 1 \times 10^{-6}\, m$$
$$C = 9.3866 \times 10^{-5}\, kg \qquad \Omega = 157.08\, rad\, s^{-1}$$
$$I = 4.9087 \times 10^{-10}\, m^4 \quad M_D = 4.43084\, kg$$
$$I_1 = 1.728 \times 10^{-6} \qquad I_2 = 1.728 \times 10^{-6}$$
$$M_1 = 4.55273\, kg \qquad M_2 = 4.67431\, kg$$
$$M_3 = 4.67446\, kg$$
$$I_{Dxx} = 7.0132 \times 10^{-3}\, kg\, m^2$$
$$I_{Dyy} = 13.3616 \times 10^{-3}\, kg\, m^2$$

5.2 Time response at different shaft stations

The equations (21), (22) and (23) are used to calculate the time response in the transverse directions. Using these equations in equation (9), we get the complete response of the displacement functions both as a function of space and time. From that relation, we can compute the time response at different shaft stations.

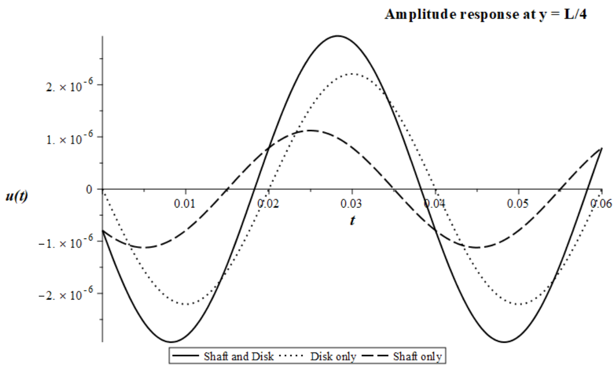

Figure 5: Transverse displacement at y = L/4 in horizontal direction for $\Omega = 1500\, RPM$

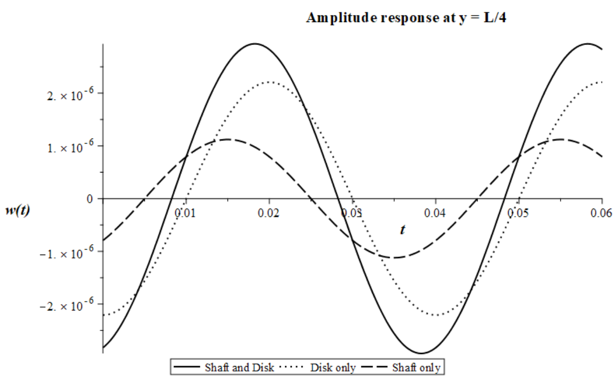

Figure 6: Transverse displacement at y = L/4 in vertical direction for $\Omega = 1500\, RPM$

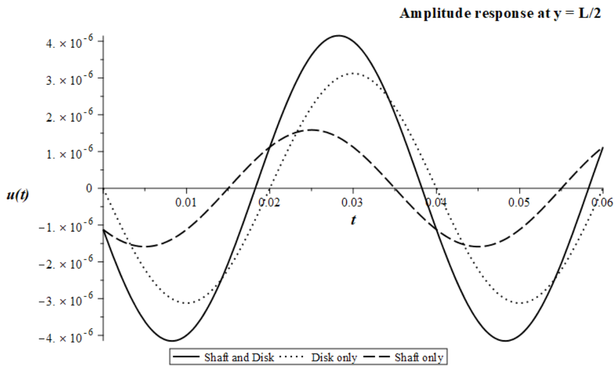

Figure 7: Transverse displacement at y = L/2 in horizontal direction for $\Omega = 1500\, RPM$

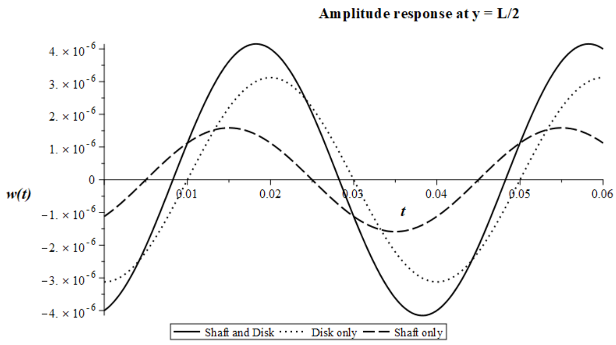

Figure 8: Transverse displacement at y = L/2 in vertical direction for $\Omega = 1500\, RPM$

The observation of plots from figures (5-8) reveals that the nature of vibrations in both tranverse directions is sinusoidal but with a certain phase difference with each other. These are the amplitude response when the system is rotating freely, for instance, by using an electric motor. These are the vibrations which will always be present in the rotating system in the absence of any external forces unless proper balancing is done.

The difference in the amplitudes of vibration can also be clearly observed by comparing the responses at the quarter length and mid length of the shaft. At the quarter length of the shaft, the influence of disk unbalance is very less and, thus, shaft unbalance is dominant there. However, as we move towards the mid-length of the shaft, the disk unbalance becomes very dominant and, hence, the amplitudes of vibration increases.

The amplitude levels of vibration observed in this way will provide us an insight on how serious is the problem of rotor balancing. If the amplitudes of vibration are too large, then, the balancing must be done as soon as possible.

6. Conclusion

In this paper, forced response of the Pelton turbine unit was studied by considering Pelton wheel as a rigid disk with lumped unbalance, and shaft, as a Euler-Bernaulli beam, with continuous eccentricity distributions in orthogonal planes. The governing differential equations for the first mode of vibration were found to be a coupled non-homogeneous ordinary linear differential equations and were solved to get the displacement functions. For the operating speed of $\Omega = 1500\,RPM$, in both tranverse directions, the peak amplitude at the midspan of the shaft was found to $3.123\mu m$ considering unbalance in the disk only, $1.586\mu m$ considering unbalance in the shaft only and $4.152\mu m$ considering unbalance in both disk and shaft.

Future Enhancements

This work can be further extended to include the unbalance in the Pelton wheel due to the loss of mass during bucket erosion. Moreover, flexible bearings and flexible disks can be considered to give more accurate results.

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