Three Dimensional Analysis of Stress and Design of Support System for the Powerhouse Cavern of Nalsyaugad Storage Hydroelectric Project

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Abstract: The presented paper is a venture to assess the stress, strain and other parameters' distribution in surrounding rocks in a powerhouse cavern excavation. Analytical solution may not be sufficient for such excavation so a numerical modeling was developed with the use of computer softwares like, ANSYS, Examine 3D and Phase2. The numerical modeling has been supported by two dimensional analytical solutions as well. The deformations obtained from both analytical and numerical methods showed allowable degree of convergence. Finally, the rock support interaction curve was obtained for a ten staged construction model using Phase 2 and was compared with that from analytical method. The support curves from both the methods showed similar trend which concluded the research affirmatively.

The three dimensional solution obtained so far is a homogeneous, isotropic and linearly elastic one while the two dimensional analysis includes both elastic as well as plastic analysis.

Keywords: cavern, Hoek-Brown Failure Criterion, Numerical Modeling, boundary conditions

1. Introduction

The powerhouse cavern is a special type of tunnel with usually larger width and height but with shorter length than the latter. Designing a tunnel itself is a critical task as it demands a complete understanding of the stress conditions, deformations and the material properties around the opening of the tunnel. The powerhouse cavern is even more complex. While the long tunnel can be analyzed as a two dimensional plane strain problem, powerhouse cavern needs a volumetric three dimensional approach for analysis. Basically, the measurement of deformations during excavation, prediction of stress from theory of elasticity and the empirical design of support system are employed during the design of tunnels. In case of cavern these may not just be sufficient. The peculiarity of properties of the rock masses further intensify with increased control volume while shifting from tunnels to caverns. Similarly, internal supports like steel sets are commonly used in tunnel as a support system while this might not be of any help in the cavern. For the design, therefore, three dimensional evaluation of stress around the opening and the interaction of the support are necessary in order to provide the stability to the opening.

1.1 Objective

The primary objective of this research work is to analyze stress and deformation pattern and thereby establish a design process for the support system in the powerhouse cavern. The analysis presented shall have specific 3D stress and related deformation features. So, the analysis in three dimensions shall be the prime focus of the research. The design process however, shall be completely two dimensional based on the stress and deformations obtained from FEM modeling supported by analytical two dimensional solution.

2. Stress analysis in rock

Rock is a relatively complex material to analyze than usual materials like steel or soil. While the latter behave much in an anticipated manner the former has a huge number of variables that make its engineering behavior almost impossible to be governed by just a few analytical equations. Following major methods have been discussed to analyze the engineering problems related to rock and excavations in it.

2.1 Analytical methods:

2.1.1 Classical stress analysis:

It is most precise method used in engineering problems. Analysis proceeds in terms of displacements, strains and stresses induced by excavation in a stressed medium, and the final state of stress is obtained by superposition of the field stresses. The conditions to be satisfied in any solution for the stress and displacement distributions for particular problem geometry and loading conditions are:

- (a) The boundary conditions for the problem;
- (b) The differential equations of equilibrium;
- (c) The constitutive equations for the material;
- (d) The strain compatibility equations.



Figure 1: An opening in a medium subject to initial stresses, for which is required the distribution of total stresses and excavation-induced displacements.

Thick walled cylinder:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\frac{\partial^2 \sigma_{xy}}{\partial x \partial y} = -\frac{\partial^2 \sigma_{xx}}{\partial x^2} = -\frac{\partial^2 \sigma_{yy}}{\partial y^2}$$

$$3$$

For plane strain conditions and isotropic elasticity, strains are defined by

Where,

$$E' = \frac{E}{1 - v^2}$$
$$v' = \frac{v}{1 - v}$$

The strain compatibility equation in two dimensions is given by

Substituting the expressions for the strain components, in equation 7, and then equation 3 in the resultant expression yields following after simplification:

$$\frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial^2 \sigma_{xx}}{\partial y^2} + \frac{\partial^2 \sigma_{yy}}{\partial x^2} + \frac{\partial^2 \sigma_{yy}}{\partial y^2} = 0$$

Equation 8 demonstrates that the two-dimensional stress distribution for isotropic elasticity is independent of the elastic properties of the medium, and that the stress distribution is the same for plane strain as for plane stress. Also, this equation demonstrates that the sum of the plane normal stresses, $\sigma_{xx} + \sigma_{yy}$, satisfies the Laplace equation

the Laplace equation.

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The problem is to solve equations 1 and 8, subject to the imposed boundary conditions. The method suggested by Airy introduces a new function U(x, y), in terms of which the stress components, defined by

$$\sigma_{xx} = \frac{\partial^2 U}{\partial y^2}.$$

$$\sigma_{yy} = \frac{\partial^2 U}{\partial x^2}.$$

$$\sigma_{xy} = -\frac{\partial^2 U}{\partial x \partial y}.$$
9.1
9.1
9.2
9.3

These expressions for the stress components satisfy the equilibrium equations 1, identically. Introducing them in equation 8 gives

$$\nabla^4 \mathbf{U} = \mathbf{0} \qquad 10$$

Where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

Equation 10 is called the biharmonic equation.

Several methods may be used to obtain solutions to particular problems in terms of an Airy stress function. The equations 8 and 9 can be transformed to cylindrical polar co-ordinates, and a solution procedure can be illustrated by referring to a thick-walled cylinder subject to internal and external pressure, as shown in Figure 2. For this axisymmetric problem, the biharmonic equation assumes the form

for which a general solution for U is given by

$$U = Alnr + Br^2lnr + Cr^2 + D = 0$$
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Figure 2: A thick-walled cylinder of elastic material, subject to interior pressure, p_i, and exterior pressure, p_o

In the expression above, the constants A, B, C, D are determined by considering both the requirement for uniqueness of displacements and the pressure boundary conditions for the problem. It can be shown that uniqueness of displacements requires B = 0, and that the stress components are then given by

$$\sigma_{\rm rr} = \frac{A}{r^2} + 2C \,. \tag{13.1}$$

$$\sigma_{\theta\theta} = -\frac{A}{r^2} + 2C \qquad 13.2$$

$$\sigma_{r\theta} = 0.$$
 13.3

Where,

$$A = \frac{a^{2}b^{2}(p_{i} - p_{o})}{a^{2} - b^{2}}$$
$$2C = \frac{p_{o}b^{2} - p_{i}a^{2}}{b^{2} - a^{2}}$$

In these expressions, a and b are the inner and outer radii of the cylinder, and pi and p_o are the pressures applied to its inner and outer surfaces.

Solution of a Typical Circular Excavation:

The preceding discussion has established the analytical basis for determining the stress and displacement distributions around openings with two-dimensional geometry



Figure.3: A circular excavation stress analysis in polar coordinate system for deducing the stress and displacement distribution around a circular excavation in a biaxial stress field

Figure 3a shows the circular cross section of a long excavation in a medium subject to biaxial stress, defined by $p_{yy} = p$, and $p_{xx} = Kp$. The stress distribution around the opening may be readily obtained from equations 9, by superimposing the induced stresses associated with each of the field stresses p and Kp. The complete solutions for stress and displacement distributions around the circular opening, originally due to Kirsch (1898), are

$$\sigma_{rr} = \frac{p}{2} \left((1+K) \left(1 - \frac{a^2}{b^2} \right) - (1-K) \left(1 - 4\frac{a^2}{r^2} + 3\frac{a^4}{r^4} \right) \cos 2\theta \right)$$
 14.1

$$\sigma_{\theta\theta} = \frac{p}{2} \left((1+K) \left(1 + \frac{a^2}{b^2} \right) + (1-K) \left(1 + 3\frac{a^4}{r^4} \right) \cos 2\theta \right)$$
 14.2

$$\sigma_{r\theta} = \frac{p}{2} \left((1 - K) \left(1 + \frac{2a^2}{r^2} - 3\frac{a^4}{r^4} \right) sin 2\theta \right)$$
 14.3

$$u_r = -\frac{pa^2}{4Gr} \left((1+K) - (1-K) \left(4(1-v) - \frac{a^2}{b^2} \right) cos2\theta \right)$$
 15.1

$$u_{\theta} = -\frac{pa^2}{4Gr} \left((1+K) - (1-K) \left(4(1-v) - \frac{a^2}{b^2} \right) \cos 2\theta \right)$$
 15.2

Where, K=ratio of horizontal to vertical insitu stress in rock mass

 u_r = displacement in rock due to stress redistribution in radial direction

 $u_{\theta} = \mbox{ displacement in rock due to stress redistribution in } \theta \mbox{ direction }$

2.2 Numerical methods

It can be seen that even for simple two-dimensional excavation geometry, such as a circular opening, quite complicated expressions are obtained for the stress and displacement distributions. Many design problems in rock mechanics practice involve more complex geometry. Other conditions which arise which may require more powerful analytical tools include nonhomogeneity of the rock mass in the problem domain and non-linear constitutive behavior of the medium. These conditions generally present difficulties which are not amenable to solution by conventional analysis.

2.3 Analysis tools: ANSYS, Examine3D and Phase2

ANSYS software has been used here as a validation of the research obtained from other geotechnical analysis and geotechnical softwares namely Phase2 and Examine3D. Particularly, the difference in rock behavior when it follows Hoek-Brown failure criteria adopted by Phase2 or Examine3D and that when it does not follow it while analyzing with ANSYS can show the difference.

Examine 3D is an engineering analysis program for underground excavations in rock. The software is mainly used for 3D stress analysis and deformations in elastic and isotropic rock masses. Since the software has been developed especially for the study of stresses in excavations in rock, the Hoek and Brown as well as Mohr-Columb criterion is followed by the software.

Phase2 is another powerful 2D finite element stress analysis program for underground or surface excavations in rock or soil. Unlike others it can be used for both elastic and plastic analysis of excavations.

2.4 Rock-Support Interaction

If the motion at any point on the rock-support interface differs from the motion that would occur at the same point in the free field if the support was not present, there is an interaction between the rock and the support which is referred to as rock-support interaction. (Rosenblueth, 1980)



Figure 4: Sectional view of a three-dimensional finite element model of tunnel

The figure 4 shows the failure and deformation of the rock mass surrounding the face of an advancing circular tunnel. The plot shows displacement vectors as well as the shape of the deformed tunnel profile

From Figure 4 it can be easily discovered that as the tunnel advances the deformation starts getting broad. So, in order to limit the deformation we need to provide the supports. Then the interaction occurs between the support and the excavation face and the support helps the tunnel to get stable.

To have a clear understanding about how the support system helps in the stabilization of the deformation in the tunnel let us observe next example.



Figure 5: Distribution near the face, of the radial elastic displacement, ui,

Figure 5 gives the idea of rock support interaction in which excavation is bring advanced by conventional drill and blast methods. The rock behavior in tunnel is illustrated in this example. In this paper as well the rock support interaction curve shall be obtained by analytical and numerical method for the case presented.

2.5 Rock Mass Strength and Hoek-Brown Failure Criteria

The characteristic of rock mass is that it behaves very different from the intact rock. So assessment of rock mass strength just by assessing the intact rock strength is quite impossible. There have been only very few tests around the world to access the in situ rock mass strength. So, rock scientists around the world rely largely upon empirical relations to define or estimate rock mass strength. Hoek and Brown failure criteria is one of those attempts made to calculate strength by defining failure criteria as discussed below:

The Hoek and Brown Failure criterion is stated as:

Where,

$$m_{b} = m_{i} exp \left[\frac{GSI - 100}{28 - 14D} \right]$$
$$s = exp \left[\frac{GSI - 100}{9 - 3D} \right]$$
$$a = \frac{1}{2} + \frac{1}{6} \left[e^{-GSI/15} - e^{-20/3} \right]$$

GSI = Geological Strength Index

$m_i = rock material parameter of intact rock$

D is a factor which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses.

3. Analysis Approach

3.1 Data collection

For most stress strain analysis superficial geological data has also proved to be very suggestive. So, surface geological data with supportive core drilled data has been used in the research. The related data was obtained from the Project Development Department of Nepal Electricity Authority.

The geotechnical data were obtained for the phyllite was as follows:

Intact Uniaxial compressive strength, σ_{ci} : 75Mpa

Geological Strength Index, GSI: 30

Rock material parameters: $m_i = 7$

Intact Modulus, E_i= 22500 Mpa

Modulus Ratio, M_r= 300

Unit weight, $\gamma = 0.026 \text{ MN/m}^3$

Cohesion, c= 1.095 Mpa

Angle of friction, $\phi = 30.43^{\circ}$

Insitu Stresses: σ_1 =13Mpa, σ_3 =13Mpa

3.2 Numerical modeling

Numerical modeling is the major tool of analysis in this venture. Basically, Finite Element Method in ANSYS and Phase2 while Boundary Element Method is utilized by EXAMINE 3D program. The basic methodology is to make stress and deformation analysis from EXAMINE 3D, check it with ANSYS and then design support system in Phase2.

3.2.1 Finite Element Method:

Defining the problem domain: This includes the identification of the effective size of the control volume for the analysis, which is taken to be roughly three times the radius of the cavern while analyzing by ANSYS or Phase2.

Generation of Mesh: Generation of mesh governs the preciseness of the analysis.



Figure 6: Mesh Generation in ANSYS



Figure 7: Mesh Generation in Phase2

3.2.2 Boundary Element Method

In BEM only the surface of the structure to be analyzed is discretized by elements. Hence, it is much easier to generate element models, when compared to other solution technique such as finite element method (FEM) which require modeling the volume of the domain.



Figure 8: Mesh Generation in Examine3D

3.3 Analytical solution

The solution though might not be obtained for all sorts of physical conditions of the field. In this research the semi analytical method suggested in Underground Excavations in Rock, E. Hoek & E.T. Brown (1980) shall be used for this purpose.

3.4 Comparison and interpretation of results

The rational of this research work lies in the comparison of the results obtained from the three dimensional analysis of stress and deformation with that obtained from the plane strain 2 dimensional one. So, it has been intended to perform a comparative study between the two. Furthermore, the plane strain finite element results shall be compared to with the available analytical solutions.

3.5 Design appropriate support system

Finally, the cavern has been designed with the most suitable support system. Both the analytical and FEM based design shall be incorporated. The support system so obtained shall be able to withstand the deformation and convergence of the cavern internal surface further inside. Convergence of the results from two different methods shall prove the adequacy of the design procedure and analysis adopted to achieve so.

4. Analytical and FEM results

The elastic and plastic analysis has shown that the deformation is much more in plastic mode as anticipated. The results from two methods have shown coherence to acceptable limits. For the analysis of the cavern behavior, analytical calculations using the semi-empirical methods given by E. Hoek have been carried out. This approach uses the Mohr-Coulomb failure criterion as the basis for analysis. However, Hoek's work has been designed for radial excavation, but the cavern under study has a complex shape. Nevertheless,

the in situ stress measurements and finite element studies carried out in an exploration tunnel shows that the stresses in rock above the roof of cavern are very similar to those which would be induced in the rock surrounding a circular tunnel. So, the following formulas were assumed to be applicable for the cavern roof as well.

Radius of plastic zone was calculated as:

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$$r_{p} = ro\left(\frac{2(po(k-1) + \sigma_{cm})}{(1+k)((k-1)pi + \sigma_{cm})}\right)^{\frac{1}{(k-1)}} \dots 17.1$$

Inward radial elastic displacement u_{ie} of the tunnel as:

Inward radial elastic displacement u_{ip} of the tunnel as:

$$u_{ip} = \left(ro \frac{(1+\nu)}{E} \right) \left[\left(2 \frac{(1-\nu)(p_o - p_{or})}{(\frac{r_p}{r_o})^2} \right) - \left(\frac{(1-2\nu)}{(p_o - p_i)} \right) \right]$$
 17.3

The uniaxial strength of the rockmass was calculated as:

$$\sigma_{cm} = \left(\frac{2c \cos\phi}{1 - \sin\phi}\right) \tag{17.4}$$

The value of k, the slope of the σ_1 versus the σ_3 line is given by:

$$k = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)$$
17.5

The critical support pressure is calculated as:

$$p_{cr} = \left(\frac{2p_o - \sigma_{cm}}{1+k}\right)_{\dots}$$
 17.6



Figure 9: Elastic deformation contours



Figure 10: Plastic deformation contours



Figure 11: Elastic deformation contours from Examine 3D



Figure12: Yielded elements percent distribution

Discussions:

The results have been summarized for roof element as follows:

Table 1: Comparison of results from different methods

Section	Method	Equival ent Radius, m	Radius of plastic zone, r _p	Elastic deforma tion, u _{ie}	Plastic deform ation, u _{ip}
Main Hall	Analyti cal	8	15.594	0.080	0.197
	FEM	8	15.851	0.073	0.212
Transfor mer Hall	Analyti cal	11	21.441	0.109	0.271
	FEM	11	18.463	0.087	0.201

Results have similar deformation characteristics between the 2D and 3D analysis in elastic analysis. The Elastic deformations are lesser than the plastic deformation which is obvious for given set of boundary conditions. The maximum deformation is higher than that in the roof in each case. The maximum deformation is at the walls in all case. Due to stress redistribution the stress contours are somewhat deflected from the roof and get concentrated along the walls. These deformations would have been minimized if the shapes were other than flat in the walls

4.1 Stress analysis:



Figure 13: First Principle Stress around the two caverns in 3 dimensions (ANSYS)



Figure 14: First Principle Stress around the two caverns sliced (ANSYS)



Figure 15: First Principle Stress (Elastic) around the two caverns in 2 dimensions (Phase2)



Figure 16: First Principle Stress (Plastic) around the two caverns in 2 dimensions (Phase2)

Discussions:

For Elastic Analysis:

The stress contours are found to be concentrated along the roof and the corners at the bottom edges.

The stress is reduced at the walls while deformation is increased in this region.

The interference of the two excavations is still interfering but is within acceptable limit i.e little more than insitu stress

For Plastic Analysis:

Unlike elastic deformation, plastic deformation shows large reduction of stress around the excavations.

The stress is reduced at the walls to almost nill stress i.e the walls are almost failed under the stress field.

The stress interference of the two excavations is quite large as compared to stress in elastic analysis. This shows that the distance to be maintained is to be taken under consideration.

4.2 Design of support system:

Finite Element Analysis for design of support system:

The design of support system was performed only with the 2-dimensional software Phase2. A 10 staged model with varying support pressure was used for the purpose. The actual phenomena of stress reduction in the in-situ stresses is represented by, factoring and gradually decreasing in magnitude in stage 2 and thereafter finally reducing to zero in the final stage 10. The factor used is 0.8, 0.4, 0.2, 0.1, 0.08, 0.04, 0.02, 0.01 and 0 from first to last stage respectively. As a result, the tunnel deformation will increase as the pressure is lowered to zero. Vlachopoulos and Diederichs method is utilized to find the tunnel deformation at the point of support installation.



Figure 17: Vlachopoulos and Diederichs chart for determining tunnel closure before first installation of support

In the main cavern, rp = 15.59 m, rt = 8 m, x = 4m, and umax = 0.227m. The distance from the tunnel face/tunnel radius = 0.38. The plastic zone radius/tunnel radius = 1.88. From the above plot, this

gives Closure/max closure approximately equal to 0.42. Therefore, closure equals $(0.227)^*(0.42) = 0.095$ m. As computed above, the tunnel roof displaces 0.095 m before the support is installed. Similarly, for transformer cavern the displacement was calculated to be 0.088m.

Lastly the ground reaction curve is generated from the respective displacement and factored pressure. Figure 20 and 21 show the comparison of the ground reaction curve from the FEM and analytical method.

From the ground reaction curve so obtained a support system is designed to stop further convergence of the cavern surface. The support system used in this analysis is as follows:

Shotcrete: Young's Modulus, 30000 MPa, Poisson's Ratio of 0.2, Compressive strength of 40 MPa, Tensile Strength of 3 MPa, Thickness of 0.2m

Rock Bolts: Type: Fully bonded, Modulus of Elasticity: 207000 Mpa, Tensile Capacity:0.1 MN, In/ Out of plane spacing: 1m, Length: 6m,Diameter: 25mm

Results after support installation:



Figure 18: Total displacement after support installation



Figure 19: Confined yielded zone after support installation

The maximum total displacement decreases to 0.115 m from 0.212 m after support installation in the main gallery and to 0.0905 from 0.201 in the transformer gallery. The supports have been proved to be effective for both the galleries. However, there is an important feature noticeable regarding the plastic zone along the

sides and bottom of both the caverns. The vertical walls are still in plastic yielding to large extent. This further clarifies the importance of shape factor in underground excavation.

There is a drastic change in the size and shape of yielded zone. This is primarily due to the load transferred to bolts and shotcrete which reduced the convergence of the rock surface further into the cavern.

The Analytical Method for Ground Support Interaction Curve and the Support System:

The ground reaction curve and the support characteristic curves have been obtained for both the caverns. The details of the calculation of the ground reaction curve from the analytical method has been shown at the annex of this research The results so obtained from both the methods have been shown as below:



Figure 20: Ground Support Interaction Curve for the main gallery

In the analytical method and the FEM based analysis both has been used to finally obtain the ground reaction curve after the excavation and the support characteristic curve



Figure 21: The Ground Support Interaction Curve for the Transformer Gallery

5. Conclusions

The paper work presented after the meticulous study of related literature and analysis of the stresses and deformations along the surface of the two underground caverns, following concluding remarks have been put forth:

Stresses in the rock mass actually develop after the redistribution of in-situ stress when stress path has been removed by excavating.

The stresses so developed can be analyzed in two dimensional aspects given that the excavation length is very long compared to the other two dimensions, so for a short length cavern section like the one in this research the two dimensional analysis cannot be sufficient.

The three dimensional analysis shows that stresses along the ends of the cavern are higher as they have limited degree of freedom, while the stresses along the mid span of the cavern show higher deformations and lesser stresses.

One important conclusion here is that at the middle span of the cavern the plane strain analysis seems valid. While the same may not be equally valid along the ends.

The rock support interaction curve, designed primarily for the radial excavation can also be valid for a non regular geometric excavation like the one in this paper. However, the curved surface with some radius of curvature, like the curved roof in this case, is required and valid for such case.

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