Proceedings of IOE Graduate Conference, 2016
pp. 147–154

Fatigue Damage Model in Plain Concrete Utilizing Damage Mechanics Theory

Aakarsha Khawas [1], Kamal Bahadur Thapa [2]

[1, 2] *Department of Civil Engineering, Pulchowk Campus, Institute of Engineering, Tribhuvan University, Nepal*
Corresponding Email: [1] aakarsha2003@yahoo.com

Abstract

A simple time-independent model for the cyclic behavior of concrete has been proposed. The formulation is cast within the framework of internal variable theory of thermodynamics and continuous damage theory is used to describe the progressive weakening behavior of the concrete. Strain space formulation is considered (elastic strain as observable variable) with anisotropic damage (represented by a single damage variable k). The formation of microcracks and microvoids destroy material bonds and affects elastic properties. The dependence of material stiffness on the damage parameter allows induced anisotropy to be captured through the components of material stiffness. Damage mechanics theory is used to model the fatigue damage and failure of concrete caused by multitude of cracks and microcracks whereby anisotropic damaging behavior is captured through the use of proper response function involving damage parameter in material stiffness tensor with further multiplying by suitable softening function. The increment of damage parameter is obtained from consistency equation in cycle dependent damage surface in strain space. The model is also capable of capturing the inelastic deformations that may arise due to misfits of cracks. By comparison with experimental data, the model shows good capability to describe the essential properties of concrete in uniaxial compressive fatigue loading.

Keywords

Fatigue — Concrete – Damage mechanics — Time-independent — Strain – Stiffness – Anisotropic

Introduction

Concrete is one of the most widely used materials in numerous civil engineering applications due to its workability and formability into various structural components. Except cement all ingredients of concrete are commonly available local materials like aggregate and water so concrete is getting more popular. Concrete is a composite material consisting of three components: the cement matrix, the aggregate and the interface between the matrix and aggregate. The cement-matrix is the weakest zone of the composite. It contains voids and microcracks even before any load has been applied. A material or a component exposed to cyclic loading leads to increase of stress concentration around the microcracks and finally leads to fracture. Forces that are required to obtain the fracture in cyclic loading are usually much less than forces that would have been required in case of monotonic loading. Phenomenon that deals with this type of fracture is called fatigue. It is caused by progressive, permanent internal structural changes in the material, which may result in microcracks and their propagation until governing macrocracks are formed.

For a reliable design, one should apply damage theory successfully as an engineering tool in the analysis of complex structural system. Many authors [1–12] have shown that continuum damage mechanics is appropriate for constitutive modeling of concrete. Continuous damage mechanics theory is mainly used to simulate the progressive degradation of material properties due to microcracking. Concrete contains numerous microcracks, even before the application of the external loads. Under applied loading, the initiation of new microcracks and the growth of existing microcracks contribute to the observed nonlinear behavior in concrete, ultimately causing failure. The existence of microcracks and their propagation cause what is termed as 'damage' to the concrete.

Bhattarai and Thapa [6] utilized continuum damage mechanics approach for modeling anisotropic inelastic behavior of concrete materials during low frequency tension-tension fatigue loading. It is assumed that within the damage surface of the given strain state, the fatigue loadings increase damage in concrete due to the growth of microcracks leading to inelastic deformations and stiffness degradation. The damage surface is modified by proposing a power function to predict the increase in damage in the material with increasing number of cycles of loading. Stress strain curve is developed where loading is done upto a fixed strain i.e. ultimate strain and then unloading is done. Sharma [9] utilized continuum damage mechanics approach for modeling fatigue damage in compression-compression fatigue. Stress strain curve is developed where loading is done upto a fixed strain i.e. ultimate strain and then unloading is done.

This paper presents stress strain curve where loading is done upto a fixed stress and then unloading is done. This makes the model very useful as it has a practical approach. In most of the experimental researches, the loading is done upto a certain stress level and then unloading is done. S-N curve has been developed. The "S-N" means stress vs. cycles to failure, which is plotted using the stress amplitude on the vertical axis and the number of cycle to failure on the horizontal axis. From the S-N curve engineers can easily read the expected lifetime of their latest creation by just inputting the stress level in the structure. The proposed model considers the development of distributed cracks in the material due to fatigue loading in uniaxial compression. As the number of cyclic loadings is increased, the concrete loses its strength due to nucleation and propagation of microcracks. During cyclic loadings the distributed cracks increase and grow to a stage where localization of cracks occurs. Modulus of elasticity of concrete degrades due to damage accumulation and the material becomes compliant. There is accumulation of inelastic strain. The deterioration of material reaches to a point where concrete cannot take further compressive loading and finally the concrete fails.

The incorporation of constitutive law based on damage mechanics theory provides a powerful analytical and computational tool to study the behavior of concrete under uniaxial compressive fatigue loading. The constitutive theory of concrete provides a coherent and comprehensive physical explanation for the observed experimental behavior. This includes such effects such as the development of distributed cracks and ensuing degradation of elastic properties. The proposed model provides a simple means of quantifying the inelastic behavior of concrete. This paper proposes a strain based constitutive model. The use of strain based model in the displacement-based finite element method enhances the computational efficiency of the numerical simulation.

1. Formulation

In this paper, it is assumed that a continuum damage mechanics approach can be taken to describe the constitutive relationship for concrete and that the fatigue loading is of low frequency so that the thermal effects could be ignored. The elastic damage behavior is characterized by the existence of a thermodynamic potential as function of the set of state variables. Strain formulation is considered (elastic strain ε as observable variable) with anisotropic damage (represented by a single damage variable k). For isothermal process, rate independent behavior and small deformations, the Helmholtz free energy (HFE) per unit volume can be deduced from Thapa and Yazdani [13] and shown as follows:

$$A(\boldsymbol{\varepsilon},k) = \frac{1}{2}\boldsymbol{\varepsilon} : \boldsymbol{E}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^i : \boldsymbol{\varepsilon} + A^i(k) \tag{1}$$

where, $\boldsymbol{E}(k)$ represents fourth order elastic stiffness tensor which depends on the state of microcracking (damage), $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{\sigma}^i$ is the stress tensor corresponding to inelastic damage. $A^i(k)$ is inelastic component of HFE associated with the surface energy of microcracks and k is the cumulative fatigue damage parameter. The colon (:) represents the tensor contraction operation.

For an inelastic damaging process, a constitutive relation between stress and strain tensors can be established utilizing fourth-order material stiffness tensor as

$$\boldsymbol{\sigma} = \frac{\partial A}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^i(k) \tag{2}$$

The rate form of equation (2) with respect to cyclic number n is given by

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{E}(k) : \dot{\boldsymbol{\varepsilon}} + \dot{\boldsymbol{E}}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^i(k) = \dot{\boldsymbol{\sigma}}^e + \dot{\boldsymbol{\sigma}}^D - \dot{\boldsymbol{\sigma}}^i(k) \tag{3}$$

where,
$\dot{\boldsymbol{\sigma}}^e$ is the stress increment in the absence of further damage in the material
$\dot{\boldsymbol{\sigma}}^D$ is the rate of stress-relaxation due to further microcracking (elastic damage)
$\dot{\boldsymbol{\sigma}}^i$(k) is the rate of stress tensor corresponding to irrecoverable or permanent deformation due to microcracking.

It is further assumed that damage during fatigue loading alters elastic properties and affects the stiffness tensor. For small deformation, the following decomposition of the fourth-order stiffness tensor $\boldsymbol{E}$ is adopted.

$$\boldsymbol{E}(k) = \boldsymbol{E}^0 + \boldsymbol{E}^D(k) \quad (4)$$

where, $\boldsymbol{E}^0$ is stiffness tensor of the undamaged or uncracked material and $\boldsymbol{E}^D(k)$ denotes the overall stiffness degradation caused by damage during fatigue loadings.

Further, $\dot{\boldsymbol{E}}(k)$ and $\dot{\boldsymbol{\sigma}}^i$ are expressed as fluxes in the thermodynamic state sense and are expressed in terms of evolutionary equations as

$$\dot{\boldsymbol{E}}^D = -\dot{k}\boldsymbol{L} \text{ and } \dot{\boldsymbol{\sigma}}^i = \dot{k}\boldsymbol{M} \quad (5)$$

where, $\boldsymbol{L}$ and $\boldsymbol{M}$ are, respectively, fourth and second order tensors that determine the direction of the elastic and inelastic damage processes. Following the Clausius–Duhem inequality, utilizing the standard thermodynamic arguments and assuming that the unloading is an elastic process, a potential function

$$\psi(\boldsymbol{\varepsilon},k) = \frac{1}{2}\boldsymbol{\varepsilon} : \boldsymbol{L} : \boldsymbol{\varepsilon} - \boldsymbol{M} : \boldsymbol{\varepsilon} - \frac{1}{2}p^2(\boldsymbol{\varepsilon},k) = 0 \quad (6)$$

is obtained as damage surface which establishes the onset of material inelasticity and stiffness deterioration where $p(\boldsymbol{\varepsilon},k)$ is interpreted as the damage function given below as

$$p^2(\boldsymbol{\varepsilon},k) = 2\left[h^2(\boldsymbol{\varepsilon},k) + \frac{\partial A^i}{\partial k}\right] \quad (7)$$

for some scalar valued function $h^2(\boldsymbol{\varepsilon},k)$. It should be noted that as long as the function $p^2(\boldsymbol{\varepsilon},k)$ is defined, the individual terms on the R.H.S. of the equation (7) need not be identified.

To progress further, specific forms of response tensor $\boldsymbol{L}$ and $\boldsymbol{M}$ must be specified. Since damage is highly directional, the response tensor should be formulated to address such directionality. In determining a proper form for the response tensor, the strain tensor is decomposed into positive and negative cones. The positive and negative cones of the strain tensor hold the corresponding positive and negative eigenvalues of the system.

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^+ + \boldsymbol{\varepsilon}^-$$

where, $\boldsymbol{\varepsilon}^+$ and $\boldsymbol{\varepsilon}^-$ represent the positive and negative cones of strain tensor respectively.

Based on the experimental observations and result for concrete materials, damage is assumed to occur in the mode II (sliding of two planes) of cracking as shown schematically in figure 1.

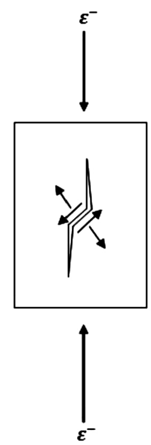

Figure 1: Schematic representation of mode II damage (sliding of two planes) under cyclic loading in compression

For cracking mode II, the following forms of response tensors are postulated for $\boldsymbol{L}$ and $\boldsymbol{M}$.

$$\boldsymbol{L} = \frac{\boldsymbol{\varepsilon}^- \otimes \boldsymbol{\varepsilon}^-}{\boldsymbol{\varepsilon}^- : \boldsymbol{\varepsilon}^-} + \frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}\eta \quad (8)$$

$$\boldsymbol{M} = \beta\boldsymbol{\varepsilon}^- \quad (9)$$

The substitution of the response tensors $\boldsymbol{L}$ and $\boldsymbol{M}$ from equations (8) and (9) into equation (6) leads to the final form of the damage surface

$$\psi(\boldsymbol{\varepsilon},k) = \frac{1}{2}\boldsymbol{\varepsilon} : \left[\frac{\boldsymbol{\varepsilon}^- \otimes \boldsymbol{\varepsilon}^-}{\boldsymbol{\varepsilon}^- : \boldsymbol{\varepsilon}^-} + \frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}\eta\right] : \boldsymbol{\varepsilon} - \beta\boldsymbol{\varepsilon}^- : \boldsymbol{\varepsilon} - \frac{1}{2}p^2(\boldsymbol{\varepsilon},k) = 0 \quad (10)$$

For uniaxial compressive loading, the damage surface equation (10) is rewritten as

$$p(\boldsymbol{\varepsilon},k) = \left[2\boldsymbol{\varepsilon}^{-} : \boldsymbol{\varepsilon}^{-}(\frac{1}{2}+\eta\upsilon^{2}-\beta)\right]^{\frac{1}{2}} \tag{11}$$

The damage function $p(k)$ was obtained from a test for concrete materials based on the experimental results and was given by Thapa and Yazdani [12] as

$$p(\varepsilon,k) = \alpha\varepsilon_{c} ln\frac{E^{0}}{E^{0}-k} \tag{12}$$

And for elastic damaging process (β=0), the limit damage surface reduces to

$$p(\varepsilon,k) = \varepsilon_{c} \tag{13}$$

where, ε_c represents the strain corresponding to uniaxial compressive strength of concrete, which is used as the reference strain and hence the result of a conventional uniaxial compressive test is needed to establish ε_c.

2. Development of Fatigue Damage Model Utilizing Bounding Surface Approach

The bounding surface approach for fatigue was proposed by Wen et al. [14] in order to predict the behavior of woven fabric composites under fatigue loading. Fatigue loading is understood as the repeated or fluctuating strain acting in a material due to which progressive permanent structural change in the form of cracks or flaws occur in the material. The material fails at stresses having a maximum value less than the compressive strength of the material. In the case of fatigue loading, as cyclic loading is applied, the limit surface is allowed to expand and to form residual strength curves. This increase in strain is caused by damage and microcracks generated during the fatigue process. As the number of load cycles increases, the strain continues to increase further and the residual surfaces also expand. The increase in strain continues to a point at which the residual strain becomes equal to magnitude of ε_c. At this point, failure surface is formed and the material cannot withstand any additional cycles resulting in failure.

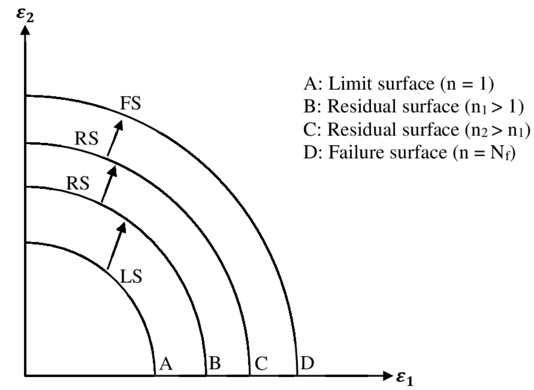

Figure 2: Schematic representation of bounding surfaces in biaxial strain space

In order to capture the described behavior of concrete under cyclic loading, an evolutionary equation is needed to predict the failure surface. To accomplish this task, the damage function is reconstructed to be product of two functions as shown below:

$$p(\boldsymbol{\varepsilon},k,n,\phi) = F(n,\phi).p(\boldsymbol{\varepsilon},k) \tag{14}$$

where,
$F(n,\phi)$ = strength softening function
n = number of cycles of loading to failure
ϕ = strain ratio (ratio of minimum strain to maximum strain)

The dependency of the function, $p(\boldsymbol{\varepsilon},k)$ on "n" and "ϕ" is supported by the experimental observation described in the previous section. By considering a fatigue uniaxial compression path and substituting equation (14) into equation (11), the following form is obtained for the softening function:

$$F(n,\phi) = \frac{\varepsilon}{\varepsilon_{c}} \tag{15}$$

where,
ε = residual strain of the concrete after specific number of cyclic loading

Equation (15) is a representation of so-called ε-n curves. Based on the researches, amplitude of strain, ε_{max}; strain ratio, ϕ; and finally the strain path all contribute to the fatigue life of concrete. While the fatigue life of concrete is adversely affected by the amplitude of loading, increasing strain ratio results in a greater fatigue life at a given strain. Moreover, the rate of reduction in concrete strength is not the same for different load paths. Guided by these findings, the following softening function is proposed in this paper as:

$$F(n,\phi) = \frac{H}{ln\frac{1}{1-\frac{n^A}{B\phi e^H}}} \tag{16}$$

where,
n = number of cyclic loading
A and B = material parameters
$H = \frac{\sqrt{2\varepsilon_c(\frac{1}{2}+\eta\upsilon^2-\beta)}}{\alpha\varepsilon_c}$

Putting equations (11), (12) and (16) in (14) and solving we get

$$k = E^0\left[\frac{n^A}{B\phi exp\frac{\sqrt{2\boldsymbol{\varepsilon}^-:\boldsymbol{\varepsilon}^-(\frac{1}{2}+\eta\upsilon^2-\beta)}}{\alpha\varepsilon_c}}\right] \tag{17}$$

By differentiating above equation with respect to n, the increment of damage in one cycle is obtained as

$$\dot{k} = E^0\left[\frac{An^{A-1}}{B\phi exp\frac{\sqrt{2\boldsymbol{\varepsilon}^-:\boldsymbol{\varepsilon}^-(\frac{1}{2}+\eta\upsilon^2-\beta)}}{\alpha\varepsilon_c}}\right] \tag{18}$$

Finally the rate of damage parameter $\dot{k}$ must be used in the simple constitutive relation of the form given by equation (3) in uniaxial compression for representing inelastic deformation, stiffness degradation and strength reduction due to fatigue cycles.

Substituting equations (8), (9) and (18) in equation (5) and then substituting equations (4) and (5) into equation (3) yields

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{E}(k):\dot{\boldsymbol{\varepsilon}} + \dot{\boldsymbol{E}}(k):\boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^i(k)$$

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{E}:\dot{\boldsymbol{\varepsilon}} - \dot{k}\left[\left(\frac{\boldsymbol{\varepsilon}^-\otimes\boldsymbol{\varepsilon}^-}{\boldsymbol{\varepsilon}^-:\boldsymbol{\varepsilon}^-} + \frac{\boldsymbol{\varepsilon}^+\otimes\boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+:\boldsymbol{\varepsilon}^+}\eta\right):\boldsymbol{\varepsilon} + \beta\boldsymbol{\varepsilon}^-\right] \tag{19}$$

3. Numerical Examples

In order to check the validity of the model and to obtain the value of the parameters introduced, numerical simulation have been performed and compared with experimental results found in the literature. The proposed model contains four parameters (A, B, α and β). Calibration of the parameters involved in modeling the behavior of concrete is carried out using the experimental data available in the literature or derived during the course of the study. Due to scarcity of experimental data in the literature for the measurement of these material parameters in the numerical simulation, analyst's judgments are required to obtain numerical results.

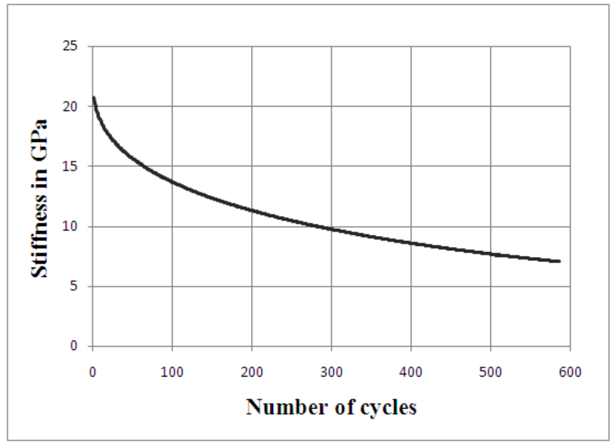

Figure 3: Model prediction of Stiffness reduction with number of cyclic loading

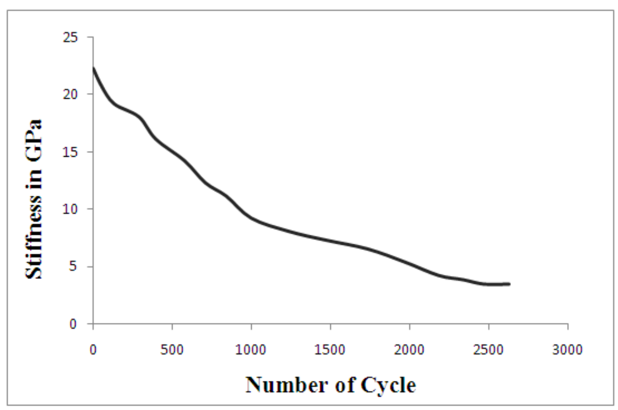

Figure 4: Stiffness vs. Number of cycles (Experimental Data)

Figure 3 shows the model prediction of modulus reduction with increasing number of cyclic loading which is found to be in similar trend of decreasing modulus as observed in the experimental work of Liu and Wang [15], shown in figure 4. The model captures the relevant features of cyclic response. For numerical simulation, the following constant were used A = 0.4, B = 15,α = 1.39 and β = 0.15.

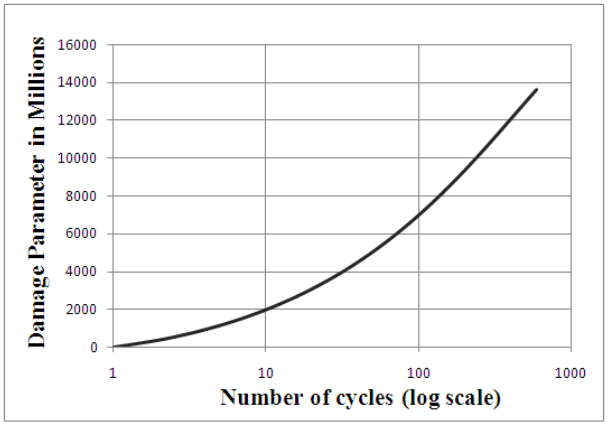

Figure 5: Model prediction of variation in damage with number of cyclic loading

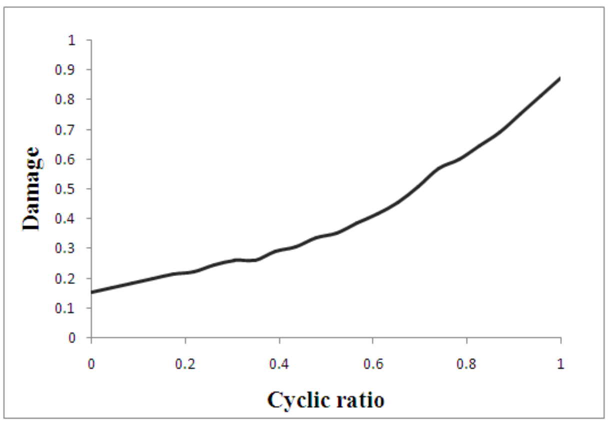

Figure 6: Damage vs. Cyclic ratio (Experimental Data)

Figure 5 shows the model prediction of increase in damage with increasing loading cycles. With the increasing loading cycles, there is initiation and propagation of microcracks which cause damage to the concrete. The theoretical model captures the similar trend of increasing damage with cyclic loading as observed in the experimental work of Lu. P. et al [16], shown in figure 6.

Figure 7 shows the model prediction of stress-strain behavior of material in monotonic loading against the experimental data of Kupfer [17].

Figure 8 shows the versatile behavior of model where the process of elastic degradation and permanent deformation is illustrated.

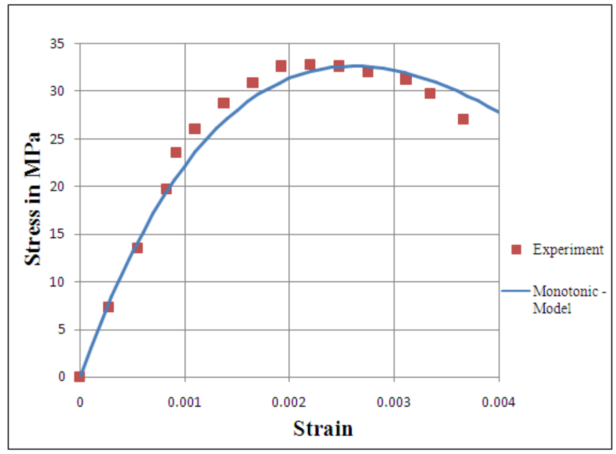

Figure 7: Stress strain behavior in monotonic loading

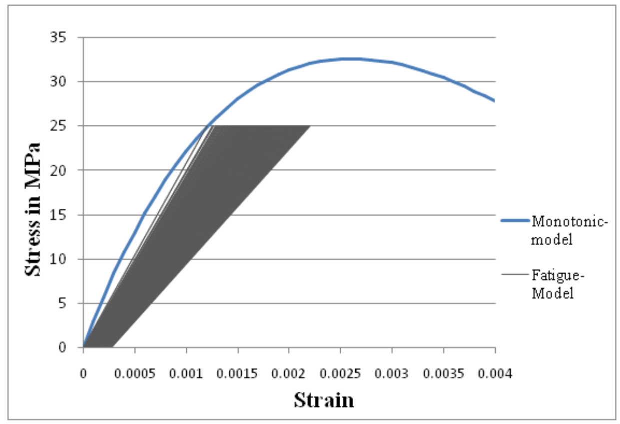

Figure 8: Stress strain behavior during inelastic damage accumulation

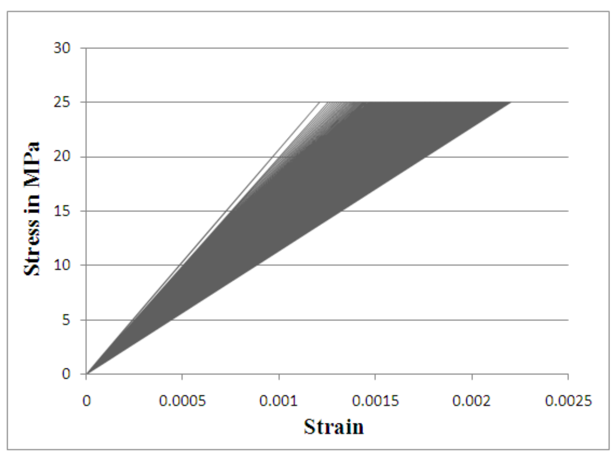

Figure 9: Stress strain behavior for elastic-damaging process

Figure 9 represents a behavior where no permanent deformation occurs even though damage is progressively accumulating as evidenced by degradation of elastic modulus. The behavior corresponds to an idealized case whereby crack faces close perfectly upon unloading and is achieved in the model by letting $\beta = 0$ (i.e.M = 0).

Figure 10 shows the increment in inelastic strain with increase in number of cyclic loading. There is accumulation of inelastic strain for each cycle of loading. Inelastic strain results in permanent deformation which is not recovered upon unloading.

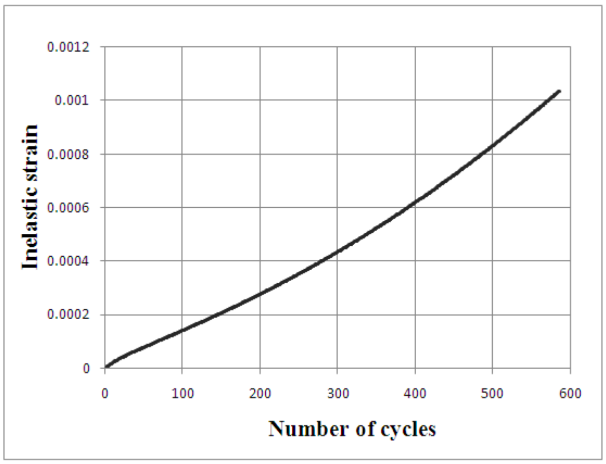

Figure 10: Inelastic strain vs. Number of cycles

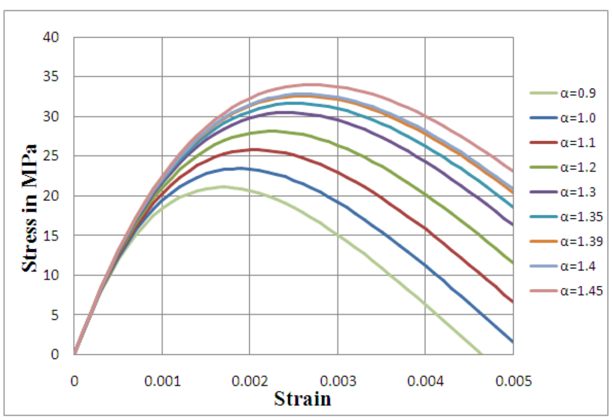

Figure 11: Stress strain behavior of concrete in uniaxial compression for different values of material parameter

Figure 11 shows the stress strain behavior of concrete in uniaxial compression for different values of material parameter α. It represents the sensitivity analysis of the material parameter α. The material parameter α has been introduced in order to calibrate the model to known peak strength.

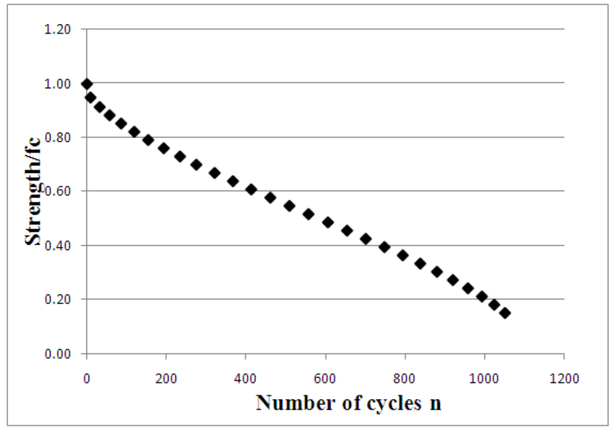

Figure 12: S-n curve for concrete under cyclic loading

Figure 12 shows the S-n curve for concrete under cyclic loading. It is clearly evident from the S-n curve that concrete lacks fatigue limit. S-n curve was plotted by finding out the number of cycles in which the concrete fails when the concrete specimen is subjected to cyclic loading with constant stress amplitude.

4. Conclusion

Fatigue is a complex phenomenon that requires special constitutive models that can take into account stiffness degradation due to cyclic loading. Fatigue damage evolution law together with the damage response functions were studied and it was found that concrete fails by sliding of two plane (Mode II) under cyclic loading in compression, which was properly addressed by $\boldsymbol{L}$ and $\boldsymbol{M}$ response tensors. The role of response tensors is to assume a damage rule, so that the evolution of the stiffness and inelastic strain tensors can be specified. Anisotropic damage can be best described by the fourth-order tensor due to its dimensional similarity with the stiffness of the material. Damage function is reconstructed by multiplying proper softening function to demonstrate the capability of the model in capturing the essential features of the concrete material, such as stiffness degradation and inelastic deformation. By comparison with experimental data, the model shows good capability to describe the essential properties of concrete in uniaxial compressive fatigue loading.

Acknowledgments

The authors are grateful to Center of Applied Research and Development (CARD) for this wonderful opportunity.

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