

# Calibration of Conventional Macroscopic Traffic Flow Models for Nepalese Roads (A Case Study of Jadibuti - Suryabinayak Section)

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**Abstract:** The importance of understanding the relationship between various traffic flow parameter, especially speed, flow and density is unquestionable from the perspective of planning, design, traffic operations and improvement of facilities. Various conventional macroscopic speed-density models namely Greenshields, Greenberg, Underwood, Drake, Pipes – Munjal, Polynomial and modified Drake and Underwood models were first calibrated and further validated for Jadibuti - Suryabinayak road section in this paper. Due to significant goodness of fit ( $R^2$  value and standard error) and realistic estimates of free flow speed, optimum speed and optimum density, Conventional Underwood model was recommended as the best fit model for the considered road section. This calibrated model can be used in predicting speed for the anticipated traffic volume, and even helps in quantifying congestion for oversaturated condition.

**Keywords:** Macroscopic traffic flow model; PCU; free flow speed; jam density; goodness of fit

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## 1. Introduction

Tremendous developments in the transportation system along with increased road network and overdependence on automobiles give rise to the evolution of various traffic flow theories. These theories illustrate traffic stream characteristics which includes behavior of vehicles on road at different circumstances, interaction of road user and roadway environment. Researchers have categorized various traffic flow models under two types viz. macroscopic and microscopic traffic flow models. Conventional mathematical models that formulate the relationships among the integrated parameter associated with traffic stream i.e. speed, flow and density are termed as macroscopic traffic flow models. Such models convert single entity level characteristics to comparable system level characteristics by integrating microscopic traffic flow model, thus looking the traffic flow from the broader perspective like Greenshield, Greenberg, Underwood, Drake model etc. Microscopic traffic flow model on the other hand deals with the traffic stream characteristics from microscopic view point which includes headway, clearance etc. via modeling driver-driver and driver-road interactions within a traffic stream for example Car following model, Pipe's model, Forbes' model etc.

The importance of understanding the relationship between speed, flow and density is unquestionable from the standpoint of highway planning, design and operation as it provides quantitative estimates of the change in speed as a function of anticipated changes in traffic demand. These models have a significant role on prediction of highway capacity, free flow speed and level of service offered by the facilities and to determine the geometric features of road so as to

provide the targeted level of service. They are equally useful in real-time traffic control, a basic component of Intelligent Transportation System (ITS) which is based on changes in traffic flow parameters. Calibration and validation of various macroscopic models is the first step towards creating theoretical background for successful planning and design.

This particular research is the first initiation towards the calibration of traffic flow models for Nepalese road, which was not done till date.

## 2. Objectives of Study

Despite being the widely accepted macroscopic traffic flow model, it is not universal that Greenshield model will always be the best fit model. Many countries had calibrated macroscopic models for different parts or even at different segment of roads but our country lacks such calibration till date. Thus the objectives of this research work is to identify conventional macroscopic traffic flow models; develop, calibrate and validate them for heterogeneous traffic flow condition and recommend the most suitable one in context of Nepalese road.

## 3. Study Area

Calibration of traffic flow models is of great importance, especially on urban areas, where large traffic flow occurs and hence Kathmandu was chosen. Various constraints such as time, manpower and fund led to confine the study area to Jadibuti – Suryabinayak road section along the Araniko Highway. It serves significant traffic from Kathmandu to Bhaktapur, and

also the forms a part of Asian Highway 42 running towards the Chinese border. It has one of the highest traffic volumes among the national roads in Nepal. Moreover, it is anticipated that the road will get much more congested than at present once the Sindhuli Road is completed. At the same time this road section is characterized by operating traffic signals which is rare in Kathmandu valley, and thus it will be useful in actuated traffic signal timing design. Hence, this particular road section was considered.

#### 4. Literature Review

In general, traffic streams is not uniform but vary over both space and time therefore it is a difficult to describe traffic flow phenomenon without the use of a common set of terms: volume or flow rate, speed, and density or concentration.

Average speed ( $V$ ) is defined as average spot speed of several vehicles passing a specific location. Flow or volume ( $Q$ ) is defined as the number of vehicles crossing a particular section within stated period of time. Volume and flow are interchangeably used in some literature whereas in Highway Capacity Manual (HCM), flow rate corresponds to equivalent hourly rate at which vehicles pass over a given point or section of a roadway during a given time interval of less than 1 hour usually 15 minute. Density or concentration ( $K$ ) is defined as the number of vehicles occupying the particular section of road.

For multilane roads, flow need to be expressed as vehicle per unit time per lane, but due to the lack of lane discipline in most of the developing countries having mixed traffic condition, flow is expressed as vehicles per unit time per total width of road (Arasan and Krishnamurthy, 2008) and so does density which is also expressed in terms of total width.

##### *Heterogeneous Traffic and Passenger Car Unit*

The real world consists of vehicles having varying physical as well as operating characteristics rather than stream of uniform vehicles. Assuming homogeneous traffic stream makes calculations much simpler but doesn't reflect the real scenario. The dividing line between homogeneous and heterogeneous is not clear cut. Arasan and Krishnamurthy (2008) suggest heterogeneous traffic to exist when the percentage of the dominant mode is less than 80% of the traffic while Fazio, Hoque, and Tiwari (1999) suggest the value to be slightly higher at 85%. Only few models are develop in the context of mixed traffic condition, some model convert heterogeneous traffic to equivalent passenger car-units and then applying procedures for homogeneous traffic.

In case of heterogeneous traffic flow condition, various types of vehicles need to be converted to equivalent vehicle using Passenger Car Unit (PCU) or Passenger Car Equivalent (PCE) factor. The PCU value of a given type of a vehicle is related to its lane occupancy relative to a standard passenger car, whose PCU value is taken as 1. These equivalency factors depend on the traffic flow parameters and thus, are subjected to variations at various traffic flow conditions. Therefore it may not be precisely correct to adopt a constant set of PCE values under different roadway and traffic conditions (Justo and Tuladhar, 1984). But, however, NRS 2070 has provided constant PCU values for different types of vehicles, with assumption that it remains same at most of the traffic flow condition.

##### *Greenshield's Model*

The first and the simplest classical macroscopic traffic flow model, till date was proposed by Greenshield's in 1935, which assume that under uninterrupted flow conditions, speed and density are linearly related. This model is considered as the powerful tool in the field of traffic flow modeling due to its simplicity and reasonable goodness of fit but yet its acceptability is not universal as it don't yield reasonable goodness of fit to data corresponding to congested and extremely low density road. Mathematically:  $U = U_f * \left\{1 - \frac{K}{K_j}\right\}$

Where,  $U$  represents speed corresponding to density  $K$ ;  $U_f$  and  $K_j$  represents free flow speed and jam density respectively. It satisfies both boundary condition ( $U=0$  at  $K=K_j$ , and  $U=U_f$  at  $K=0$ ).

##### *Greenberg Model*

Greenberg (1959) observed traffic flow in the north tube of the Lincoln Tunnel at New York City and developed a logarithmic speed-density model, assuming that the traffic flow along a roadway can be considered as a one-dimensional fluid.

Mathematically,

$$U = U_c \times \ln \left( \frac{K_j}{K} \right)$$

Where,  $U$  represents Speed corresponding to density  $K$ ;  $K_j$  and  $U_c$  represents jam density and speed corresponding to capacity. This model shows strong goodness of fit under congested conditions but does not satisfy the boundary condition at low density regime i.e. fails to predict free flow speed.

##### *Underwood's Exponential Model*

Underwood (1961) proposed another exponential form of macroscopic traffic flow model so as to overcome the drawback of Greenshield model. Mathematically:

$$U = U_f \times e^{(-K/K_0)}$$

Where, U represents speed corresponding to density K;  $U_f$  and  $K_0$  represent free flow speed and optimum density (density at capacity). This model gives better fit than the Greenshields and Greenberg models for the uncongested traffic conditions, but fails to do so for congested conditions. It doesn't satisfy the boundary condition at high density regime.

#### **Underwood Model with Taylor Series Expansion**

Exponential function of Underwood model can be expanded in a Taylor series for the numerical approximation of jam density, which can't be obtained from conventional Underwood model.

Mathematically,

$$U = U_f e^{-k/k_c} = U_f \left( 1 - \frac{k}{k_c} + \frac{k^2}{2k_c^2} - \frac{k^3}{6k_c^3} + \frac{k^4}{24k_c^4} - \dots \right)$$

Considering up to third degree of k yields

$$U = U_f e^{-k/k_c} = U_f \left( 1 - \frac{k}{k_c} + \frac{k^2}{2k_c^2} - \frac{k^3}{6k_c^3} \right)$$

The realistic estimate for the jam density ( $K_j$ ) can be obtained from above equation by substituting  $U = 0$ .

#### **Drake Model (Bell-Shaped Curve Model)**

Drake (1961) improved Underwood model by proposing bell shaped model after analyzing all model from statistical point of view. He estimate density from speed and flow data, fitted the speed vs. density function and transformed the speed vs. density function to a speed vs. flow function to validate his model. Mathematically expressed as:

$$U = U_f \exp \left[ -\frac{K^2}{2 \cdot K_c^2} \right]$$

Where, U represents speed corresponding to density K;  $K_c$  and  $U_f$  represents density corresponding to maximum flow and free flow speed respectively. Drake model generally yields a best fit than other models for uncongested conditions, but fails to provide a good fit for congested conditions.

#### **Pipes-Munjjal model**

Pipes proposed a model with introduction of new parameter (n) to provide a more generalized modeling approach shown by the following equation. When  $n=1$ , Pipe's model resembles Greenshields' model. Thus by varying the values of 'n', a family of models can be developed. Mathematically,

$$U = U_f * \left\{ 1 - \left( \frac{K}{K_j} \right)^n \right\}$$

Where symbol have their usual meanings

#### **Polynomial Model**

Polynomial models of order 2, 3, 4 and onwards are calibrated based on the data and are checked for their goodness of fit and other estimate. Density and speed relationship can be expressed mathematically on quadratic form (polynomial of order 2) as:

$$U = U_f + b * K + c * K^2$$

Where, b and c are constant model parameters.

#### **Modified Greenberg Model**

Ardekani and Ghandehari (2008) proposed a Modified Greenberg model introducing a non-zero average minimum density,  $K_o$ , assuming that there is always some vehicles on the freeway.

$$U = U_c * \ln \left( \frac{k_j + k_o}{k + k_o} \right)$$

Where  $U_c$  and  $K_o$  represents speed corresponding to maximum flow and Average minimum density respectively. This model yields a finite free flow speed of  $U_f = U_c * \ln(1 + k_j/k_o)$  when density approaches zero. Density at capacity of this model will be  $k_c \approx 0.4k_j$ , a value close to that of the classical Greenberg model, ( $k_c \approx 0.368k_j$ ) obtained by solving for  $dq/dk=0$ .

#### **The Drake Model with Taylor Series Expansion**

Conventional Drake model can be expanded using Taylor series expansion to obtain a numerical approximation for the jam density, as follows:

$$U = U_f \left( 1 - \frac{k^2}{2k_c^2} + \frac{k^4}{8k_c^4} - \frac{k^6}{48k_c^6} \right)$$

Where symbol have their usual meanings.

## **5. Methodology**

### **A. Site Selection**

The traffic and physical characteristics is almost similar throughout the Jadibuti – Suryabinayak road section and after proper inspection, the section just ahead of Radhe Radhe chowk between Sallagahri intersection and Sankhadhar Shakhwa Chowk was selected. The considered road segment was not perfectly straight but yet, the curvature was not that steep enough to cause reduction of traffic speed. As all three data; speed, density and flow were targeted to be extracted from the same video; one kilometer distance can't be considered. Taking lesser distance might not be realistic and considering larger distance may lead to parallax error while extracting speed and density data, thus after trade-off 200m stretch separated by two speed limit traffic signs was considered. There were two lane each

of 3.5m in either direction along with 2m shoulder width.



Figure 1: Site selected for study

### B. Data Collection and Extraction Methodology

Video graphic survey, as a part of data collection method had been conducted for six days, covering both peak and off peak period including saturday, from mornings 8 am to evening 7 pm. The real field scenario was trapped on video by installing the video camera at third floor of Laxmi steel building and were analyzed to extract the required traffic flow parameters viz. average speed, flow and density. The road geometry including number of lanes, lane width, shoulder width and total width were noted.

Classified traffic volume, categorized into six types viz. two wheeler, car, micro, bus, tipper/light truck and truck, were counted for every 15 minute interval as suggested by HCM and was expressed as PCU/15 minute/total width as suggested by Arasan and Krishnamurthy (2008). Insignificant number of non-motorized vehicles and different operating characteristics of vehicle moving on shoulder lead them to be excluded from the volume count. The PCU value as suggested in NRS 2070 was adopted.

Table 1: Passenger Car Unit (PCU) value as per NRS 2070

Vehicle	Bike	Car/ jeep	Micr obus	Bus	Light truck	Truck
PCU value	0.5	1	1.5	3	1.5	3

Density value was extracted for every 30<sup>th</sup> second. The average value of 15 minute interval was termed as average density and was expressed as PCU/200m/total width. As the different mode have different range of speed, the result obtained from random sampling may be misleading and thus stratified random sampling

based on the volume proportion of different modes was considered. To ensure the representative sample, 10% of total volume of each mode observed in 15 minute interval was considered for sampling and finally mean space mean speed ( $V_s$ ) was calculated. The vehicle arriving at every 30<sup>th</sup> second irrespective of mode and lane was considered for speed calculation and time taken to pass the considered section was noted down until required sample size was achieved. If the required sample for a particular mode was completed then that mode is excluded and the other remaining mode was considered for speed calculation. If the extracted sample was less than the required sample then the vehicle arriving at every 15<sup>th</sup> and 45<sup>th</sup> second was considered for data extraction and the process was continued.

Among two directional data, Kotheshwor bound direction data were used for model calibration and Shallagari bound direction data were used in validation process. Twelve hour of data, altogether 48 data (each representing data corresponding to 15 minute period) had been extracted for calibration process covering all range of time. As twelve hours of extracted data covers almost all range of speed, flow and density, further extraction of data was considered ineffective; so 12 hours of data was used for calibration. Six hour data, i.e. 24 points were extracted for validation purpose.

## 6. Data Analysis

### A. Model Calibration

Extracted data were analyzed, and based on it, various macroscopic traffic flow models were calibrated. Conclusions drawn from the models are summarized on Table 2, which gives the estimate of free-flow speed ( $V_f$ ), jam density( $K_j$ ), speed at capacity( $V_c$ ), density at capacity( $K_c$ ) and goodness of fit ( $R^2$  value).

The free flow speed will be about 60 kmph or slightly above that, as the road section is characterized by maximum speed limit of 60kmph and strict enforcement. This is further supported by the fact the 85<sup>th</sup> percentile speed of mixed traffic condition was found to be 58 kmph. The length of car is around 3m and they opt to have about 2m clearance. This yield around 40 passenger cars in the considered 200m section, but with 2 lanes, the section considered may accommodate 80 PCU/200m in each direction.

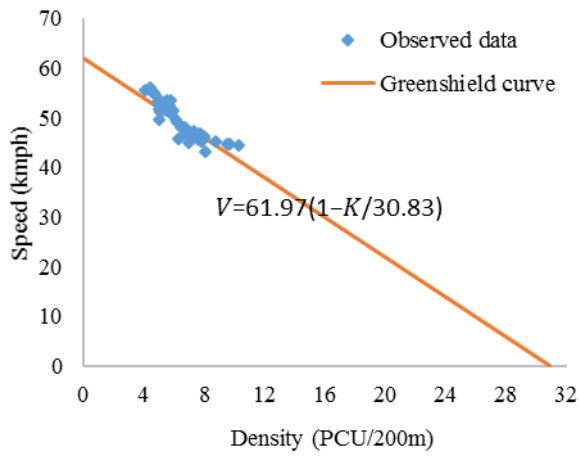


Figure 1: Calibrated Greenshields model

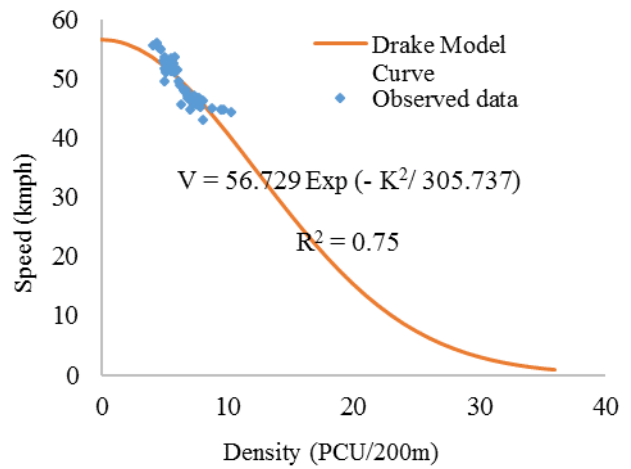


Figure 4: Calibrated Conventional Drake Model

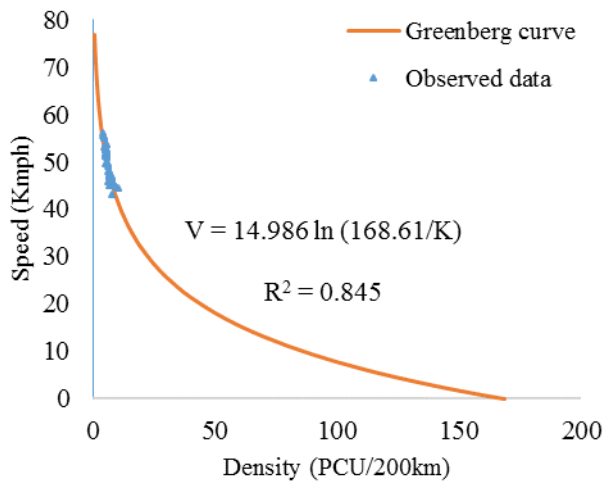


Figure 2: Calibrated Greenberg Model

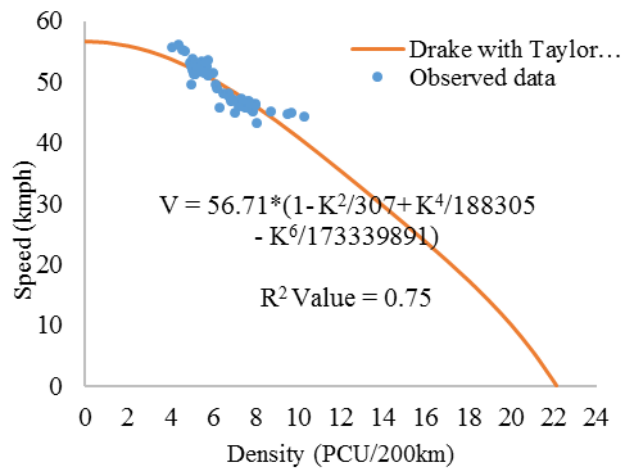


Figure 5: Calibrated Drake Model with Taylor Series Expansion

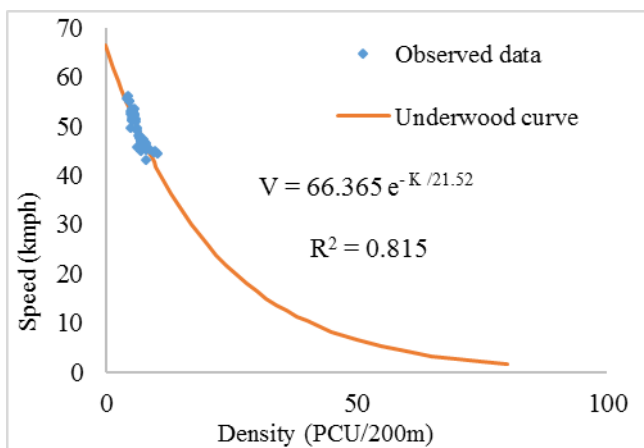


Figure 3: Calibrated Underwood Model

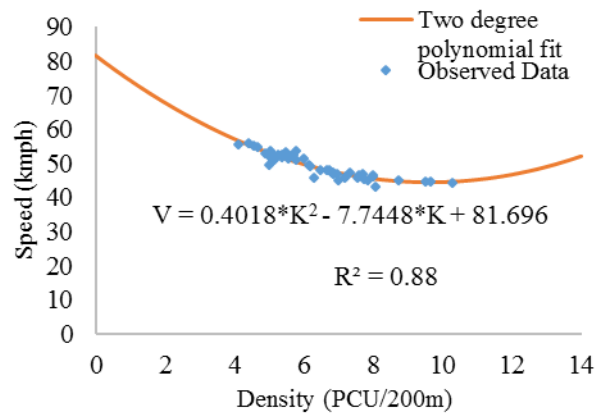


Figure 6: Calibrated Two degree polynomial model

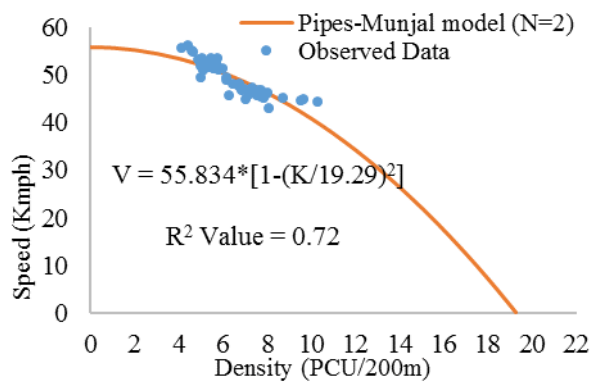


Figure 7: Calibrated Pipes – Munjal Model for N = 2.

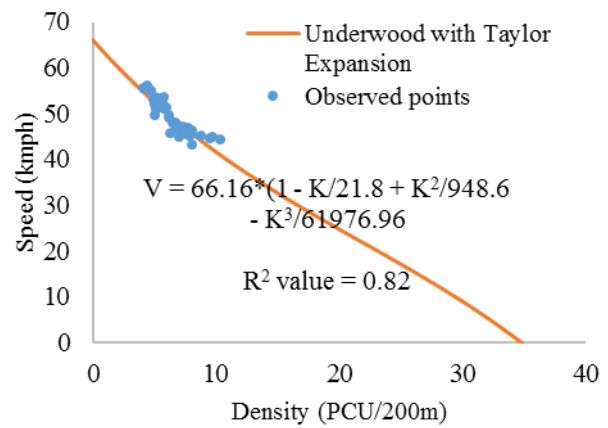


Figure 8: Calibrated Underwood Model with Taylor Series Expansion

Table 2: Calibration of various macroscopic traffic flow model

Model	Calibrated Equation	V <sub>f</sub> (kmph)	V <sub>c</sub> (kmph)	K <sub>j</sub> (PCU/200m)	K <sub>c</sub> (PCU/200m)	R <sup>2</sup>
Greenshields	$V = 61.97 \left(1 - \frac{K}{30.83}\right)$	61.97	30.98	30.83	15.42	0.80
Greenberg	$V = 14.99 \times \ln \frac{168.61}{K}$	-	14.99	168.61	62.03	0.84
Conventional Underwood	$V = 66.36 \times e^{-K/21.52}$	66.36	24.42	-	21.52	0.82
Underwood with Taylor Expansion	$V = 66.16 * \left(1 - \frac{K}{21.8} + \frac{K^2}{948.6} - \frac{K^3}{61976.96}\right)$	66.16	22.05	34.81	21.8	0.82
Drake model	$V = 56.73 * e^{-\frac{K^2}{305.737}}$	56.73	34.41	-	12.36	0.75
Drake with Taylor Series Expansion	$V = 56.71 * \left(1 - \frac{K^2}{307} + \frac{K^4}{188305} - \frac{K^6}{173339891}\right)$	56.71	34.26	22.13	12.39	0.75
Pipes-Munjal(N=2)	$U = 55.83 * \{1 - (K/19.29)^2\}$	55.83	37.22	19.29	11.14	0.72
Polynomial (N=2)	$V = 0.4018 * K^2 - 7.7448 * K + 81.696$	81.69	-	-	-	0.88

Table 3: Re-calibration of recommended models based on validation data

Model	Re-Calibrated Equation	V <sub>f</sub> (kmph)	V <sub>c</sub> (kmph)	K <sub>j</sub> (PCU/200m)	K <sub>c</sub> (PCU/200m)	R <sup>2</sup>
Greenshields	$V = 62.93 \left(1 - \frac{K}{30.11}\right)$	62.93	30.11	31.46	15.06	0.87
Greenberg	$V = 12.94 \times \ln \frac{290.33}{K}$	-	12.94	290.33	106.81	0.89
Conventional Underwood	$V = 64.94 \times e^{-K/23.59}$	64.94	23.89	-	23.59	0.88

Despite high goodness of fit ( $R^2 = 0.88$ ), two degree polynomial model is excluded from the list of best fit models as it doesn't yield realistic free flow speed as well as the speed beyond the current density range. Similarly, due to low goodness of fit and unrealistic jam density; Pipes-Munjaj, Conventional Drake and Drake model with Taylor series Expansion were excluded from the list of best fit models. As both conventional Underwood and Underwood with Taylor series expansion yield same result, only conventional Underwood was considered for validation process. As illustrated above, all three model except Greenberg model (which doesn't yield any finite free flow speed) give more or less a realistic value of free flow speed (around 60kmph). The jam density in Greenberg model is 168.61 PCU/200m, which seems too high. Though Underwood model doesn't yield any definite jam density, assuming 2 kmph as speed at jam density it yields jam density of about 75 PCU/200m.

Taking realistic jam density and free flow speed and reasonable goodness of fit as parameter, Underwood model seems to be the best fit model. But new approach of validation was adopted and all three suitable models viz. Greenshield, Greenberg and conventional Underwood were considered for validation and after two step validation process the best fit model was recommended.

### B. Validation of Model

Three recommended models were validated using Shallaghari bound directional data. Validation process was conducted in two phases. First, all three recommended calibrated models were checked for their goodness of fit based on validation data. And secondly, recommended models were again recalibrated and their corresponding goodness of fit, jam density, density at capacity, speed at capacity and free flow speed were analyzed. After screening through these two process, finally the best fit model is recommended.

**Table 4: Validation of calibrated models**

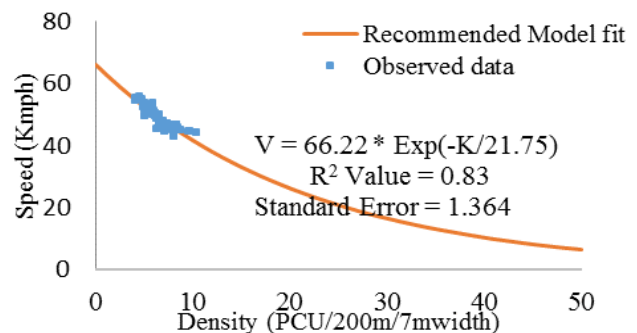
Model	Calibrated Equation	R <sup>2</sup> Value	St. Error
Greenshield	$V = 61.97 \left(1 - \frac{K}{30.83}\right)$	0.868	0.978
Greenberg	$V = 14.99 \times \ln \frac{168.61}{K}$	0.9	0.841
Underwood	$V = 66.36 \times e^{-K/21.52}$	0.878	0.942

Table 3 summarizes the goodness of fit and estimate of various terms associated with the recommended models after recalibration, based on validation data.

The goodness of fit of all three recommended calibrated models, based on validation data are tabulated in Table 4. The goodness of fit follows the same earlier pattern with Greenberg having the high goodness of fit. Considering goodness of fit, Greenberg model is marginally better than other two, but the estimates of jam density, speed and density at capacity doesn't seem to be realistic compared to other two. Similarly, Greenshield model provides comparatively better result than the Greenberg model for free flow speed and speed at capacity but the prediction of jam density and density at capacity is questionable. Now coming to the conventional Underwood model, all the values i.e. free flow speed, speed and density at capacity seem to be realistic and thus it is recommended as the best fit model for the study area.

### C. Recommended Best Fit Calibrated Model

Using both calibration and validation data i.e. altogether 72 points, Conventional Underwood model was calibrated and recommended as the best fit model.



**Figure 9: Recommended Calibrated Conventional Underwood model**

## 7. Summary and Conclusion

Various conventional macroscopic traffic flow models; Greenshields, Greenberg, conventional Underwood, Underwood model with Taylor series expansion, Polynomial model, Pipes-Munjaj model, conventional Drake and Drake model with Taylor series expansion were calibrated. After screening via validation process, conventional Underwood was found to be the most appropriate model due to its ability to yield far better and realistic estimate of free flow speed, speed and density at capacity and approximation of jam density compared to other models.

The recommended best fit model, i.e. conventional Underwood based on both directional data, having goodness of fit of about 83% is:  $V = 66.22 \times e^{-K/21.75}$ . Based on the model, the free flow speed is 66.22 kmph and density at capacity is

21.75 PCU/200m/7m width. The calibrated model can be used for predicting the future scenario of speed and density with anticipated change in traffic volume. Using the calibrated model, the level of service offered by the facilities at the considered roadway segment can be obtained.

## 8. Further Research Area

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This research paper only covers the calibration of macroscopic traffic flow models for freeway section of Kathmandu valley, via case study of Jadibuti-Suryabinayak section. This is just an initiation for calibration of traffic flow models for Nepalese roads and the research need to be carried out further. Thus, it is recommended to study and calibrate the macroscopic traffic flow model for different types of roads, especially for urban roads within Kathmandu valley. Similarly lane specific macroscopic traffic flow model is recommended for future study. Also, study regarding calibration of various microscopic traffic flow models is also suggested to be carried out in future.

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