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A Strain-Based Fatigue Damage Model for Woven Fabric Composites

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Abstract

An anisotropic fatigue damage model for glass/epoxy woven fabric composites is developed in tension-tension fatigue loading utilizing strain based damage mechanics theory. The proposed model of distributed damage is capable of accommodating fully anisotropic elastic degradation and inelasticity in woven fabric composites that develop in the fatigue environment. The formation and propagation of multitude of microcracks under fatigue loading destroys material bonds and alters elastic properties as observed in the reduction of material stiffness. In this paper, the proposed theory has shown that glass-epoxy woven fabric composites during a tension-tension fatigue loading are highly affected by the directionality nature of damage leading to the splitting failure modes. To reflect damage in the material, fourth-order elastic stiffness tensor is used. The evolution of cycle-dependent damage parameter is described by the second invariant of the positive cones of the strain tensor, since the damage is assumed to take place in the direction of the applied tensile strain (i.e., cleavage mode of cracking). The model is capable of capturing the stiffness degradation and inelastic deformation caused by fatigue induced cracking. The model is also checked against experimental data for tension-tension fatigue loading.

Keywords

Glass-epoxy woven fabric composites – Damage – Fatigue – Anisotropic – Inelasticity

1. Introduction

Fiber-reinforced composites are increasingly used in numerous aerospace, automotive and construction industries because of their high specific strength and stiffness. Woven fabric composites are widely used in civil engineering, because they possess superior balanced properties in the fabric plane such as damage tolerance, dimensional stability, impact resistance, high strength and better performance during bi-axial stress state as compared to conventional unidirectional lamina. The ease of manufacturing and handling adds its beauty more.

The fatigue behavior of composite materials has been a topic of active research in recent years. Widespread studies and researches on fatigue have concluded that as many as 90% of all the material failures are caused by fatigue [1]. Number of experiments and researches on fatigue of fabric composite have occurred in past. For homogeneous monolithic materials with isotropic material properties, fatigue failure is governed by a single crack that propagates in a direction perpendicular to the cyclic loading axis [2]. However, in composite materials, Rim Ben Toumi [3] in his experiment observed that the failure is characterized by multiple damage modes such as matrix cracking, fiber-matrix delamination and debonding, fiber breakage etc. Multitude of cleavage type distributed cracks which are highly directional were observed by Hansen [4]. In order to account for progressive distributed damage, Continuum Damage Mechanics which dates back Kachanov 1958, has been considered to be very useful for composite fatigue and has been followed by many researchers [2, 4, 5, 6, 7, 8]. The strain space approach is accepted to be very advantageous for numerical simulation and for complete stress-strain behavior [9, 10, 11].

The idea of using the changes in the material stiffness E as indirect measure of crack growth, internal changes and energy dissipation due to damage process during fatigue has been widely used in past researches [2, 3, 4, 12, 13]. One such approach was used by

Hansen [4] in his fatigue damage model for woven fabric composite.

Hansen [4] performed experiments and observed damage initiation and growth in glass-reinforced woven composites both in static and fatigue environment. Three types of specimens were taken for experiment, namely: undamaged, barely visible impact damaged (BVID) and penetrated damage. Infrared thermography was used as non-destructive inspection technique for detecting damage initiation and growth. Hansen [4] observed that fatigue produces continuously changing non-uniform stress fields due to impact induced local stress raiser and stress redistribution effects. Further three damage phases were noticed by Hansen [4]. In phase I, material property was degraded sharply and was caused by “knee-effect”, whereby glass-epoxy debonding and failures were observed in transverse fiber bundles resulting in microcracks. Saturation of these cracks indicated the end of phase I and beginning of phase II. In Phase II, friction between fibers and some delamination were observed that continued the damaging process keeping the microcrakcs still progressively distributed. Almost 92% of the fatigue life was governed by phase I and II. Similarly, phase III was characterized by localized damage, fiber breakage and failure of the specimen. Hansen [4] has developed constitutive model for the fatigue damage for phase I and II using Continuum Damage Mechanics. The material compliance tensor was considered as an internal variable evolving with damage. The developed model was multi axial but could not address anisotropic effects of cracking and was unable to capture permanent deformation due to fatigue environment.

Wen and Yazdani [6] has developed anisotropic and inelastic fatigue damage model for glass-epoxy woven fabric composites during tension-tension fatigue based on damage mechanics theory which was capable of incorporating multitude of matrix cracking and interfacial debonding. The model was capable of capturing the elastic degradation process as well as the permanent deformation due to damage and anisotropic characteristics also. The model predictions were compared with Hansen’s experimental works. However, the formulation relied on stress based approach and subsequently the static stress-strain behavior has not been illustrated. Cyclic stress-strain behavior has also not been able to satisfy the developed S-N plot. The variation of inelastic strain accumulation with number of cycle has not been discussed. Further, Wen et al. [7] has developed generalized unified bounding surface approach guided by isotropic hardening/softening theories of plasticity and damage mechanics to predict the fatigue behavior of the woven composites under biaxial loadings, however, it still was stress space formulation.

Thapa and Yazdani [9] has developed a damage model for structural concrete in strain space within the continuum thermodynamics framework. The equivalency of the stress and the strain space formulation was established. Rate independent behavior, infinitesimal deformations and isothermal conditions were assumed and Helmholtz Free Energy (HFE) was utilized as an energy potential to develop damage surface. Anisotropy caused by induced cracking was captured by developing and using kinetic relations which was developed by adopting additive decomposition of the stiffness tensor. The prepared damage model was capable of capturing the general mechanical behavior of concrete in both tension and compression. However, the model lacks addressing the material non linearity under large confining pressure which actually can be captured using a plasticity type approach.

This paper aims to perform the extension of the strain space formulation of Thapa and Yazdani [9] in fatigue damage modeling of glass/epoxy woven fabric composites, because such formulation involves iterative procedure which actually reduces processing time and also optimizes the data storage. Further with such formulation the complete stress-strain behavior of such material can be illustrated and it would also be possible to use the model in displacement based finite element method in order to enhance the computational efficiency of the numerical simulation.

2. Formulation

The strain space formulation of continuum damage mechanics theory within the framework of widely accepted approach of thermodynamics with internal variables by Coleman & Gurtin [14] and Lubliner [15] is used for the general formulation of this research i.e. to describe the constitutive relation for glass woven fabric composite. Here, the low frequency fatigue loading is assumed so that the thermal effects could be ignored. For such isothermal process with rate independent behavior and small deformations (for brittle material), the Helmholtz Free Energy (HFE) per unit volume can be deduced from Thapa and Yazdani [9] as below,

$$A(\boldsymbol{\varepsilon},k) = \frac{1}{2}\boldsymbol{\varepsilon} : \mathbf{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^i : \boldsymbol{\varepsilon} + A^i(k) \tag{1}$$

where, the stress and strain tensors are given by $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ respectively, A denotes Helmholtz Free Energy, $\mathbf{E}$ denotes the material stiffness tensor that depends on the state of microcracks and k is the cumulative scalar fatigue damage parameter. Here, the tensor contraction operation is designated by (:) and the stress tensor corresponding to the inelastic damage is given by $\boldsymbol{\sigma}^i$.

For an inelastic damaging process, a constitutive relation between stress and strain tensors can be established utilizing fourth-order material stiffness tensor as,

$$\boldsymbol{\sigma} = \frac{\partial A}{\partial \boldsymbol{\varepsilon}} = \mathbf{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^i(k) \tag{2}$$

Assuming N being the fatigue cycle number, taking rate form of equation (2) by differentiating with respect to N,

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}(k) : \dot{\boldsymbol{\varepsilon}} + \dot{\mathbf{E}}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^i(k) = \dot{\boldsymbol{\sigma}}^e + \dot{\boldsymbol{\sigma}}^D - \dot{\boldsymbol{\sigma}}^i(k) \tag{3}$$

where, $\dot{\boldsymbol{\sigma}}^e$ denotes the stress increment when further damage in the material is prevented, $\dot{\boldsymbol{\sigma}}^D$ is the stress relaxation rate due to further micro cracking (elastic damage) and $\dot{\boldsymbol{\sigma}}^i(k)$ designates for the stress tensor rate corresponding to the irreversible deformation due to micro cracking. It is further assumed that, damage during fatigue loading degrades elastic properties and affects the stiffness tensor. The damage is recorded in the fourth order material stiffness tensor $\mathbf{E}$. To account for induced anisotropy, the following additive decomposition of $\mathbf{E}$ is adopted,

$$\mathbf{E}(k) = \mathbf{E}^O + \mathbf{E}^D(k) \tag{4}$$

where, $\mathbf{E}^O$ is the stiffness tensor of the initial undamaged or uncracked material and $\mathbf{E}^D(k)$ is the overall stiffness degradation caused by damage during fatigue loadings. Further, the rates of stiffness tensor $\dot{\mathbf{E}}(k)$ and that of the inelastic stress tensor $\dot{\boldsymbol{\sigma}}^i$ are expressed as fluxes in thermodynamics state sense and are expressed in terms of evolutionary equations as,

$$\dot{\mathbf{E}}^D = -\dot{k}\mathbf{L} \qquad and \qquad \dot{\boldsymbol{\sigma}}^i = \dot{k}\mathbf{M} \tag{5}$$

where, $\mathbf{L}$ and $\mathbf{M}$ are the fourth and second order response tensors that determine the direction of the elastic and inelastic damage processes.

To progress further, specific forms of response tensor $\mathbf{L}$ and $\mathbf{M}$ must be specified. Since the damage is highly directional (i.e. anisotropic), the response tensor should be formulated to achieve such anisotropy. For addressing anisotropy and formulating response tensor, lets decompose the strain tensor into its positive and negative cones. The positive and negative cones of the strain tensor holds the corresponding positive and negative Eigen values of the system. Note that $\varepsilon = \varepsilon^+ + \varepsilon^-$. Guided by the experimental results and observations for glass/epoxy woven fabric composite materials in tension by Hansen [4], where majority of damage is shown to take place in the direction of applied strain and in tension regimes (i.e. the cleavage mode of cracking as shown in Figure 1), and further assuming that there is no coupling between cleavage type cracks in orthogonal direction, the following form of response tensors are proposed.

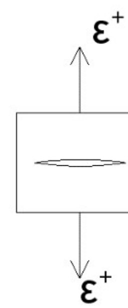

Figure 1: Cleavage mode of cracking

$$\mathbf{L} = \frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+} \tag{6}$$

$$\mathbf{M} = \frac{\alpha \boldsymbol{\varepsilon}^+}{(\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+)^{1/2}} \tag{7}$$

where, the symbol "⊗" is the tensor product operation. Here, it should be noted that the material used in Hansen's work was assumed to be quasi-isotropic laminate, with the lay-up sequence of $[(+45^0\#{-45^0})/(90^0\#0^0)]_s$, made up of plain woven glass/epoxy prepreg fabrics. So, the strength values in different directions are considered equal initially.

In this paper, an alternate and new damage evolution law has been proposed based on the second invariant of the positive cone of the strain tensor considering the fact that the damage in each cycle depends on the following factors,

- Current elastic stiffness E
- Number of cycles N
- Second invariant of the positive cone of strain tensor

Therefore, the cumulative damage can be written as,

$$k = E\int_0^N \mathrm{A}\left(\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}{\varepsilon_o{}^2}\right)^{\mathrm{m}} N^B \, \mathrm{d}N \tag{8}$$

where, A, B and m are material constants, N is the number of cycles and ε_o is the reference strain level. Differentiating with respect to number of cycles N, the increment damage in one cycle is given as,

$$\dot{k} = EA\left(\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}{\varepsilon_o{}^2}\right)^m N^B \tag{9}$$

Substituting equations (6), (7) and (9) into equation (5), yields,

$$\dot{\mathbf{E}}^D = -EA\left(\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}{\varepsilon_o{}^2}\right)^m N^B \frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+} \tag{10}$$

$$\dot{\boldsymbol{\sigma}}^i = EA\left(\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}{\varepsilon_o{}^2}\right)^m N^B \frac{\alpha\boldsymbol{\varepsilon}^+}{(\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+)^{1/2}} \tag{11}$$

Substituting equations (4) and (5) into equation (3), we get,

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}} + \left[-EA\left(\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}{\varepsilon_o{}^2}\right)^m N^B \frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}\right] : \boldsymbol{\varepsilon}$$

$$-EA\left(\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}{\varepsilon_o{}^2}\right)^m N^B \frac{\alpha\boldsymbol{\varepsilon}^+}{(\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+)^{1/2}} \tag{12}$$

3. Numerical Simulation and Results

The proposed model has four material parameters A, B, m and α. In order to check the validity of the model and to obtain the values of the parameters introduced, numerical simulation have been performed and the comparison is done with the experimental results by Hansen [4]. Due to scarcity of experimental data in the literature for the measurement of these parameters in the numerical simulation, judgments are used to obtain the numerical results.

On the basis of the experiment carried out by Hansen [4], the following constants are used for numerical simulation. A = 0.03, B = -0.74, m = 1.4, and α=0.0035. These parameters are estimated by comparing predicted results and experimental results over a range of applied stresses. The initial strain tensor is taken $\boldsymbol{\varepsilon}^+$ = [0.007 0 0] corresponding to the applied far-field stress tensor of $\boldsymbol{\sigma}^+$ = [155 0 0] MPa. The reference strain is fixed at 0.0156. In the case of BVID specimen, the initial uniform impact damage is considered to be 2.5% prior to fatigue.

The complete static monotonic stress-strain behavior is illustrated and compared with experimental data by Hansen [4] in Figure 2. The similar trends are observed. In the model prediction, the maximum stress level is obtained to be 310MPa giving a strain of 0.02. After this peak value, the stress goes on decreasing as strain keeps on increasing. The material is said to be reached at failure strain at the value of 0.025 as also described in the experiment. The behavior is non-linear.

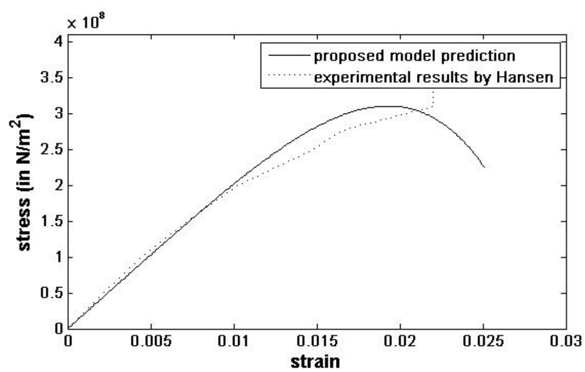

Figure 2: Static monotonic stress-strain curve

Similarly, Figure 3 clearly shows that the proposed model captures the initial damage due to the knee effect quite well. The model is useful in prediction of Phase I and Phase II but could not predict Phase III. Figure 4 is a curve that is plotted between damage and number of cycles. It shows that, damage goes on increasing with the number of cycles rapidly during first few cycles and then it progresses slowly during phase II.

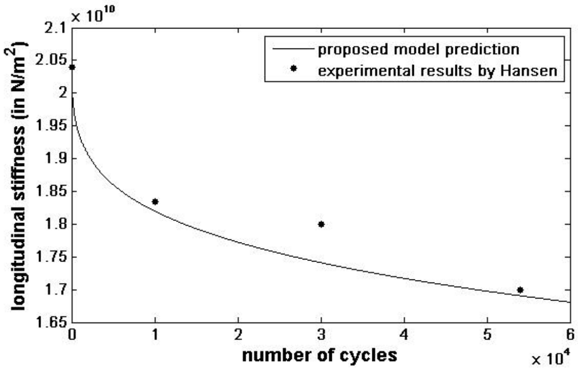

Figure 3: Longitudinal stiffness reduction with number of cycles

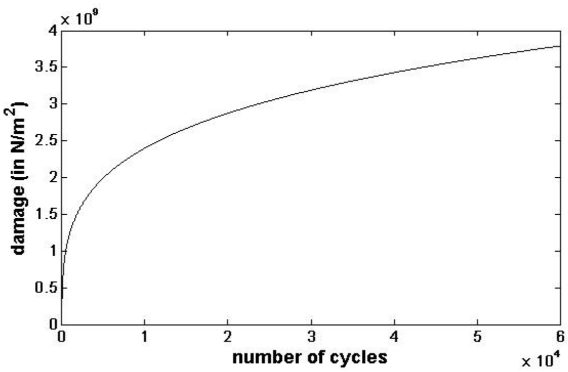

Figure 4: Damage vs number of cycle

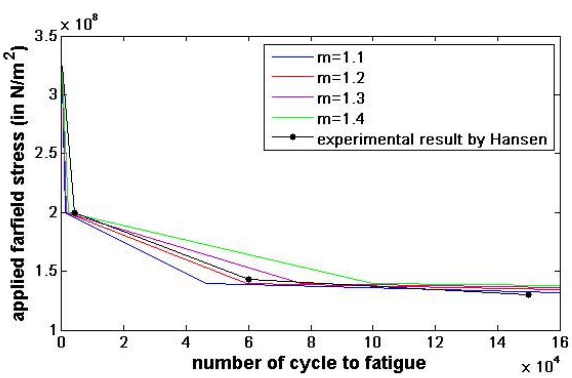

Figure 5: Comparison of experimental S-N curve with predicted S-N curves

The predicted stress (S) vs number of cycles (N) i.e. S-N curve is compared with the experimental S-N curve by Hansen [4] for BVID specimen in Figure 5 and the effect of material parameter "m" on predicted S-N curve is also shown. It can be seen that the trend of predicted and experimental S-N curves matches each other which is quite satisfactory.

The fatigue stress-strain curves are shown in Figures 6 and 7. Figure 6 illustrates the elastic degradation as well as a permanent deformation due to inelastic behavior. However, Figure 7 illustrates the nature when the composite is treated as perfectly elastic with only reversible deformations, though the damage is progressively accumulating as evidenced by degradation of longitudinal stiffness. This behavior corresponds to an idealized case whereby crack faces close perfectly upon unloading and is achieved in the model by letting $\alpha = 0$. But in most of the heterogeneous materials like glass-fiber composites, permanent deformations take place which has been replicated in Figure 6. The inelastic damage accumulation i.e. the cumulative permanent deformation under fatigue loading is illustrated in Figure 8.

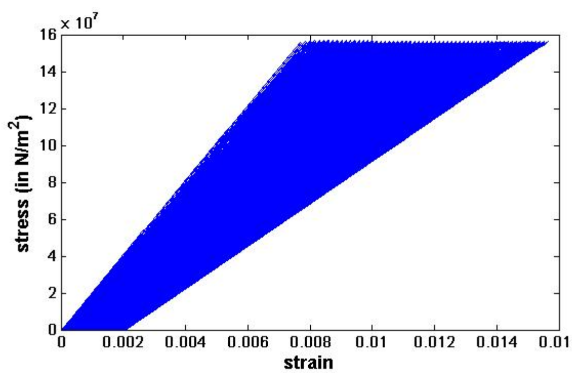

Figure 6: Stress-strain behavior of woven composite during inelastic damage accumulation ($N_f = 48993, \varepsilon_f = 0.0156$, $\sigma_f = 155MPa$)

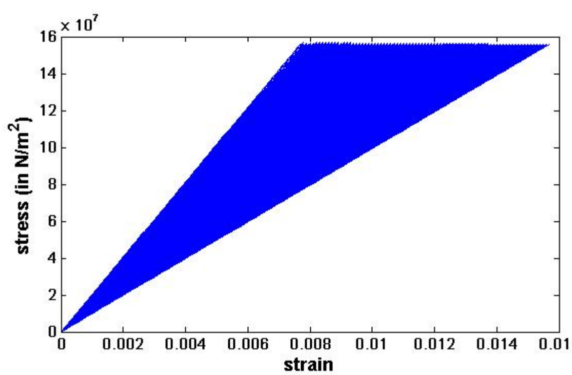

Figure 7: Stress-strain behavior for elastic damaging process ($N_f = 129355$, $\varepsilon_f = 0.0156$, $\sigma_f = 155MPa$)

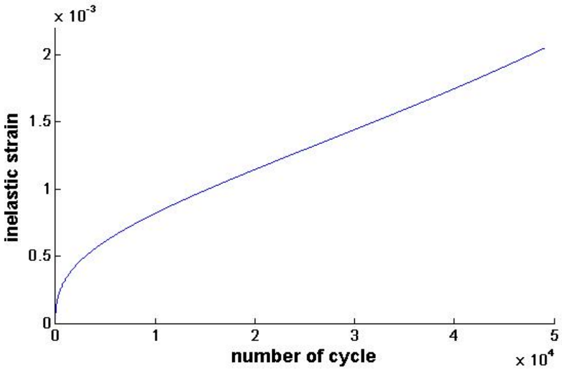

Figure 8: Inelastic damage accumulation in fatigue environment

4. Conclusion

An anisotropic-inelastic fatigue damage model is established for glass/epoxy woven fabric composite in tension-tension fatigue utilizing Continuum Damage Mechanics formulation in strain-space. The distributed progressive weakening of the composite due to the formation of microcracks and microvoids is reflected through the reduction of fourth-order material stiffness tensor. The damage mechanics formulation is cast within the general framework of the internal variable theory of thermodynamics considering rate-independent behavior, infinitesimal deformations and Helmholtz Free Energy (HFE) as an energy potential. The proposed damage evolution law consists four damage parameters. For the numerical simulation, the parameters A, B, m and α are determined from the sensitivity study and the values are selected such that they best fit the experimental data.

With this predicted model one can find the fatigue life of a glass/epoxy woven fabric composite for a given value of working stress amplitude by looking into the developed S-N curve. However, this model is quite simple as it considers only uniaxial loading condition and has a potential to be extended for multiaxial loading conditions.

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