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Dynamic Response of Pelton Turbine Unit for Forced Vibration

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Abstract
The study of vibration is concerned with the oscillatory motions of bodies and the forces associated with them. Rotor dynamics is the branch of engineering that studies the lateral and torsional vibrations of rotating shafts, with the objective of predicting the rotor vibrations and containing the vibration level under an acceptable limit. This research presented the modeling of excitation force imparted by water jet in the form of Fourier series and determined the forced response of Pelton turbine unit analytically. The mathematical model was developed by calculating the kinetic energy and potential energy of the disk and shaft. Lagrange's equation, Rayleigh-Ritz method and virtul work method were used to derive the equation of motion of forced vibration condition. The developed methodologies were followed to find the analytical solution of dynamic response of selected Pelton turbine unit of 2 kW with single nozzle rated at 1500 RPM. A rigid disk (runner and buckets assembly) was situated midway along the length of flexible shaft with rigid and undamped simply supported bearings at both ends of the shaft. It was found that summation of first five Fourier components began to represent the actual shape of pressure pulse. Hence, minimum five Fourier components were to be considered in analysis for meaningful representation of a forcing function. Analytically, the amplitudes of forced vibration of Pelton Turbine unit with single nozzle in X direction (the direction of water jet) and Z direction were found to be 3.3 μm and 6.6 $\times 10^{-5} \mu$m respectively. Hence, this methodology can be applied to find the forced response in micro-hydro power plants and other hydropower plants to calculate the acceptance level of vibration analytically and can compare the vibration level during the operation period in long run by measuring the amplitudes using vibration measuring devices.

Keywords
Pelton – Dynamic – Response – Fourier – Vibration – Force

1. Introduction

Vibration is the motion of particle or a body or a system of connected bodies displaced from position of equilibrium. Most of the engineering machines and structures experience vibration to some degree hence their design requires consideration of their oscillatory behavior. Most vibrations are undesirable in machines and structures because they produce increased stresses, energy losses, causes added wear, increase bearing loads, induce fatigue, create passenger discomfort in vehicles and absorb energy from the system [1]. All bodies having mass and elasticity are capable of vibration. In engineering systems, harmonic excitation is encountered and commonly produced by unbalance in rotating machinery. Vibrations can be free and forced depending upon the application of force. The failure of major structures such as bridges, buildings, or airplane wing is a possibility under resonance. Thus, the calculation of natural frequencies is a major importance in the study of vibrations [2]. Accurate analysis of vibration characteristics is crucial in design stage of hydraulic machinery for their performance improvements, reliability, life of the components and safety.

Rotor dynamics is the branch of engineering that studies the lateral and torsional vibrations of rotating shafts, with the objective of predicting the rotor vibrations and containing the vibration level under an acceptable limit [3]. The dynamic analysis of the Pelton turbine and assembly was studied to obtain the natural frequency of the system [4]. The mathematical model was developed and solved analytically to find the critical frequency and validated it numerically. The critical frequency of the system was found to be 192.69 Hz and 192.73 Hz in the

X and Y direction and 137.86 Hz and 137.98 Hz numerically. The natural frequency of the turbine when coincides with actual frequency of the turbine causes the formation of the resonance which increase in chances of failure of the turbine by buckling of deformation of the shaft. From the literature review it was seen that, very less work has been done in the field of the dynamic behavior of pelton wheel turbine and their effects in design and operation [5]. The Computational Fluid Dynamics (CFD) analysis of Pelton turbine of Khimti Hydropower in Nepal was presented [6]. They determined torque generated by the turbine and pressure distributions in bucket for further work on fatigue analysis. The pressure distribution was found maximum at bucket tip and runner Pitch Circle Diameter (PCD). The torque generated by the middle bucket was replicated over time to determine total torque generated by Pelton turbine.

Lots of research have been done related to dynamic response on the turbine blade but most of these research are limited to thermal (gas and steam) turbines. Although few research have been carried on the dynamics of water turbine, they are focused either on the dynamic response due to generator and bearing behavior or the fluid and blade surface interaction. Pelton turbines are widely used in large hydropower plants all around the countries. Pelton turbines are also being used in several micro hydro power plants. In MHP plants, the turbine is usually designed and manufactured by local manufacturers within the country. Most of the research being conducted are limited to improvement of designs for sediments and erosion. However, less work has been done in the field of dynamic response of turbines assembly and their effects in design and operation. Forced dynamic response of Pelton turbine has not been studied so far.

Therefore, this research is mainly focused to model the excitation force imparted by water jet in the form of Fourier series to obtain the function in its exact form and to determine the forced response of Pelton turbine analytically.

2. Methods and Materials

2.1 Mathematical Model Development

The complete mathematical model has been developed in four different phases. The basic elements considered for the mathematical model development are disk, shaft and bearings. The equation of motion for the system has been developed using energy method.

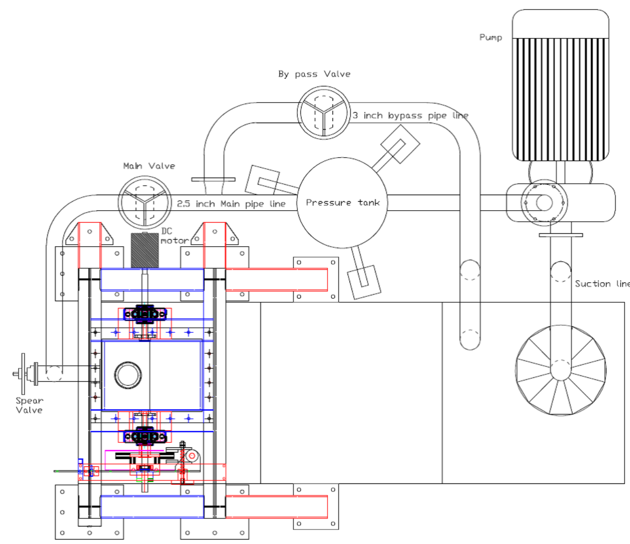

Figure 1: Schematic diagram of turbine setup

2.1.1 Phase I: Total energy of the system

The rigid disk, flexible shaft and rigid, undamped and simply supported bearings were considered. Therefore, kinetic energy of both disk and shaft were computed but potential energy for only shaft was computed given by its strain energy as shaft is only the flexible element. Three reference frames are used in this work.

$x_d\ y_d\ z_d$: Frame fixed on the disk center

$x_s\ y_s\ z_s$: Frame fixed with shaft

X Y Z: Fixed inertial frame

The Disk

The disk is considered rigid and it is characterized solely by its kinetic energy. The coordinate of the disk center O is u (y, t), v and w (y, t) with reference to fixed inertial frame XYZ; the coordinate along Y-axis remains constant. Then, the position vector of the disk center O in the XYZ coordinate system can be written as;

$$\vec{R} = \left\{ \begin{array}{c} u \\ v \\ w \end{array} \right\} \tag{1}$$

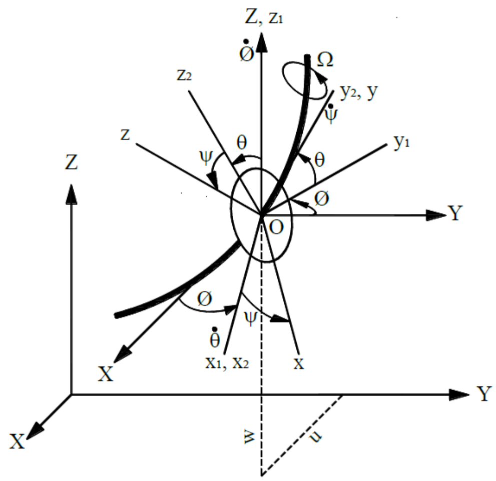

Figure 2: A disk on a rotating flexible shaft with reference frames

The xyz coordinate system is related to XYZ coordinate system through three set of Euler's angles ϕ, θ and ψ. To achieve the orientation of the disk, the disk is first rotated by an angle ϕ about Z axis; then it is rotated about the new x-axis x_1 by an angle θ and lastly by an angle ψ about the final y axis. The instantaneous angular velocity vector of the xyz frame is,

$$\omega = \dot{\phi}Z + \dot{\theta}x_1 + \dot{\psi} \tag{2}$$

Through the coordinate transformation, the components of angular velocities in the direction of principal axes xyz can be expressed as,

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -\dot{\phi}\cos\theta\sin\psi + \dot{\theta}\cos\psi \\ \dot{\phi}\sin\theta + \dot{\psi} \\ \dot{\phi}\cos\theta\cos\psi + \dot{\theta}\sin\psi \end{bmatrix} \tag{3}$$

Where, $\dot{\phi}$, $\dot{\psi}$ and $\dot{\theta}$ are rate of spin, rate of precession and rate of nutation respectively. For the system considered, the spinning axis is Y axis and angular motion about X and Z axes are comparatively small. Thus, $\cos\theta \approx 1, \cos\phi \approx 1, \sin\theta \approx \theta$ and $\sin\phi \approx \phi$ [7] then the angular velocities becomes

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -\dot{\phi}\sin\psi + \dot{\theta}\cos\psi \\ \dot{\phi}\theta + \dot{\psi} \\ \dot{\phi}\cos\psi + \dot{\theta}\sin\psi \end{bmatrix} \tag{4}$$

The kinetic energy of the disk is the summation of translation kinetic energy and rotational kinetic energy and given by [8]

$$T_D = \frac{1}{2}M_D\left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}\left(I_{Dxx}\omega_x^2 + I_{Dyy}\omega_y^2 + I_{Dzz}\omega_z^2\right) \tag{5}$$

Where, M_D is the mass of the disk, I_{Dxx}, I_{Dyy}, and I_{Dzz} are the moment of inertia about the principal axes X, Y and Z axes respectively. As the disk is assumed to be symmetrical, $I_{Dxx} = I_{Dzz}$

$$\begin{aligned} \therefore T_D = \frac{1}{2}M_D\left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}I_{Dxx}\left(\dot{\theta}^2 + \dot{\phi}^2\right) \\ + \frac{1}{2}I_{Dyy}\left(\lambda^2 + 2\dot{\phi}\theta\lambda\right) \end{aligned} \tag{6}$$

The Shaft

The shaft is considered as a flexible element with constant circular cross-section. Hence, it has both kinetic energy and potential energy, given by the strain energy of the shaft. The K.E. of the shaft is given by [9]

$$\begin{aligned} T_s = \frac{\rho A}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dy + \frac{\rho I_{sxx}}{2}\int_0^L \omega_x^2 dy + \frac{\rho I_{syy}}{2} \\ \int_0^L \omega_z^2 dy \end{aligned} \tag{7}$$

Where, T_S is the kinetic energy of the shaft, ρ is mass per unit volume, A is the cross-sectional area of shaft, I is the second moment of inertia of the shaft cross-section about the neutral axis and it is equal to $\frac{\pi D^4}{64}$ [3], D is the diameter of shaft.

$$\begin{aligned} \therefore T_s = \frac{\rho A}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dy + \frac{\rho I}{2}\int_0^L \left(\dot{\theta}^2 + \dot{\phi}^2\right) dy \\ + \rho I L \lambda^2 + 2\rho I \lambda \int_0^L \dot{\phi}\theta dy \end{aligned} \tag{8}$$

And the strain energy of the shaft is given by [9]

$$U_s = \frac{EI}{2}\int_0^L \left[\left(\frac{\partial^2 u}{\partial y^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2\right] dy \tag{9}$$

Where, E is the Young's Modulus of Elasticity.

Hence, the total kinetic energy of the system is expressed as

$$T = \frac{1}{2}M_D\left(\dot{u}^2+\dot{w}^2\right)+\frac{1}{2}I_{Dxx}\left(\dot{\theta}^2+\dot{\phi}^2\right)$$
$$+\frac{1}{2}I_{Dyy}\left(\lambda^2+2\dot{\phi}\theta\lambda\right)+\frac{\rho A}{2}\int_0^L\left(\dot{u}^2+\dot{w}^2\right)dy \quad (10)$$
$$+\frac{\rho I}{2}\int_0^L\left(\dot{\theta}^2+\dot{\phi}^2\right)dy+\rho IL\lambda^2+2\rho I\lambda\int_0^L\theta\dot{\phi}dy$$

And, the total potential energy of the system is expressed as

$$U_s = \frac{EI}{2}\int_0^L\left[\left(\frac{\partial^2 u}{\partial y^2}\right)^2+\left(\frac{\partial^2 w}{\partial y^2}\right)\right]dy \qquad (11)$$

2.1.2 Phase II: Mathematical Model for Equation of Motion

Lagrange's Equation

The Lagrange's Equation is expressed as [1]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial U_s}{\partial q_i}=F_{q_i} \qquad (12)$$

Where, N is the number of degrees of freedom ($1 \le i \le N$), q_i are the system's generalized independent coordinates, F_{q_i} are the generalized forces, T is the kinetic energy and U_s is the potential energy of the system.

Rayleigh-Ritz Method of Analytical Solution

Rayleigh-Ritz method is also known as assumed modes method. For proper description of the lateral vibration behavior of the rotor, it is necessary to write the displacement u and w of the rotor in terms of shape function f(y), before applying the expressions obtained in the Lagrange's equation. The first mode of vibration for the shape function is assumed as [7]

$$f(y) = \sin\left(\frac{\pi y}{L}\right) \qquad (13)$$

The expressions for the displacements in x and z directions are expressed as,

$$u(y,t) = f(y)U(t) = U\sin\left(\frac{\pi y}{L}\right) \qquad (14)$$

$$w(y,t) = f(y)W(t) = W\sin\left(\frac{\pi y}{L}\right) \qquad (15)$$

Where, U and W are generalized independent coordinates. As the angular displacements ϕ and θ are small, they are approximated as,

$$\theta = \frac{\partial w}{\partial y} = \frac{df(y)}{dy}W = g(y)W = \frac{\pi}{L}W\cos\left(\frac{\pi y}{L}\right) \quad (16)$$

$$\phi = -\frac{\partial u}{\partial y} = -\frac{df(y)}{dy}U = -g(y)U = -\frac{\pi}{L}U\cos\left(\frac{\pi y}{L}\right) \quad (17)$$

With the disk situated at the midway along the length of shaft and applying the displacement function, the total kinetic energy of the system is expressed as

$$T = \frac{1}{2}m\left(\dot{U}^2+W_2^2\right)-a\lambda\dot{U}W+\rho IL\lambda^2 \qquad (18)$$

Where,

$$m = M_D+\frac{\rho AL}{2}+\frac{\rho I\pi^2}{2L} \qquad (19)$$

$$a = \frac{\rho I\pi^2}{L} \qquad (20)$$

Similarly, the total potential energy of the system is expressed as

$$U_s = \frac{1}{2}K\left(U^2+W^2\right) \qquad (21)$$

Where,

$$K = \frac{\pi^4 EI}{2L^3} \qquad (22)$$

External forces

The virtual work done by the transverse forces is given by

$$\delta W = F_1(t)\delta u+F_2(t)\delta w \qquad (23)$$

Using the general coordinates

$$dW = F_1(t)\delta U\sin\left(\frac{\pi y}{L}\right)+F_2(t)\delta W\sin\left(\frac{\pi y}{L}\right) \quad (24)$$

Now, eliminating the virtual displacement we get the general forces as

$$F_U = F_1(t)\sin\left(\frac{\pi y}{L}\right)$$
$$F_W = F_2(t)\sin\left(\frac{\pi y}{L}\right) \qquad (25)$$

In Pelton turbine,the water jet strikes the bucket tangentially only in the X direction, so, $F_1(t) = F_{ext.}(t)$ and $F_2(t) = 0$ and as the buckets are at the middle along the length of shaft i.e., $y = \frac{L}{2}$

$$\begin{aligned} F_U &= F_{ext.}(t) \\ F_W &= 0 \end{aligned} \tag{26}$$

Using Lagrange's equation in the above obtained kinetic and potential energy choosing generalized co-ordinates as U and W, we get equation of motion as

$$\begin{aligned} m\ddot{U} - a\lambda\dot{W} + KU &= F_U \\ m\ddot{W} + a\lambda\dot{U} + KW &= F_W \end{aligned} \tag{27}$$

Then, the equation of motion for force response in matrix form is expressed as,

$$\begin{bmatrix} m & 0 \\ o & m \end{bmatrix}\begin{bmatrix} \ddot{U} \\ \ddot{W} \end{bmatrix} + \lambda\begin{bmatrix} 0 & -a \\ a & 0 \end{bmatrix}\begin{bmatrix} \dot{U} \\ \dot{W} \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}\begin{bmatrix} U \\ W \end{bmatrix} = \begin{bmatrix} F_{ext.}(t) \\ 0 \end{bmatrix} \tag{28}$$

2.1.3 Phase III: Force imparted by water jet in the buckets and Fourier representation

Force exerted by water jet on the buckets

In Pelton turbine, the jet strikes the bucket in the tangential direction and the magnitude of force can be calculated as [10]

$$F_j = \rho_w a_j V_1 (V_{w1} + V_{w2}) \tag{29}$$

Where, F_j is the force exerted by the jet of water in the direction of motion, ρ_w is the mass density of water, a_j is the area of jet, V_{w1} is the component of velocity of the jet V_1 in the direction of motion, and V_{w2} is the component of velocity V_2 in the direction of vane.

$$\begin{aligned} V_{w1} &= V_1 \\ V_{w2} = k(V_1 - u_1)\cos\phi_1 &- u_1 \\ u_1 &= \frac{\pi D_w N}{60} \\ V_1 &= C_v\sqrt{2gH_N} \end{aligned} \tag{30}$$

Where, k is the blade friction coefficient, ϕ_1 vane angle at outlet, u_1 is peripheral/circumferential velocity of runner, D_w is the diameter of Pelton wheel, N is speed of wheel in RPM (1500 RPM for synchronous motor), C_v is the coefficient of velocity (generally 0.98 or 0.99) and H_N is the net head on turbine.

Fourier representation of the force

In mechanical systems, most often the excitation forces are not harmonic functions but periodic in nature [11]. The response of the system to periodic excitation is determined by first determining the Fourier components of the excitation force and then determining the response of the system to each of these harmonic components. If F(t) is a periodic function with period τ, its Fourier series representation is given by [11]

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t) \tag{31}$$

Where, a_n and b_n are Fourier coefficients and can be determined by using the following relations

$$\begin{aligned} a_n &= \frac{2}{\tau}\int_0^{\tau} F(t)\cos n\omega t dt \\ b_n &= \frac{2}{\tau}\int_0^{\tau} F(t)\sin n\omega t dt \end{aligned} \tag{32}$$

The Fourier series representation of excitation force is expressed as

$$\begin{aligned} F(t) = 0.592F_1 &+ \sum_{n=1}^{\infty}\left(\frac{0.3183F_j}{n}\sin 3.7196n \cos n\omega t\right) \\ &+ \sum_{n=1}^{\infty}\left(\frac{0.3183F_j}{n}(1-\cos 3.1796n)\sin n\omega t\right) \end{aligned} \tag{33}$$

2.1.4 Phase IV: Mathematical model for analytical solution of the force response of the system

For this phase undamped, rigid and isotropic bearings will be considered. Writing the differential equation of motions for forced vibration as

$$\begin{aligned} m\ddot{U} - a\lambda\dot{W} + KU = 0.592F_1 \\ + \sum_{n=1}^{\infty}\left(\frac{0.3183F_j}{n}\sin 3.7196n \cos n\omega t\right) \\ + \sum_{n=1}^{\infty}\left(\frac{0.3183F_j}{n}(1-\cos 3.1796n)\sin n\omega t\right) \end{aligned} \tag{34}$$

$$m\ddot{W} + a\lambda\dot{U} + KW = 0 \tag{35}$$

These are two coupled linear differential equations of second order and their solution will be in the form;

$$\begin{aligned} U(t) = A_0 + A_1\sin\omega t + A_2\cos\omega t + A_3\sin 2\omega t \\ + A_4\cos 2\omega t + A_5\sin 3\omega t + A_6\cos 2\omega t + A_7\sin 4\omega t \\ + A_8\cos 4\omega t + A_9\sin 5\omega + A_{10}\cos 5\omega t \end{aligned}$$

(36)

$$W(t) = B_0 + B_1 \sin \omega t + B_2 \cos \omega t + B_3 \sin 2\omega t$$
$$+ B_4 \cos 2\omega t + B_5 \sin 3\omega t + B_6 \cos 2\omega t + B_7 \sin 4\omega t$$
$$+ B_8 \cos 4\omega t + B_9 \sin 5\omega t + B_{10} \cos 5\omega t$$

(37)

Where, $A_0, A_1, ...A_{10}$ and $B_0, B_1, ...B_{10}$ are constants.

2.2 Solution of Developed Mathematical Model

Using the Equations 34, 35, 36 and 37, the constant $A_0, A_1, ...A_{10}$ and $B_0, B_1, ...B_{10}$ can be calculated. Then the Equations 36 and 37 can be plotted with respect to time and amplitude of forced vibration in X and Z direction can be found from the graph.This gives the amplitude values in X and Z direction at the midway of shaft span and amplitude value at any position within the shaft span in X and Z directions can be found by using the Equations 14 and 15 respectively.

3. Results and Discussion

The developed methodologies were followed to find the analytical solution of dynamic response of selected Pelton turbine unit of 2 kW with single nozzle rated at 1500 RPM. The values of parameters as per manual of installed setup is listed and necessary calculations are done.

Table 1: Parameters used for finding the amplitude of force vibration

SN	Parameters	Value
1	Rated RPM, N	1500
2	PCD of Runner (mm)	155
3	Number of buckers, n	16
4	Thickness of bucket (mm)	18
5	Gap between consecutive buckets (mm)	12
6	Mass of runner, M_D (kg)	10.654
7	Diameter of Shaft, D (mm)	32
8	Length of shaft, L (mm)	519
9	Density of shaft material, ρ_s	7860
10	Modulus of Elasticity, E_s (GPa)	202
11	Spin speed, λ(rad/s)	157.07

3.1 Calculation of Force exerted by the water jet on the buckets

For determining the force imparted by water jet, some of the parameters are needed to calculate which are listed in Table 2.

Table 2: Parameters used for finding jet force calculation

SN	Parameters	Value
1	ρ_w (kg/m^3)	1000
2	u_1 (m/s)	12.174
3	V_{w1} (m/s)	27.454
4	V_{w2} (m/s)	1.847
5	a_j (m^2)	0.0024

The value of jet force is calculated using the Equation 29 and is found to be 193 N.

3.2 Calculation of Fourier components and their resultant

The fourier components are found as $a_0 = 228.5120$
$a_n = \frac{61.4338}{n} \sin 3.7196n$
$b_n = \frac{61.4338}{n} (1 - \cos 3.1796n)$
The representation of excitation force in the form of Fourier series is
$F(t) = 114.256 + \sum_{n=1}^{\infty} \left(\frac{61.4338}{n} \sin 3.7196n \cos n\omega t \right) + \sum_{n=1}^{\infty} \left(\frac{61.4338}{n} (1 - \cos 3.1796n) \sin n\omega t \right)$

For analysis of excitation force to obtain the function in its exact form, different graphs for excitation force versus time were plotted.

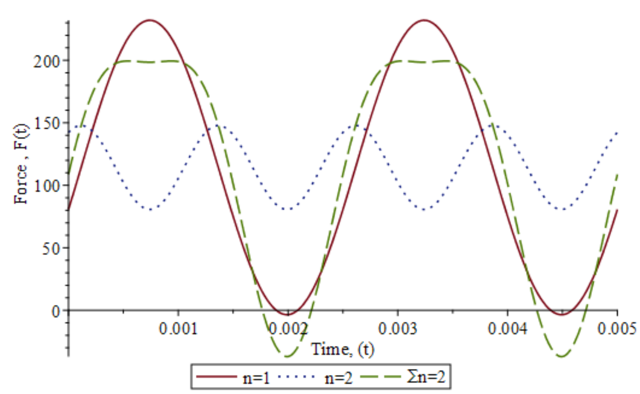

Figure 3: First two Fourier components and their resultants

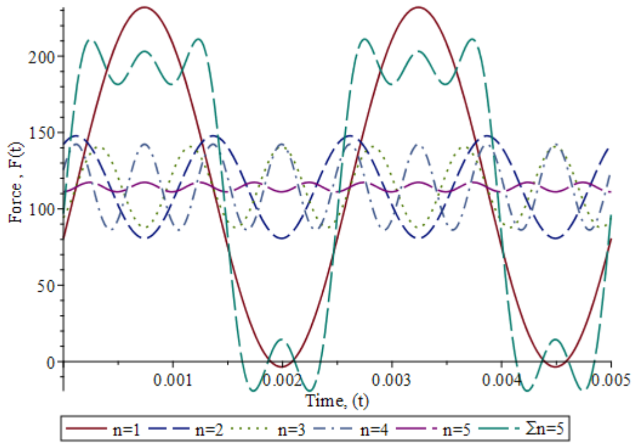

Figure 4: First five Fourier components and their resultants

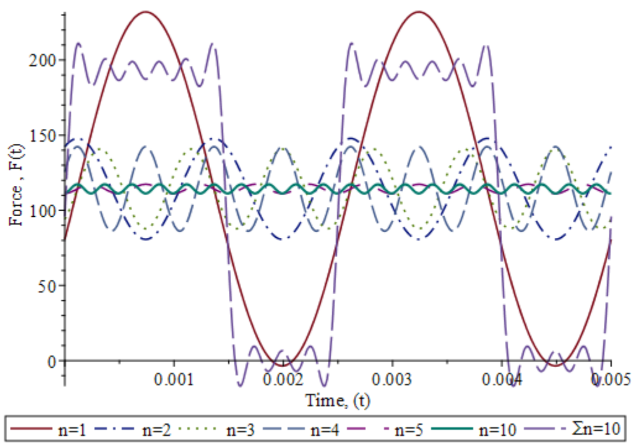

Figure 5: First ten Fourier components and their resultants

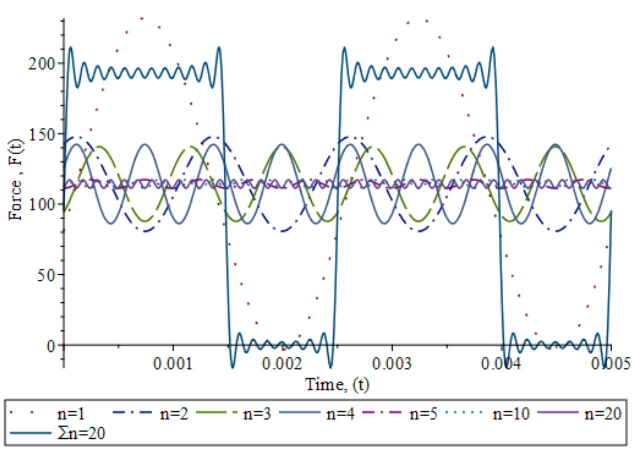

Figure 6: First twenty Fourier components and their resultants

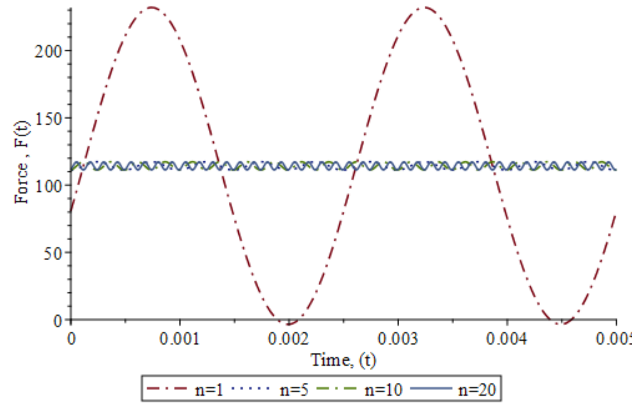

Figure 7: Fourier components for n = 1, 5, 10 and 20

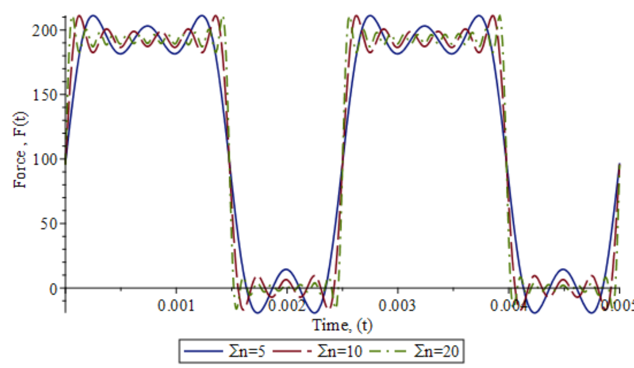

Figure 8: Resultant of Fourier components for n = 5, 10 and 20

Figures 3, 4, 5 and 6 show that the first Fourier component has the maximum amplitude, while higher order components generally decrease but the ripple frequency of higher order components increases and resultant of higher order represents more precisely the actual shape of pressure pulse. Hence, this indicates very lucidly how the higher order Fourier components contribute to the development of exact function. It is obvious that infinite number of Fourier components will have to be considered if the function in its exact form is to be obtained. The comparison of level of accuracy obtained by summation of first five, ten and twenty Fourier components is shown in Figure 8. It is clear that the first five Fourier components are enough to represent the function to obtain the solution of forced response.

3.3 Calculation of constant terms

The values of m, a and K are found using Equations 19, 20, and 22.

$A = 8.042 \times 10^{-4} m^2$
$I = 5.15 \times 10^{-8} m^4$
$m = 12.298 kg$
$a = 7.694 \times 1063 kg$
$K = 3622335 N/m$

After calculation of constant $A_0, A_1, ...A_{10}$ and $B_0, B_1, ...B_{10}$, Equations 36 and 37 are represented as

$$U(t) = -3.1543 \times 10^{-6} - 1.5243 \times 10^{-6} \sin \omega t + 4.5325 \times 10^{-7} \cos \omega t - 5.9723 \times 10^{-8} \sin 2\omega t - 9.1544 \times 10^{-8} \cos 2\omega t - 2.4655 \times 10^{-8} \sin 3\omega t + 2.9051 \times 10^{-8} \cos 3\omega t - 2.0763 \times 10^{-9} \sin 4\omega t - 9.1401 \times 10^{-10} \cos 4\omega t - 1.9914 \times 10^{-10} \sin 5\omega t$$

$$W(t) = -1.8589 \times 10^{-11} \sin \omega t - 6.2518 \times 10^{-11} \cos \omega t + 1.8109 \times 10^{-12} \sin 2\omega t - 1.1814 \times 10^{-12} \cos 2\omega t - 3.8062 \times 10^{-13} \sin 3\omega t - 3.2302 \times 10^{-13} \cos 3\omega t + 8.9609 \times 10^{-13} \sin 4\omega t - 2.0356 \times 10^{-13} \cos 4\omega t - 1.2351 \times 10^{-14} \sin 5\omega t - 1.5602 \times 10^{-15} \cos 5\omega t$$

Now, plotting U(t) and W(t) with respect to time to determine the amplitudes of forced vibration as,

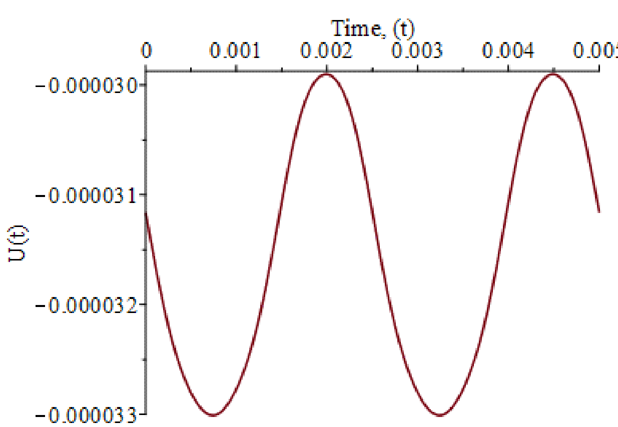

Figure 9: Vibration amplitude in X-direction at the middle of shaft span

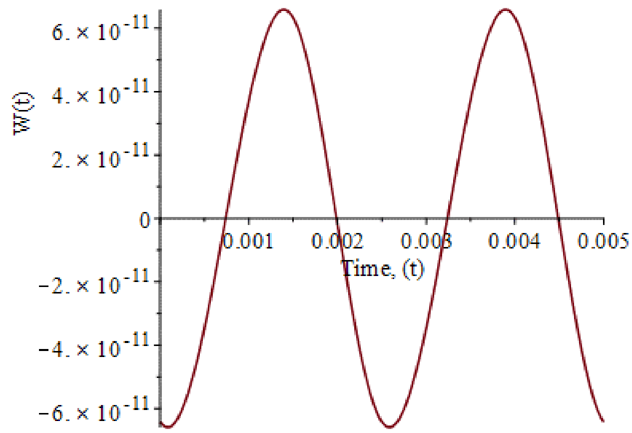

Figure 10: Vibration amplitude in Z-direction at the middle of shaft span

From the Figures 9 and 10, the amplitudes of vibration of selected Pelton turbine unit with single nozzle are found to be 3.3 μm and 6.6 $\times 10^{-5} \mu m$ in X and Z directions respectively at the midway along the length of the shaft.

4. Conclusion

This paper presented the methodologies to study the dynamic response of Pelton turbine unit as a shaft-disk system. Hence, the mathematical model for dynamic response of the Pelton turbine unit was formulated and the analytical solution of amplitude of forced vibration was found. The Fourier analysis for the excitation force showed the minimum number of Fourier components to be considered to obtain the solution in its exact form.

Thus, the analysis showed the summation of first five Fourier components began to represent the actual shape of pressure pulse. Hence, minimum five Fourier components are to be considered in analysis for meaningful representation of a forcing function. The amplitudes of forced vibration of the selected Pelton turbine unit of 2 kW with single nozzle rated at 1500 RPM in X (the direction of water jet) direction and Z direction were found to be 3.3 μm and 6.6 $\times 10^{-5} \mu m$ respectively analytically.

This methodology can be applied to find the dynamic force response in MHP and other hydropower plants to calculate the acceptance level of vibration analytically and can compare the vibration level during the operation period in long run by measuring the amplitudes using vibration measuring devices.

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