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Dynamic Response of Overhung Pelton Turbine Unit for Forced Vibration

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Abstract

This research work was carried out to model the excitation force imparted by water jet in the form of Fourier series and to determine the forced response of cantilever type Pelton turbine unit analytically by mathematical model development. Also, the analytically calculated amplitude of forced vibration was compared with result from the ANSYS simulation and the effect of rotational speed of shaft to the amplitude of vibration was evaluated.The mathematical model was developed by calculating the kinetic energy of the disk and both kinetic energy and potential energy of the shaft. Rayleigh-Ritz method was used to exact natural mode of vibration and Lagrange's equation to derive the equation of motion for forced condition. Fourier analysis was done to obtain the function in its exact form. The developed methodologies were followed to find the analytical solution of dynamic response of selected Pelton turbine unit of 1 kW rated at 1500 RPM. A rigid disk (runner and buckets assembly) was situated along the end of flexible shaft with rigid and undamped simply supported bearings at another end of the shaft. Analytically, the amplitudes of vibration of Pelton Turbine unit with single nozzle in X direction (the direction of water jet) and Z direction were determined and they are compared with the values generated from the ANSYS simulation.

Keywords

ANSYS SOFTWARE, RAYLEIGH-RITZ METHOD, DYNAMIC RESPONSE FOURIER

1. Introduction

Rotors and rotating parts are key components with vital role in various engineering applications like pump, compressors, turbines, generators, fans, marine drives and many more. The study of dynamic behavior of these rotors and rotating parts have significant role to understand the operating mechanism and failures associated with vibration. Rotor dynamics study the lateral/transverse (bending), longitudinal (axial), and torsional vibration of the rotating shafts with the objective of limiting the vibration under an acceptable range. Transverse is mode of rotor dynamics associated with bending of the rotor and similarly longitudinal is the motion in axial direction and torsional is twisting around its own axis of the rotor [1].

Parts of rotor-blade systems are subjected to highly hostile working conditions. Thus, the design and manufacturing challenge is concerned with the improvement in performance, life span and weight reduction without loss of reliability. The possible forces responsible for the vibration increase in a hydro turbine may be mechanical, hydraulic or electrical [2]. These forces may be mechanical excitations, centrifugal forces due to imbalance of the rotating mass i.e., runner, shaft, and generator rotor, elastic force of the shaft due to incorrect shaft alignment, frictional forces, oil-film instability in bearing, hydraulic excitations, flow through waterways; non-uniform velocity distributions in various waterways of the turbine cause hydraulic unbalance and pressure fluctuations in the penstock and electrical excitations etc [3]. Under such condition, their behavior can be studied and predicted to some extent by appropriate analysis of their dynamic response through proper mathematical modeling of physical system.

Hydro powers are most prominent sources of renewable energy for electricity generation in various water resources rich countries and must remain major source of energy for the sustainable development of country as it is renewable and clean source. For sustainable supply of the electricity from hydropower, efficiency of the overall system must be maximized; the reliability and life of the each component should be increased. Thus, study of dynamic response of the Pelton turbine unit, one of the widely used water turbines worldwide, may contribute for the improvement in the performance as well as reliability, stability and the life span of the components of the hydraulic power system.

The research work is mainly targeted to do the following:

- To develop the governing equation for the shaft of an overhang Pelton turbine unit for forced vibration by applying Lagrange's Equation.
- To determine the vibration amplitudes for operating conditions of the unit by analytical methods and software model then compare the results obtained under forced vibration condition.

Assumptions

- The model is assumed to be linear and discrete system for most of the models.
- Pelton turbine unit is the combination of shaft-runner-buckets-flexible bearings system but flexible shaft-rigid disk-rigid bearing are considered for model development to reduce the level of complexities.

2. Methods and materials

2.1 Literature Review

The past research works which are published on the international and national journals and the past theses on the related field will be collected and studied thoroughly from internet and libraries.

2.2 Mathematical Model Development and solution

Governing equation for the dynamic behavior of the Pelton turbine unit will be developed by using Lagrange's Equation. The simple approach to rotor dynamics study is rotor/bearing system that is generally known as the Fö ppl/Jeffcot rotor, or simply Jeffcot rotor, which is often used to evaluate the complex rotor-dynamic systems in the real world. The model will be developed considering the shaft – disk (runner)/buckets - bearing system as Föppl/Jeffcot rotor, i.e. single rigid disk mounted on axial center of a circular flexible shaft, which is supported by rigid bearings at each ends. Buckets are uniformly distributed to the periphery of runner/wheel of the Pelton turbine, and the impact, velocity, acceleration distribution profile and time interval between the jets striking the buckets is almost constant. Thus, runner - buckets combination is treated as rigid disk.

2.2.1 Rotational matrix

Any rotation can be described by three successive rotations about linearly independent axes and these rotations are Euler angles. The positions, angular velocities and angular accelerations of a body that rotates about a fixed point, such as a gyroscope, and body that rotates about its center of mass (an aircraft, shaft of turbine etc.) can be described by Euler's angles [4].

X, Y and Z is fixed inertial frame and x, y and z is the body fixed axis. Firstly, the rotation is counter clockwise from an initial XYZ system about the Z, z_1 axis as shown in Figure 2.1(a) into x_1, y_1, z_1 system by an angle ϕ.

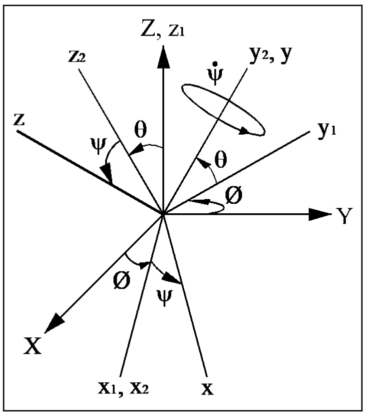

Figure 1: rotational matrix for 312 euler angles

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$$\begin{bmatrix} \cos\phi\cos\psi - \sin\phi\sin\theta\sin\psi & \sin\phi\cos\psi \\ -\sin\phi\cos\theta & \cos\phi\cos\theta \\ \cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi - \cos\phi\sin\theta\cos\phi \end{bmatrix}$$

$$\begin{bmatrix} -\cos\theta\sin\phi \\ \sin\theta \\ \cos\phi \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (1)$$

For the system considered, the spinning axis is Y and angular motion about X and Z axes are comparatively small (A time-domain). Thus, $\cos\theta \approx 1, \cos\phi \approx 1, \sin\theta \approx \theta \sin\phi \approx \phi$, and then,
angular velocity of XYZ Frame [5]

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -\dot{\phi}\sin\psi + \dot{\theta}\cos\psi \\ \dot{\phi}\theta + \dot{\psi} \\ \dot{\phi}\cos\psi + \dot{\theta}\sin\psi \end{bmatrix} \quad (2)$$

2.3 Method of analytical solution

Finite-element methods (FEM) are based on some mathematical physics techniques and the most fundamental of them is the so-called Rayleigh-Ritz method which is used for the solution of boundary value problems. In the Rayleigh-Ritz (RR) method we solve a boundary-value problem by approximating the solution with a linear approximation of basic functions [6]. The method is based on a part of mathematics called calculus of variations. In this method we try to minimize a special class of functions called functional.

2.4 Shape function

[5] The shape function is the function which interpolates the solution between the discrete values obtained at the mesh nodes. Therefore, appropriate functions have to be used and, as already mentioned, low order polynomials are typically chosen as shape functions. In this work trigonometric shape functions are used. $f(y)=$
$\{\cosh\beta y - \cos\beta y\} - \frac{\cosh\beta l + \cos\beta l}{\sinh\beta l + \sin\beta l}(\sinh\beta y - \sin\beta y)$
Where,
$\beta l = 1.875$

2.4.1 Mathematical modelling of the jet force

$F_j = \rho_w \times A_j \times V_1 (V_{w1} + V_{w2})$
ρ_w is the density f water= 1000 Kg/m^3
A_j is the area of the water jet
V_1 is the velocity of the water jet
V_{w1} is the component of the velocity of the jet in the direction of motion
V_{w2} is the component of the velocity of the jet in the direction of vane And,
then,
$V_{w1} = V_1$
$V_{w2} = K(V_1 - u_1)\cos\phi_1 - u_1$

'k' is the blade friction coefficient
'ϕ_1'is the vane angle at oulet which is 15 degree in or case
'u_1' is the circumferential velocity of the runner
$f(t) = 478.4 + \sum_{n=0}^{\infty}\left[\frac{23920}{n\omega} \times \sin 0.04 n\omega . \cos n\omega t - \frac{23920}{n\omega}\{(\cos 0.04 n\omega - 1) \times \sin \omega t\}\right]$ Considering the first five terms of the fourier series the forcing function is given as [7] :
$f(t) = 478.4 + \frac{23920}{\omega} \times \sin 0.04\omega \times \cos\omega t + \frac{11960}{\omega} \times \sin 0.08\omega \times \cos 2\omega t$
$+ \frac{7973.33}{\omega} \times \sin 0.12\omega \times \cos 3\omega t + \frac{5980}{\omega}\sin 0.16\omega \times \cos 4\omega t + \frac{4784}{\omega}\sin 0.12\omega \times \cos 5\omega t$
$- \frac{23920}{\omega}(\cos 0.04\omega - 1)\sin 4\omega t - \frac{11960}{\omega}$
$(\cos 0.08\omega - 1)\sin 2\omega t$
$- \frac{7973.33}{\omega}(\cos 0.12\omega - 1)\sin 3\omega t$ $-$
$\frac{5980}{\omega}(\cos 0.16\omega - 1)\sin 4\omega t - \frac{4784}{\omega}(\cos 0.2\omega - 1) \times \sin 5\omega t$

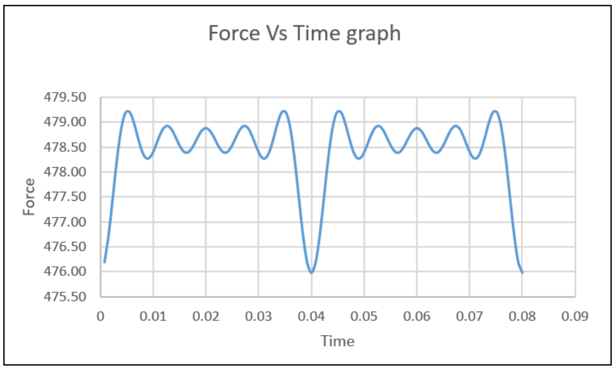

Figure 2: Graphical Representation of the Jettforce

2.5 Kinetic energy of the disk

[8] Thus, the kinetic energy of the disk is given by,
$T_D = \frac{1}{2}M_D(\dot{u}^2 + \dot{w}^2) + \frac{1}{2}\left[I_{Dxx}\omega_x^2 + I_{Dyy}\omega_y^2 + I_{Dzz}\omega_z^2\right]$
Where,
T_D – the kinetic energy of disk.
M_D –the mass of the disk.
I_{Dxx}, I_{Dyy}, and I_{Dzz} are the moment of inertia about the principal axes X, Y and Z respectively.
As the disk is assumed to be symmetrical, $I_{Dxx} = I_{Dyy}$.

2.6 Shaft

2.6.1 Kinetic Energy of the shaft

The K.E. of the shaft is defined for an element and integrated over the length of the shaft 'L'. The K.E. of

the shaft is given by the expression [8],
$T_s = \frac{\rho A}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dy + \frac{\rho l_{xx}}{2}\int_0^1 \omega_x^2 dy + \frac{\rho l_{yy}}{2}\int_0^1$

$$\omega_y^2 dy + \frac{\rho l_{zz}}{2}\int_0^1 \omega_z^2 dy \quad (3)$$

Where,
T_S – kinetic energy of the shaft,
ρ – the mass per unit volume,
A – the cross-sectional area of shaft and it is assumed to be constant,
I – the area moment of inertia of the shaft cross-section about the neutral axis and it is also supposed to be constant. Total kinetic energy of shaft and disc
$T = \frac{1}{2}M_D\left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}l_{Dxx}\left(\dot{\theta}^2 + \dot{\phi}^2\right) + \frac{1}{2}I_{Dyy}\left(\Omega_2 + 2\dot{\phi}\theta\Omega\right) + \frac{\rho A}{2}\int_0^1 \left(\dot{u}^2 + \dot{w}^2\right) dy +$

$$\frac{\rho l}{2}\int_0^1 \left(\dot{\theta}^2 + \dot{\phi}^2\right) dy + \rho IL\Omega^2 + 2\rho l\Omega\int_0^1 \theta\dot{\phi} dy \quad (4)$$

2.6.2 Potential Energy of the Shaft

Total potential energy of system is
Total potential energy of the system is [9],
$U = \frac{EI}{2}\int_0^1 \left[\left(\frac{\partial^2 u}{\partial y^2}\right) + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] dy$

$$U = \frac{EI}{2}\left[\int_0^L h^2(y)\, U^2 dy + \int_0^L h^2(y)\, W^2 dy\right] \quad (5)$$

= $\frac{EI}{2} \times 5623.26\left(U^2 + W^2\right)$
Where,
k eq =5623.26 EI
The bearings at the support are considered as rigid and isotropic with negligible damping

2.7 Total kinetic energy of shaft disc assembly

Therefore, the kinetic energy of the disk-shaft assembly is,
$T = T_D + T_s$
$=\left[\frac{1}{2}(4M_D + 450.3I_{Dxx} + 0.101\rho A + 35.7\rho I)\left(\dot{U}^2 + \dot{W}^2\right)\right.$ $-(450.3I_{Dyy} + 71.5\rho I)\,\Omega\dot{U}\dot{W}$

$$T = \frac{1}{2}m\left(\dot{U}^2 + \dot{W}^2\right) - a\Omega\dot{U}\dot{W} \quad (6)$$

Meq. = ($4M_D$ + 450.3 I_{Dxx} + 0.101 ρ A +35.76ρ I)
a = 450.3 I_{Dyy}+ 71.52 ρ I

2.8 Lagrangian equation

$$L = T_T - U_s \quad (7)$$

Where, T is the kinetic energy and U is the potential energy of the system.

The constant terms appearing in the expressions of K.E. and P.E. are systematically removed as their contribution to the Lagrange equation is nil.

Using Lagrange's equation for the system of rigid bodies in the form [10] ;

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{U}}\right) - \frac{\partial L}{\partial U} + \frac{\partial U}{\partial U} = F\dot{U} \quad (8)$$

Where,
N is the number of degrees of freedom $(l \leq i \leq N)$.
qi are the system's generalized independent coordinates.
Fqi are the generalized forces.
denotes differentiation with respect to time t.

2.9 Equation of Motion

Using Lagrange equation

$$Meq\ddot{U} - a\lambda\dot{W} + kU = F \quad (9)$$

This is the equation of motion in X direction

$$Meq\ddot{W} + a\lambda\dot{U} + kW = 0 \quad (10)$$

This is the equation of motion in Z direction

2.10 Solution of the equation of motion

Let the displacement of the system in the X-axis is given by:

$U(t) = A_0 + A_1\sin\omega t + A_2\cos\omega t + A_3\sin 2\omega t + A_4\cos 2\omega t + A_5\sin 3\omega t + A_6\cos 3\omega t + A_7\sin 4$

$$\omega t + A_8\cos 4\omega t + A_9\sin 5\omega t + A_{10}cos5\omega t \quad (11)$$

the Coefficient of this expression are listed below:

$A_1 = -\dfrac{23290(k-2m.\omega^2)(1-\cos 0.04\omega)}{\omega(a^2.\lambda^2.\omega^2-(k-2m.\omega^2))}$
$A_2 = \dfrac{23920.\sin 0.04\omega(k-2m.\omega^2)}{\omega((k-2m.\omega^2)^2-a^2.\lambda^2.\omega^2)}$
$A_3 = -\dfrac{11960(k-8m.\omega^2)(1-\cos 0.08\omega)}{\omega(6a^2.\lambda^2.\omega^2-(k-8m.\omega^2))}$
$A_4 = \dfrac{11960.\sin 0.08\omega(k-8m.\omega^2)}{\omega((k-8m.\omega^2)^2-4a^2.\lambda^2.\omega^2)}$
$A_5 = -\dfrac{7973.33(k-18m.\omega^2)(1-\cos 0.12\omega)}{\omega(9a^2.\lambda^2.\omega^2-(k-18m.\omega^2))}$
$A_6 = \dfrac{7973.33.\sin 0.12\omega(k-18m.\omega^2)}{\omega((k-18m.\omega^2)^2-9a^2.\lambda^2.\omega^2)}$
$A_7 = -\dfrac{5980(k-32m.\omega^2)(1-\cos 0.16\omega)}{\omega(16a^2.\lambda^2.\omega^2-(k-32m.\omega^2))}$
$A_8 = \dfrac{5980.\sin 0.16\omega(k-32m.\omega^2)}{\omega((k-32m.\omega^2)^2-16a^2.\lambda^2.\omega^2)}$
$A_9 = -\dfrac{4784(k-50m.\omega^2)(1-\cos 0.2\omega)}{\omega(25a^2.\lambda^2.\omega^2-(k-50m.\omega^2))}$
$A_{10} = \dfrac{4784.\sin 0.2\omega(k-50m.\omega^2)}{\omega((k-50m.\omega^2)^2-25a^2.\lambda^2.\omega^2)}$

The Deflection of the Shaft for the operating speed $(1500rpm)$ is as shown in figure below:

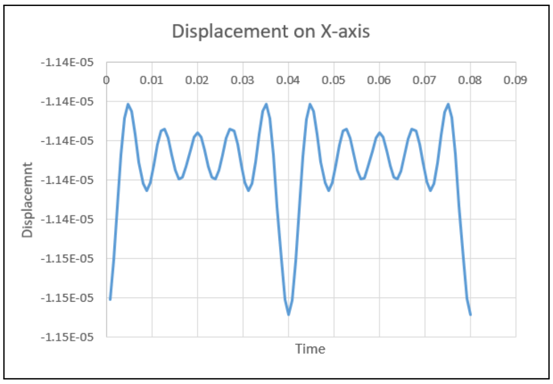

Figure 3: Deflection on X-Direction

Again the displacement in the Z-axis is given by $W(t) = B_0 + B_1 \sin\omega t + B_2 \cos\omega t + B_3 \sin 2\omega t + B_4 \cos 2\omega t + B_5 \sin 3\omega t + B_6 \cos 3\omega t + B_7 \sin 4\omega t +$

$$B_8 \cos 4\omega t + B_9 \sin 5\omega t + B_{10} cos 5\omega t \tag{12}$$

The Coefficient of this expression are listed below:

$B_1 = \dfrac{23920.a.\lambda \sin 0.04\omega}{((k-2m.\omega^2)^2-a^2.\lambda^2.\omega^2)}$
$B_2 = \dfrac{23920.a.\lambda(1-\cos 0.04\omega)}{(a^2.\lambda^2.\omega^2-(k-2m.\omega^2)^2)}$
$B_3 = \dfrac{23920.a.\lambda \sin 0.08\omega}{((k-8m.\omega^2)^2-4.a^2.\lambda^2.\omega^2)}$
$B_4 = \dfrac{23920.a.\lambda(1-\cos 0.08\omega)}{(6a^2.\lambda^2.\omega^2-(k-8m.\omega^2)^2)}$
$B_5 = \dfrac{23920.a.\lambda \sin 0.12\omega}{((k-18m.\omega^2)^2-9.a^2.\lambda^2.\omega^2)}$
$B_6 = \dfrac{23920.a.\lambda(1-\cos 0.12\omega)}{(9a^2.\lambda^2.\omega^2-(k-18m.\omega^2)^2)}$
$B_7 = \dfrac{23920.a.\lambda \sin 0.16\omega}{((k-32m.\omega^2)^2-16.a^2.\lambda^2.\omega^2)}$
$B_8 = \dfrac{23920.a.\lambda(1-\cos 0.16\omega)}{(16a^2.\lambda^2.\omega^2-(k-32m.\omega^2)^2)}$
$B_9 = \dfrac{23920.a.\lambda \sin 0.2\omega}{((k-50m.\omega^2)^2-25.a^2.\lambda^2.\omega^2)}$
$B_{10} = \dfrac{23920.a.\lambda(1-\cos 0.2\omega)}{(25a^2.\lambda^2.\omega^2-(k-50m.\omega^2)^2)}$

Substituting the value of $U(t), W(t)$ and the forcing function F in the equation of motion of X-direction we get :

The Deflection of the Shaft for the operating speed $(1500rpm)$ in X-Direction is as shown in figure below:

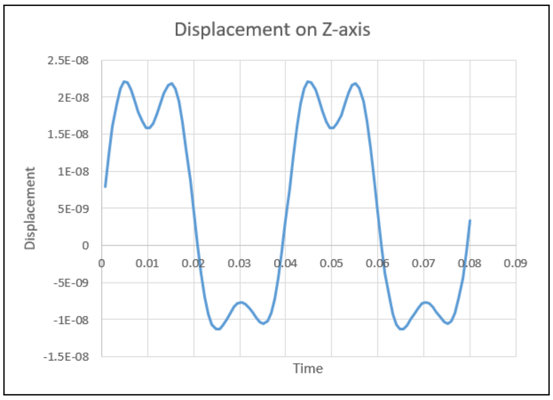

Figure 4: Deflection on Z-Direction

2.11 Software model development

The software model of the pelton turbine model is generated on the solidworks and it is then imported on the ANSYS workbench for the harmonic response analysis. Currently preliminary simulation of the model has been completed. The displacement for the X-axis at the rated rpm is found to be 6.59 μm. Similarly, the displacement on the Z axis was found to be 5.85 *nm*.

The result from the ANSYS simulation of the model has been tabulated in the following table:

Table 1: Deflection on X-axis

S.No.	Frequency (Hz)	Amplitude (mm)
1	3.2	6.54E-03
2	6.4	6.54E-03
3	9.6	6.54E-03
4	12.8	6.55E-03
5	16	6.56E-03
6	19.2	6.56E-03
7	22.4	6.58E-03
8	25.6	6.59E-03
9	28.8	6.60E-03
10	32	6.62E-03
11	35.2	6.63E-03
12	38.4	6.65E-03
13	41.6	6.67E-03
14	44.8	6.70E-03
15	48	6.72E-03
16	51.2	6.75E-03
17	54.4	6.78E-03
18	57.6	6.81E-03
19	60.8	6.84E-03
20	64	6.87E-03
21	67.2	6.91E-03
22	70.4	6.95E-03
23	73.6	6.99E-03
24	76.8	7.03E-03
25	80	7.08E-03
26	83.2	7.13E-03

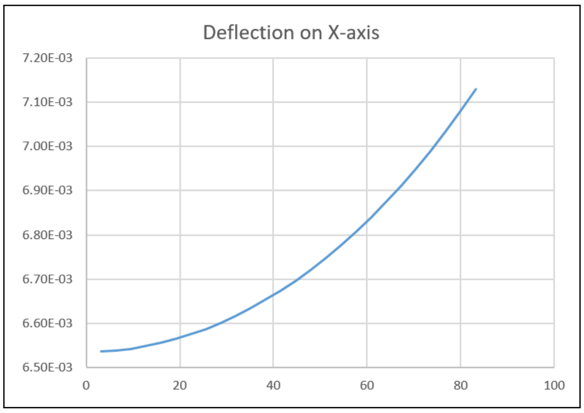

Figure 5: Deflection on X-Axis

Table 2: Deflection on Z-axis

S.No.	Frequency (Hz)	Amplitude(mm)
1	3.2	5.78E-05
2	6.4	5.78E-05
3	9.6	5.79E-05
4	12.8	5.79E-05
5	16	5.81E-05
6	19.2	5.82E-05
7	22.4	5.83E-05
8	25.6	5.85E-05
9	28.8	5.87E-05
10	32	5.90E-05
11	35.2	5.93E-05
12	38.4	5.96E-05
13	41.6	5.99E-05
14	44.8	6.02E-05
15	48	6.06E-05
16	51.2	6.10E-05
17	54.4	6.15E-05
18	57.6	6.20E-05
19	60.8	6.25E-05
20	64	6.30E-05
21	67.2	6.36E-05
22	70.4	6.42E-05
23	73.6	6.49E-05
24	76.8	6.56E-05
25	80	6.63E-05
26	83.2	6.71E-05

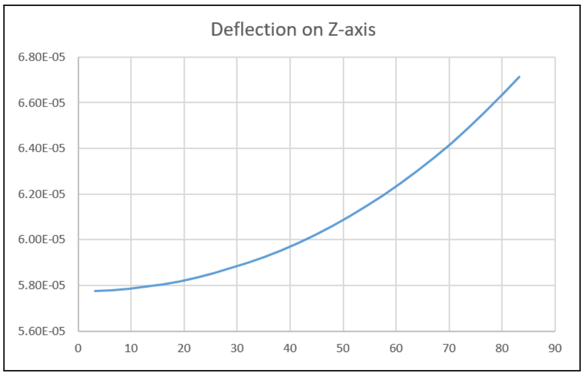

Figure 6: Deflection on Z-Axis

3. Parameters

The values of parameters as per manual of installed setup is listed and necessary calculations are done.

Table 3: Pameter Values

Parameters	Value
Outpur Power	1000 W
Rated RPM	1500 rpm
Pitch Circle Diameter of Runner	180 mm
Runner Material	Bronze
No. of Buckets	16
Thickness of Buckets (Distance) from front face (splitter) of bucket to back face of bucket)	18 mm
Gap between consecutive Buckets	12.434 mm
Density of Bucket Materials (Casted Brass)	8300 kg/m^3
Total Mass of Runner-Buckets Assembly	10.654 kg
Diameter of Shaft	40 mm
Material of Shaft	Mild Steel
Density of Shaft Material, ρ_s	7860 kg/ m^3
Young's Modulus of Elasticity of the Shaft Material, E_s	220 GPa
Diameter of Nozzle opening, d_a	26 mm
Jet area, a_j	0.0000504 m^2

4. Conclusion

This paper presented the methodologies to study the dynamic response of Pelton turbine unit as a shaft-disk system. Hence, the mathematical model for dynamic response of the Pelton turbine unit was formulated and the analytical solution of amplitude of forced vibration was found. The Fourier analysis for the excitation force showed the minimum number of Fourier components to be considered to obtain the solution in its exact form. Thus, the analysis showed the summation of first five Fourier components began to represent the actual shape of pressure pulse. Hence, minimum five Fourier components are to be considered in analysis for meaningful representation of a forcing function. The amplitudes of forced vibration of the selected Pelton turbine unit of 1 kW with single nozzle rated at 1500 RPM in X in X (the direction of water jet) direction and Z direction were found to be 7.29 μ m and 39 *nm* respectively analytically. Similarly, the amplitude of vibration of in X in X (the direction of water jet) direction and Z direction were found to be 6.59 μ m and 58.5 *nm* respectively by ANSYS simulation.This methodology can be applied to find the dynamic force response in MHP and other hydropower plants to calculate the acceptance level of vibration analytically and can compare the vibration level during the operation period in long run by measuring the amplitudes using vibration measuring devices.

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