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Mathematical Modeling for the Design of Francis Runner

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Abstract
There exists lots of designing software and methodologies for designing Francis runner but they require huge software skills, complicated primary calculations and they are costlier to purchase. This paper reports the process of Mathematical modeling for the calculation of major dimensions of Francis runner. This paper uses the MATLAB program to arrange the mathematical model in order and calculate the required dimensions. MATLAB program starts with taking input as net head and volume flow rate . MATLAB program contains the provision for varying energy distribution along streamline to vary the beta distribution to obtain the optimum designed blade. This paper mainly focuses for mathematical modeling of Francis runner and obtains the Three Dimensional (3D) model of runner. This runner can be subjected to Computational Fluid Dynamics (CFD) simulation to obtain the blade efficiency and for further optimization.

Keywords
Francis Turbine, MATLAB, ANSYS, CATIA, 3D, Simulation

1. Introduction

Nepal has a huge hydro power potential but only about 2 % has been extracted yet [1]. Nowadays, there are lots of under construction and construction survey undergoing for developing hydro power in Nepal. According to [2], 88% (995MW) of planned and proposed hydro power are going to use Francis turbine. So, there is a huge scope for Francis turbine manufacturing in Nepal. But the design process of Francis turbine is site specific. This process involves lots of iterative tedious calculation, which leads to delay in design and manufacturing. The classic design through grapho-analytical method is cumbersome and did not put-in evidence to the usual positive and negative aspect of designing option [3]. So, it is necessary to develop Computer Aided Design (CAD) technique for designing Francis runner.

There are many studies about Francis turbine design in the literature. Some of them are direct method, Inverse method, Curve fitting method, Bovet and Con formal mapping method. One of the most important design approaches of Francis turbine design is direct method. Here, the designing process begins with the inlet conditions and based on the inlet conditions, various dimensions of the runner is calculated and based on the runner dimension various dimension of other component are calculated. Kacak, E. et al, used the Bovet and Con formal mapping method for preliminary design of Francis Turbine. In Bovet method, the dimensionless Specific speed value is the main parameter to determine whole turbine dimensions. The author modeled the 3D view of the runner using bladegen module and performed CFD analysis for design verification [4]. Milos, T. et al, presented the step by step computer aided design technique for the Francis turbine design. The author used Bovet and Con formal mapping method [3]. Fatma, A. et al, used in house MATLAB codes to determine the major dimensions of Francis turbine especially for low head system and use CFD based Design methodology for design optimization [5]. Choi, H. et al, describes the method of CFD validation of performance improvement of 500kw Francis turbine. Here the author concluded that the CFD prediction efficiency generally agrees well with the experimental data and the discrepancy lies within 1.5% [6]. Shrestha, K. et al, describes the alternative optimized design of Francis turbine runner for large sediment load. Here the author used in-house MATLAB software called “khoj” for obtaining various dimensions of Francis runner and used PRO/Engineer software for 3D modeling and used ANSYS CFX for design optimization. The author compared the result with the test experiment of Jhimruk HPS [7]. Khanal, K. et al, explain the methodology for designing the Francis runner blade to find minimum sediment erosion using CFD. Here the author had taken the case of Devighat HPS. The CFD analysis was performed to obtain optimum blade angle distribution [8]. There are various methodology and software exist for designing the Francis turbine but most of them are very limited to use, some requires high software skills, complicated primary calculations and very expensive. It demands the simple, economic and user friendly technique to design the Francis runner. The main objective of this paper is to purpose the step by step methodology for mathematical modeling to obtain the Francis runner. This paper also aims to present the MATLAB results and 3D model of Francis runner. Devighat Hydro Power Stationas(HPS) (head=39 m and mass flow rate= 14.3 m^3/s) located at Nuwakot district, Nepal, has been used to obtain 3D view of the runner. The in-house MATLAB software code used for primary data calculation of major dimension of Francis runner is completely original and is in a continuous process of up-gradation.

Nomenclature

C	absolute velocity (m/s)
C_m	meridional component of C (m/s)
C_u	tangential component of C (m/s)
U	peripheral velocity (m/s)
W	relative velocity (m/s)
β	angle between relative and peripheral Velocity (degree)
η	efficiency
n	synchronous speed
D	runner diameter
1	inlet section
2	outlet section

Underline shows the reduced parameter

2. Methodology

2.1 Mathematical Modeling

The mathematical modeling was started with inlet conditions (net head (H) and volume flow rate (Q)). The different formulas for mathematical models were obtained from various articles in journals [3–6, 8, 9] , Technical Manuals [10] , Thesis [11] and books [12] . The collected formulas were placed in order to proceed one after another. First, main dimensions were calculated and then dimensions for axial view of runner and after that provision of changing energy distribution to calculate beta distribution, Radial view and 3D view of the runner and finally thickness of the runner .

2.1.1 Main dimensions

This includes the calculations of outlet dimensions, characteristic number and inlet dimensions of the runner blades. The dimensioning of the outlet was started with assuming no rotational speed at best efficiency point (BEP) that is Cu2 =0. The unknown parameters were obtained from various relations.
Assumptions:

$$c_{u2} = 0; 13 \le \beta_2 \le 22;$$
$$35 \le U_2 \le 42; 0.7 \le \underline{u_1} \le 0.75;$$

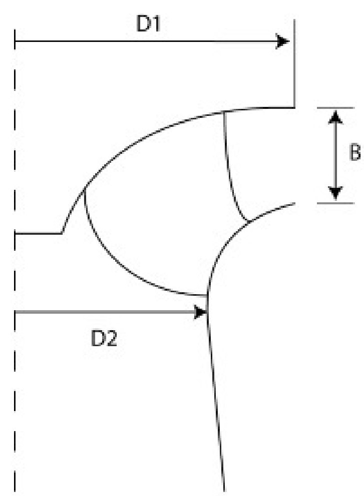

Figure 1: axial view

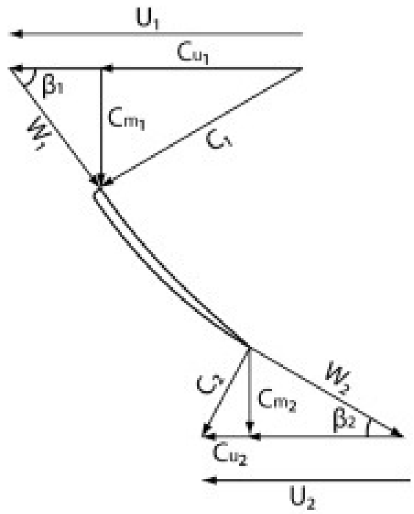

Figure 2: velocity diagram

Figure 1 shows the axial view of the runner with inlet and outlet diameter and height. Figure 2 shows the velocity diagram of inlet and outlet of runner.
Outlet relations:

$$D2 = \sqrt{\frac{4Q}{(\pi \times C_{m2})}}; \tag{1}$$

$$n = \frac{(60 \times U_2)}{(\pi \times D_2)}; \tag{2}$$

$$C_m 2 = U2 * tan\beta 2; \tag{3}$$

$$Z_p oles = \frac{3000}{n} \tag{4}$$

The obtained values of Z_{poles} should be Integer, so the result from equation (4) should be approximated to integer value and based on that the corrected value of n, C_{m2}, D_2 need to be calculated and assigned to original symbol. The relations for correction are listed below:

$$n_{corrected} = \frac{60 \times f}{Zp}; \tag{5}$$

$$D_{2,corrected} = (\frac{n \times D_2^3}{n_{corrected}})^{\frac{1}{3}}; \tag{6}$$

$$U_{2,corrected} = \frac{(\pi \times D_{2,corrected} \times n_{corrected})}{60}; \tag{7}$$

$$\beta_{2,corrected} = \tan^{-1}(\frac{(240 \times Q)}{(\pi^2 \times D_{2,corrected}^3 \times n_{corrected})}; \tag{8}$$

$$C_{m2,corrected} = U_{2,corrected} \times \tan(\beta_{2,corrected}) \tag{9}$$

Speed number (Ω):

$$\Omega = \underline{\omega} \times \sqrt{\underline{Q*}} \tag{10}$$

where,

$$\underline{\omega} = \frac{\omega}{\sqrt{2 \times g \times H}}$$
$$\underline{Q} = \frac{Q}{\sqrt{2 \times g \times H}}$$

Inlet dimensions:

$$\eta_h = 2 * \underline{u_1} * \underline{C_{u1}}; \tag{11}$$

where,

$$\underline{C_{u1}} = \frac{C_{u1}}{\sqrt{2 \times g \times H}};$$

$$\underline{u_1} = \frac{U_1}{\sqrt{2 \times g \times H}}$$

$$D_1 = \frac{60 \times U_1}{n \times \pi}; \tag{12}$$

$$\tan\beta_1 = \frac{C_{m1}}{U_1 - C_{u1}}; \tag{13}$$

$$C_{m2} = 1.1 \times C_{m1}; \tag{14}$$

$$B_1 = \frac{(1.1 \times {D_2}^2)}{(4 \times D_1)} \tag{15}$$

2.1.2 Runner axial view

The elliptical profile for hub or shroud was considered and the profile of it was divided into number of equal slot of each length, ds. The start and end point of each streamline were obtained by divided inlet and outlet height into equal points.

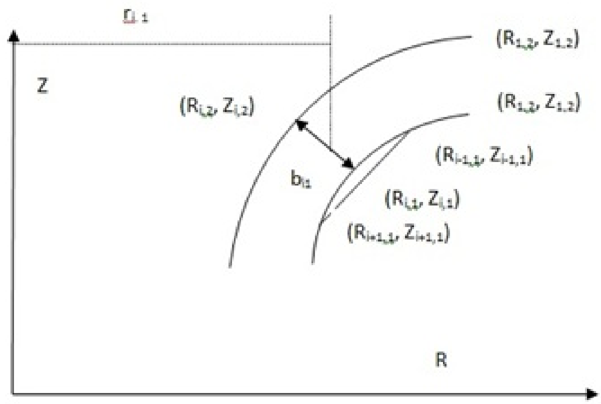

Figure 3: finding a new point on streamline

The co-ordinates of each point on starting stream line (hub or shroud) were obtained by dividing it into number of small elements and by using the interpolation, the co-ordinate of other stream lines were obtained as shown in Figure 3. The equation of ellipse is given as:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \tag{16}$$

where,

$$a = \frac{(D_1 - D_2)}{2}$$
$$b = 0.75 \times a$$

$$s = \int_0^{slutt} \sqrt{(1 + (\frac{dy}{dx})^2 dx}; \tag{17}$$

$$\frac{dy}{dx} = \frac{b}{2} \times \frac{(-2x)}{a^2} * (1 - (\frac{x}{a})^2)^{0.5} \tag{18}$$

$$\alpha_{i,1} = \tan^{-1}\frac{(Z_{(i-1,1)} - Z_{(i+1,1)})}{(R_{(i-1,1)} - R_{(i+1,1)})}; \tag{19}$$

$$r_{i,1} = \frac{(R_{(i,1)} + R_{(i,2)})}{2}; \tag{20}$$

$$b_{i,1} = \frac{(R_{(i,2)} - R_{(i,1)})}{\sin(\alpha_{i,1})}; \tag{21}$$

$$R_{i,2} = \sqrt{R_{i,1}^2 + \frac{A_{i,1} \times \sin\alpha_{i,1}}{\pi}}; \tag{22}$$

$$Z_{i,2} = Z_{(i,1)} - b_{(i,1)} \times \cos\alpha_{i,1}; \tag{23}$$

$$A_{i,1} = 2 \times \pi \times r_{i,1} \times b_{i,1}; \tag{24}$$

In the above set of equations, $R_{i,2}$ and $Z_{i,2}$ gives the point on the second streamline, similarly other points of other streamlines for axial view were obtained using relations as above.

2.1.3 Runner energy distribution

After axial plane was constructed, the energy distribution was chosen, assuming the distribution to be equal for all streamlines. So, beta distribution along the stream line was obtained from,

$$\beta = \tan^{-1}(\frac{C_m}{(U - C_u)}); \tag{25}$$

2.1.4 Runner radial view

Before finding the radial view, the G, H plane was obtained, which had simplified the process of going from axial view to radial view of the runner,

$$G_{i,1} = G_{(i-1,1)} + \sqrt{((R_{(i-1,1)} - R_{(i,1)})^2 + (Z_{(i-1,1)} - Z_{(i,1)})^2)} \quad (26)$$

$$\triangle H = \frac{\triangle G}{\tan(\beta)}; \quad (27)$$

$$d\theta = \frac{\triangle H}{R}; \quad (28)$$

with increasing the θ by d θ , the values of R and corresponding θ is obtained and (R, θ)is converted to Cartesian co-ordinate form. These are the co-ordinate for x and y.

2.1.5 3D view of runner

The co-ordinate obtained from axial view are x, z and co-ordinate from radial view are x, y. On combining these axial and radial views, the 3D co-ordinate of each point of runner blade (x, y, z) are obtained.

2.1.6 Thickness of runner

To simplify the runner analysis, runner was considered as I-section with thickness t, maximum bending moment σ, height b, and differential pressure $\triangle p$.

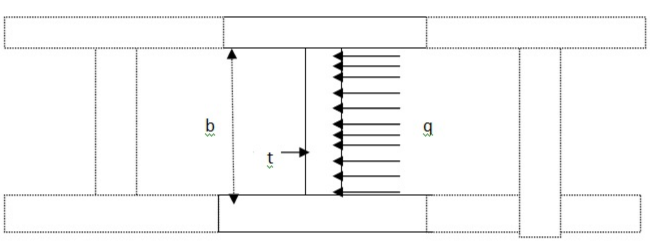

Figure 4: simplified figure of runner blade zone

Figure 4 shows simplified runner blade sections.

$$q = \triangle r \times \triangle p; \quad (29)$$

$$t = \sqrt{\frac{2 \times b^2 \times \triangle p}{\sigma}} \quad (30)$$

2.2 MATLAB Coding

The mathematical formulations were written in terms of codes in MATLAB script files. It contains main program m-file, runner m-file and input m-file. The main program calls the input m-file and runner m-file. After running the main program, it asks for inlet conditions (net head and net volume flow rate in SI unites). After providing the asked values, it was programmed to store the data in Excel file which contains data of 3D view of runner. The MATLAB program was also designed to obtain the graphs of different views during building of 3D model.

3. Results and discussion

Devighat HPS located at Charghare VDC, Bidhur Municipality, and Nuwakot District, Nepal is taken as reference. It is the Cascade of Trishuli Hydro Power Station with 14.1 megawatt installed capacity.

Table 1: parameter values obtained from in-house MATLAB code

Parameters	symbols	values	units
Estimated power output	P	5.2417	MW
Outlet diameter, runner	D	1.358	m
Outlet angle, runner	β_2	32.9431	degree
Outlet meridional velocity	C_{m2}	9.8732	m/s
Outlet peripheral velocity	U_2	15.2365	m/s
Speed number, runner	Ω	0.58249	-
NPSH	NPSH	7.3238	m
Inlet diameter, runner	D_1	2.2218	m
Inlet height, runner	B_1	0.22825	m
Inlet blade angle, runner	β_1	41.5068	degree
Inlet meridional velocity	C_{m1}	8.9756	m/s
Inlet peripheral velocity	C_{u1}	24.8957	m/s

Devighat HPS contains three Francis turbine each of capacity 5.03 megawatt, 39 m head and 14.3 m^3/s flow rate. These head and flow rate are given as input to obtain the Francis runner.

The developed software gives the various dimension which is listed in Table 1. This software code also gives the Excel data in excel data sheet and the curves which are shown in different figures below.

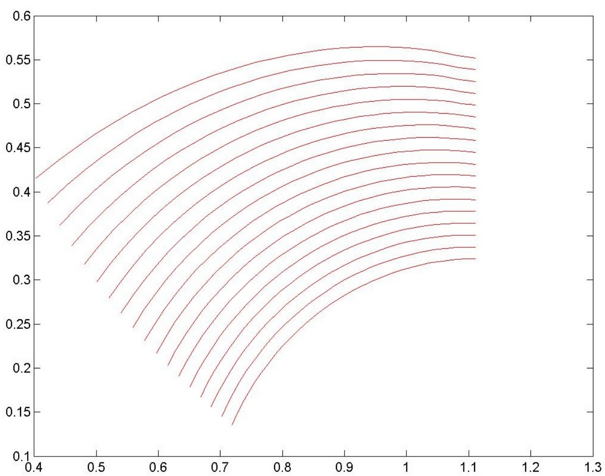

Figure 5: Axial view of runner

Figure 5 shows the axial view of the runner in the x-z plane. Here the vertical line represents the height and the horizontal line represents the length along x axis.

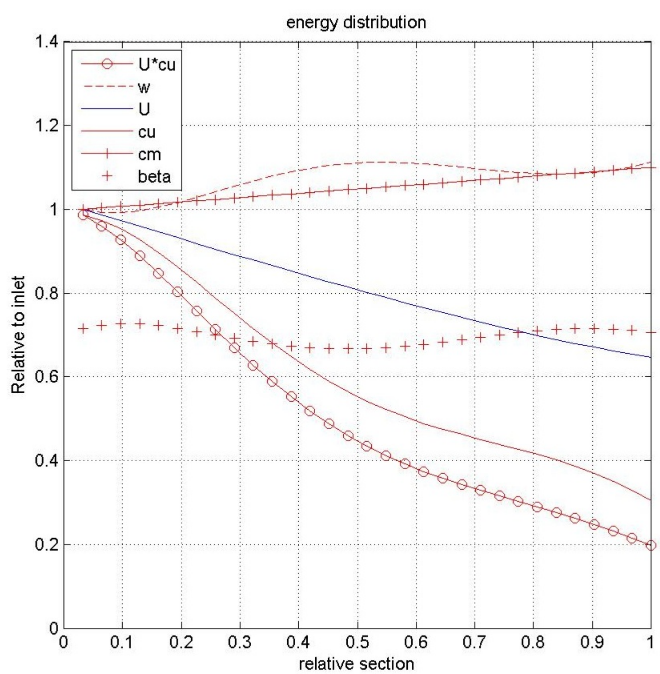

Figure 6: Energy distribution along streamline

Figure 6 gives the energy distribution along the stream lines. These energy distribution can be changed to get different blade curvature. This is set by the designer.

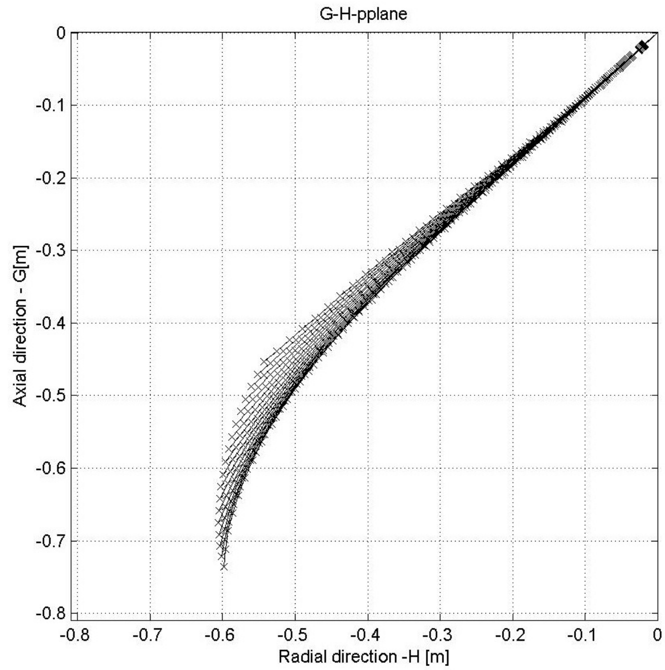

Figure 7: Plot along G-H plane

Figure 7 shows the G-H plane that is, the vertical line shows axial directions and horizontal lines shows the distribution along Radial direction. This plot helps for easy transition between axial view to radial view.

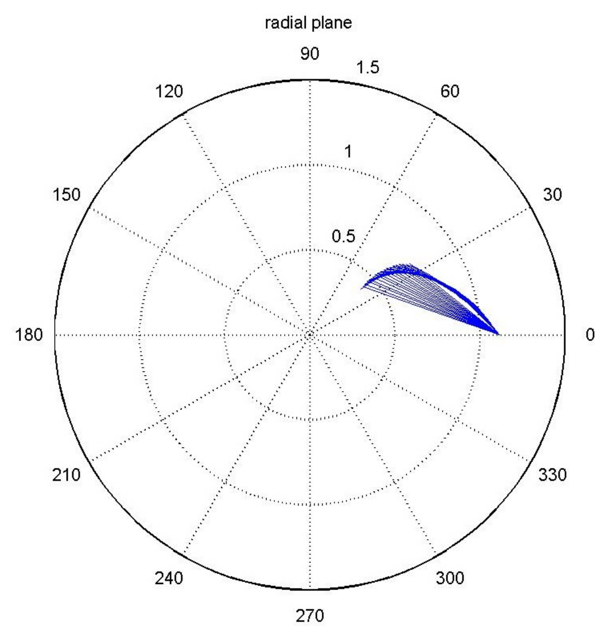

Figure 8: Plot along Radial plane

Figure 8 shows radial view. This gives the twisting of the blades in the radial directions.

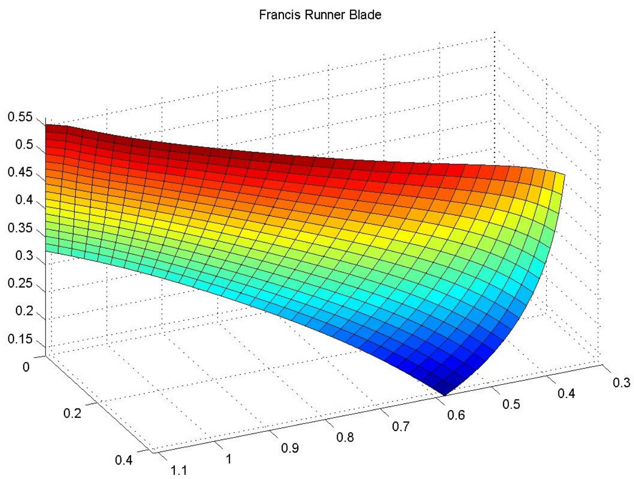

Figure 9: 3D plot of single blade

Figure 9 shows 3D view of the single runner blade. This view is obtained by combining the co-ordinate along x, y, z directions.

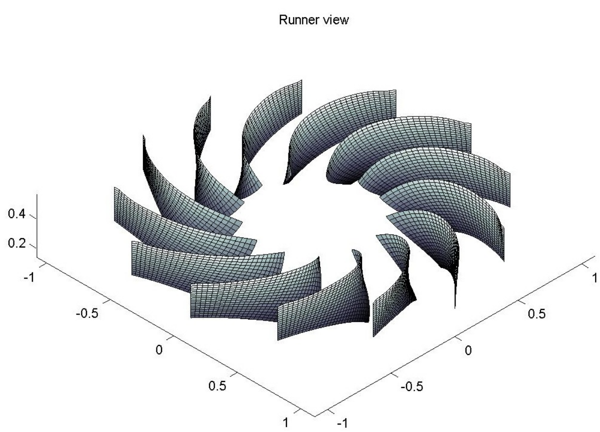

Figure 10: isometric view of runner blades

Figure 10 shows isometric view of Francis runner blade assembly, this contains the 13 numbers of blades obtained from the developed software.

The excel data on exporting to CATIA software and 3D modeling of it and analysing it in ANSYS CFX has given the hydraulic efficiency of turbine runner to be 59.6% at the single cycle iteration, with k-w SST modeling, steady state flow. This value of efficiency is close to papper [13] , under similar conditions. The hydraulic efficiency of designed Francis turbine obtained in ANSYS CFX is not very high, it is because of the reason that the simulation was carried out for coarse mesh and for steady state and also for only one cycle iteration simulation (means no further modification of blades was carried out). This efficiency can be improved by changing energy distribution assumption along streamlines , we considered earlier, and followed by simulation until the optimum results is obtained.

4. Conclusion

This study has showed a methodology for mathematical modeling of designing Francis runner blade by using the direct method. Here, the runner blade of Devighat hydro power Station is considered as reference blade to obtain the 3D model of optimized blade using developed software (MATLAB code). The developed software generated the Francis runner with hydraulic efficiency 59.6% which demands for further optimization by changing the energy distribution, the software includes this provision. The reference blade has installed capacity of 5.03 MW, but the developed software has given the capacity of 5.2471 Mw which means, the capacity of reference blade may be further improved. From the result and discussion section, it is concluded that the hydraulic efficiency of obtained Francis runner is acceptable for given condition, thus this modeling technique and software is reliable and could be accepted and followed for designing Francis runner in most of the hydro -sites.

5. Recommendation

The developed in-house MATLAB code is based on direct method of designing Francis turbine. The result obtained from this method could be compared with results from other methods like Bovet method, curve fitting method, Inverse method etc., within the program and select the best one. In addition to this, only one cycle simulation had been carried out to obtain the optimized Francis runner, however, number of simulation cycles could be carried out with changing energy distribution and other parameters and comparing the results to obtain the best one. Furthermore, the sophisticated experiment could be done to verify the obtained 3D design of Francis runner obtained from developed software so that the designed model could be more reliable for further practical applications.

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