

Numerical Study of Wake-Induced Vibration on a Square Prism with a Splitter Plate, Placed Downstream of a Larger Circular Cylinder for Energy Harvesting Applications

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Abstract

Flow-induced vibration for energy harvesting is a developing technology, especially at low velocity and small dimensional regimes where conventional turbines are found ineffective. This study numerically explores the wake-induced vibration and interference between galloping and vortex-induced vibration phenomenon on a square prism (12mm) with a splitter plate (60mm length) setup that is placed downstream of another larger circular cylinder (40mm) for piezoelectric energy harvesting applications. Introducing an upstream cylinder created oscillating vortices on the downstream square prism that would otherwise have non-oscillating lift and drag forces. Distance between the bluff bodies was studied at 5D and 3D, D being the diameter of circular cylinder at 4 m/s velocity, and sinusoidal coefficient of lifts with maximum values of 0.85 and 0.98 respectively were obtained. Using splitter plate, the coefficient of lift increased drastically to 2.3 from 0.9. CFD simulation was run from 0.5 m/s to 10 m/s. As the velocity increases, coefficient of lift as well as the vortex shedding frequency increases reaching the maximum of 4.6 and 43.33 Hz at 7 m/s. It was found to follow Strouhal law with an average Strouhal number of 0.213 which agrees with experimental values. But at around 7.5 m/s due to shear layers reattachment between the two bluff bodies, no more oscillating lift is observed, and it is a lock-out velocity for VIV. A bistable region can be expected near this velocity region. Beyond it, periodic vortices cease to exist but galloping phenomenon may be observed that requires multi-physics simulation between the fluid and solid domain for further study. FSI Simulation was done for an inlet velocity of 4 m/s which generated sinusoidal emf with peak voltage of 1.2V and 1.59 V for piezoelectric plates of 1mm and 0.3mm thickness respectively. Experiment was carried out on wind tunnel at Pulchowk Campus at 2.18 m/s air velocity that confirmed the sinusoidal pattern with matching frequency but experimental voltage obtained was much lower than that obtained from the simulation. From the study, we can conclude that the proposed energy harvester has the potential for piezoelectric energy harvesting at the studied low velocity and small dimensional regimes and can be further enhanced with future studies.

Keywords

Wake Induced Vibration, Vortex Induced Vibration, Galloping, Fluid Structure Interaction, Piezoelectric Energy Harvesting, Computational Fluid Dynamics, Multi-physics simulation

1. Introduction

Conventional wind or water turbines are ineffective for low-power applications and at small-scale size and require a minimum flow velocity, thus encouraging to seek new technologies for energy harvesting like flow-induced vibrations. Fluid-solid interaction phenomenon such as Vortex-Induced Vibrations (VIV), transverse or torsional galloping, flutter, and wake-induced vibrations have been considered recently to extract energy from such flow-induced vibrations. Among them, VIV and galloping-based energy harvesting systems are easier to design due to the possible dependence on one degree of freedom: bending of the structure.

VIV is a fluid-structure interaction phenomenon that occurs when fluid flows past a bluff body, such as a sphere, cylinder, prism, etc. at certain flow velocities. As the fluid flows around the cylinder, vortices are shed in its wake, creating periodic changes in pressure and flow velocity. These alternating forces can excite the secondary square cylinder's natural modes of vibration, leading to large oscillations transverse to the fluid flow. But this phenomenon is effective only in its lock-in range

(synchronization region where the Strouhal vortex shedding frequency is close to one of the natural frequencies of the energy harvester), while it becomes ineffective if the wind speed is greater or lower.

Compared to VIV, galloping has a higher amplitude and lower frequency. The most famous occurrence of galloping was the collapse of the original Tacoma Narrows Bridge on 7th November 1940 at 68 km/hr. wind. A structure consisting of only a circular cylinder in cross-flow is immune to galloping. For some geometries, instantaneous velocity and force can be in the same direction. In such case, the simplest representation of galloping by 1DOF motion, the criterion for the occurrence of galloping instability, commonly attributed to Den Hartog[1], is galloping will occur when $F_y/d\alpha > 0$; where $\alpha = \tan^{-1}(\dot{y}/V)$ and $F_y = -L \cos \alpha - D \sin \alpha$ such that \dot{y} is the transverse velocity of the body, V is the velocity of fluid, L and D , are the instantaneous lift and drag forces.

Energy harvesting through flow-induced vibration involves the use of piezoelectric materials attached to the vibrating structure that can produce an electrical charge when subjected to mechanical stress. By adjusting the structural and geometric parameters of the square prism with splitter, it

is possible to match its natural frequency with the frequency of the vortices shed behind the primary circular cylinder. With careful design, a galloping response can be achieved just after the lockout from VIV region thus maximizing energy harvesting and increasing velocity bandwidth for energy harvesting. The governing equations for this study are the Navier Stokes equation, the equation of motions of vibration, the Euler-Bernoulli beam theory, electric equation for the piezoceramic layer.

2. Literature Review

While the Reynolds number signifies whether the flow is laminar or turbulent, the Strouhal number indicates the vortex shedding frequency. The Strouhal number ($St = f \cdot D / U$) is defined as the ratio of the frequency of vortex shedding to the product of the characteristic length of the body and the velocity of the fluid. Based on the existing experimental knowledge, the Strouhal number for the cylindrical body is constant within our study range (for $300 < Re < 1.5 \times 10^5$, St is approximated as 0.21).

The circular cylinder is a classic choice for generating vortex-induced vibration. However, as discussed by Den Hartog (1956)[1], galloping doesn't occur for circular cylinders due to its symmetrical structure. But with a splitter attached behind the cylinder, galloping can be obtained. Rectangular shapes are utilized for galloping energy harvesting. Also, the velocity range for galloping to occur is higher than VIV. This study aims to design an energy harvester with enhanced effectiveness and a wider range utilizing both VIV and galloping phenomenon. For this study, a square prism with a splitter plate attached behind it is placed in the wake region of the circular cylinder.

A similar configuration to this study was experimentally investigated by Wang et al. (2021), [2] using wind tunnel tests. A rectangular plate of width $1D$, $2D$, and $3D$ was used and the distance between plate and cylinder was varied from $0.1D$ to $2D$, where D is the diameter of a circular cylinder. Their experimental results showed that for a specific plate height, full interference and partial interference between galloping and VIV, and then lock-in-type response were observed successively with increasing the space between the cylinder and the plate. The most effective design scheme among the tested cases was a $2D$ -high plate placed around $0.2D - 0.4D$ downstream of the circular cylinder.

Liu et al. (2020) [3] conducted a wide range of experiments with different configurations and provided a promising result with higher voltage output at even low wind speed harvesting utilizing wake flow induced by using double upstream flat-plates. They found out that even at a low wind speed, placing the double plates upstream of a cylinder can change the vibration response from VIV to galloping. When the ratios of the horizontal and vertical distances to the windward width of the cylinder were 1 and 0.5, respectively, the cut-in wind speed was as low as 1.5 m/s. For the square prism, after the double plates are placed upstream, the cut-in wind speed decreased from 3.5 m/s to 1 m/s and the output voltage increased significantly from 1 V to 12 V at just 1.5 m/s.

Qin et al. (2019) [4] designed a cantilever beam with two

square cylinders and a circular cylinder for scavenging wind energy through VIV and galloping. The experiment conducted showed that a large output can be obtained between wind speeds of 2 m/s to 7 m/s. Abdelkefi et al. (2013) [5] used two different upstream cylinders (square in wake of circular cylinder) each attached to vibrating plates, and experimented with a wide range of distances between the cylinders. They concluded that a complex relation exists between spacing between the two cylinders, upstream cylinder size, flow speed, load resistance, and the harvested power output. They designed a square cylinder based galloping energy harvester that can generate energy at low wind speeds (< 1.7 m/s) with a cut-in speed of 0.4 m/s. It was evident that using wake galloping can significantly widen the range of speeds over which energy can be harvested. However, the level of enhancement is dependent on the interactions of the wake effects and the spacing distance between the two cylinders.

Apelt et al. (1975) [6] from 1973 to 1975 experimented by placing a rigid plate of different lengths behind a cylinder. They found that drag coefficient and Strouhal number varied when the ratio of length of the plate to diameter of the cylinder is less than 5 (i.e. $L/D < 5$), due to modifications in the near-wake flow patterns. However, if $L/D > 5$, then vortex shedding disappeared and the drag coefficient would not change by altering the length of the plate.

Alam et al. (2003) [7] conducted an experimental study on fluctuating fluid forces acting on two circular cylinders in a tandem arrangement by varying the spacing between them at a subcritical Reynolds number of 6.5×10^4 . Significant changes in parameters like lift, drag, pressure distribution, Strouhal number, and vortex shedding patterns were observed. The well-bistable flow occurred at critical spacing of $L/D = 3$, where two values of drag coefficient (CD) are seen for two different flow patterns; reattachments flow and jump flow. The experiment showed that the lift and drag forces of the downstream cylinder are highly dependent on the spacing between the cylinders, especially before the critical spacing with two noteworthy peaks at $L/D = 0.4$ and 1.40 . However, in the case of the upstream cylinder, the variation in the coefficient of lift distribution is undulating and reached maximum at $L/D = 3$ and 6.25 , and minimum at $L/D = 4.5$ and 8 . The occurrence of the maximum and minimum values of lift are discussed as due to synchronization of the wakes behind the cylinders with 'in-phase' and 'out-of-phase' conditions, respectively. They also mentioned the work of Sakamoto et al. (1987) [8] who noted that, in the case of two square cylinders, coefficient of lift becomes maximum at the critical spacing due to synchronization of the cylinders with phase a lag of 2π (in phase).

3. Modelling and Simulation

3.1 Design concept

The primary design concept is to closely tune the mechanical resonance frequency of the harvester to that of shedding vortices generated from wake interaction of upstream cylinder with the downstream cylinder at the desired air velocity. Another important aspect is the enhanced design of the harvester such that the distance between the cylinders and the

ratio of their size produces a suitable wake interaction such that VIV and galloping phenomenon can be utilized for maximizing the amplitude of vibration and obtaining a much wider velocity range at which the energy harvester can operate.

3.2 Computational Fluid Dynamics

With the motive of engaging both VIV and galloping phenomenon, an upstream circular cylinder of diameter 40 mm and then, a square cylinder of 12mm was placed at distance L between them. The solid parts were suppressed for fluid simulation in Fluent. Sweep meshing was done with quadratic elements and inflation method and unit cell mesh thickness was kept in the Z-direction. Then the sections of inlet, outlet, fluid wall, fluid body, solid fixed wall, and fluid-structure geometry were identified and named. Pressure-based transient solver and SST k-omega viscous model are used. Inlet velocity and pressure outlet boundary conditions were defined. Dynamic meshing is used for faces that undergo fluid-solid interaction.

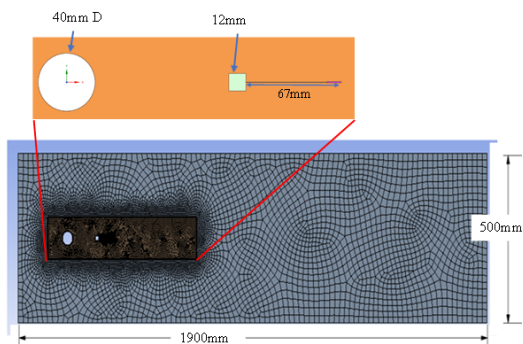


Figure 1: Fluent Geometry and Meshing.

The Turek-Hron benchmark was also successfully run to test the validity of the CFD process. The coefficient of lift and drag as well as animations of pressure and velocity contour was set up to be recorded during the simulation process. Time steps are chosen less than 1/10th of the shedding time period. For the energy harvester with natural frequency (W_n) = 10.093 Hz (chosen from CFD results) and taking Strouhal number (St) = 0.21, the following calculations as shown in Table 1 are done for inlet velocity from 0.5 m/s to 10 m/s.

Table 1: Different case set up from 0.5 m/s to 10 m/s

Case No.	Inlet velocity, U (m/s)	Reduced velocity, U_r (m/s)	Re	Shed. Freq. (Hz)	Time period, T (ms)	Sim. time step (ms)
1	0.5	1.24	1,354	2.625	380.95	20
2	1	2.48	2,707	5.25	190.48	10
3	2	4.95	5,414	10.5	95.238	8
4	4	9.91	10,829	21	47.619	4
5	6	14.86	16,243	31.5	31.75	3
6	7	17.34	18,950	36.75	27.21	2
7	7.5	18.58	20,304	39.375	25.40	2
8	8	19.82	21,657	42	23.81	2
9	10	24.77	27,072	52.5	19.048	1

3.3 Structural solvers

The splitter plate end downstream of the square prism is fixed. Two piezoelectric patches are attached above and below, near the end. The top and bottom faces of the PZT patches were fixed and other sides were given boundary conditions as ground (0 V). The end of the splitter was fixed. Calculations for stress and deformation due to gravity due to gravity is done in Static Structure solver and natural modes of vibrations are computed in the Modal solver which is also shown in the results section.

The Transient Structural solver in ANSYS employs the Finite Element Method (FEM) to solve dynamic structural problems over time and considers inertial and damping effects. The outer fluid domain is suppressed. Step end time is defined, large deflection is turned on and restart points are enabled. Fixed support is defined at the downstream end of splitter plate. Fluid-structure interaction is defined by faces at which force is to be imported from the fluid solver during the multi-physics simulation.

3.4 System Coupling: 2-way FSI

System Coupling is used in ANSYS to solve this multiphysics problem by connecting independent fluent and transient structure solvers such that bi-directional exchange of solution data may occur at a small timestep in the range of a microsecond (equal to time step required within fluent solver) enabling accurate capture of complex fluid-structure interaction. Force is transferred from fluent to transient structural solver and displacement from structural solver to fluent. Iterations are carried out until convergence within the solvers and at data transfer is reached to the required values.

4. Results and Discussions

4.1 Static and Modal Analysis

A maximum bending, stress, and strain of 2.8 mm, 51.67 MPa, and 0.000258 were found at the end of splitter near the fixed support. It is small compared to the fluid-induced forces on the structure. Piezoelectric and MEMS extension was added to add piezoelectric body (PZT-5A) and to measure the generated voltage. The design of the vibrating structure can be done such that the lock-in can be achieved in the desired velocity range. For this, the harvester's natural frequency should be comparable to the shedding frequency of the vortices. We know that, Shedding Frequency = $(St \cdot v) / D$
Required natural frequency at velocity v, $W_n = v / (Ur \cdot D)$

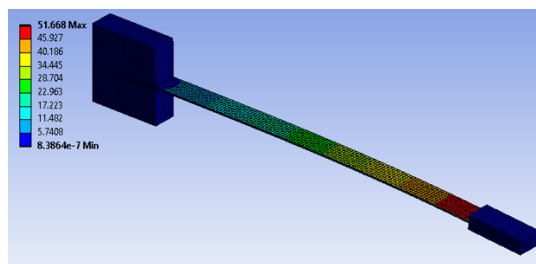


Figure 2: Stress due to gravity

During the modal analysis, it was found that piezoelectric patch attached to the structural body, if not fixed or constrained to any relative motion would not significantly affect the natural frequency of the structure. In agreement with the solid mechanics, the following method can be employed to obtain the desired natural frequency of the structure as per our requirement.

1. Increasing diameter of upstream cylinder also decreases shedding frequency.
2. Increasing size of downstream square cylinder decreases natural frequency.
3. Increasing length of splitter plate reduces natural frequency.
4. Increasing thickness of splitter plate significantly increases natural frequency.

With these manipulations, we can create the structure with the desired natural frequency. However, phenomena like added mass, added stiffness, and added damping can slightly change the effective natural frequency and then the lock-in frequency. The structure was designed with the first natural mode of vibration of 10.093 Hz followed by other modes at 131.45 Hz, 138.39 Hz, and 157.97 Hz.

4.2 Computational Fluid Dynamics

It was observed that using the circular bluff body upstream provided oscillating vertices as shown in Figure 3, on the square prism that would otherwise have non-oscillating small lift and drag forces only, thus enabling the possibility of piezoelectric energy harvesting. Two trials were carried out, keeping the bluff bodies at 5D and 3D distances that produced numerical results for oscillating coefficient of lifts with maximum values of 0.85 and 0.98 respectively.

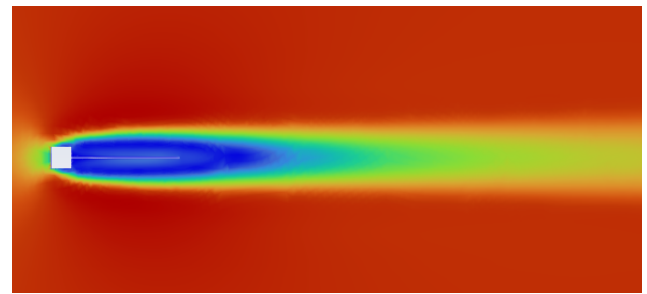


Figure 3: Coefficient of Lift, Drag and Velocity contour for square cylinder without cylindrical bluff body upstream for inlet velocity of 4 m/s..

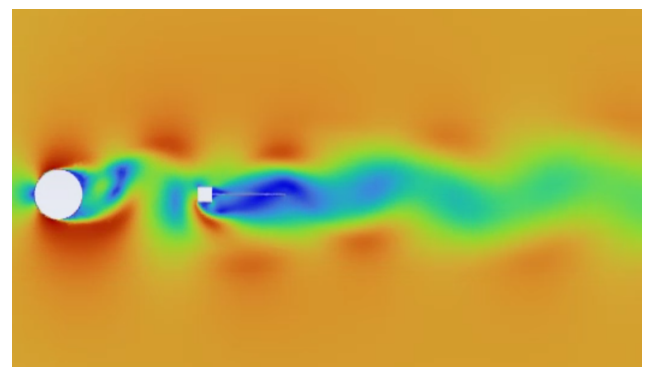
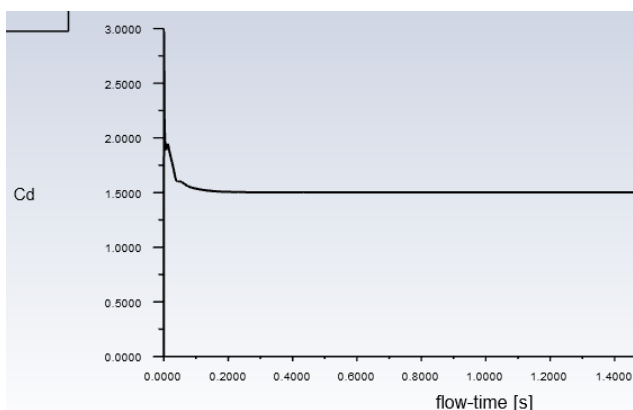
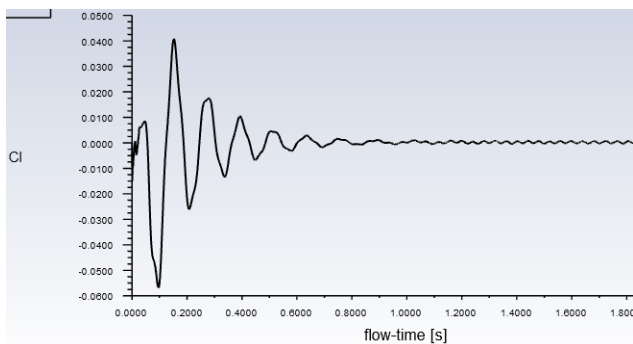
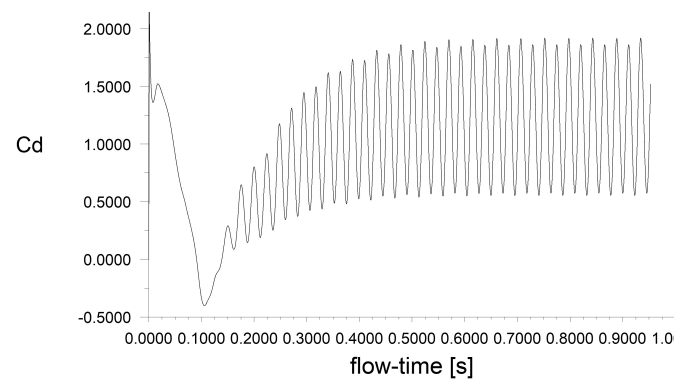
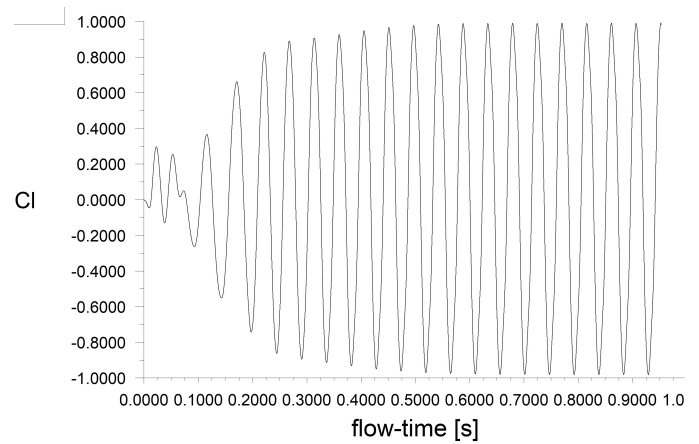


Figure 4: Coefficient of Lift, Drag and Velocity contour for square cylinder with cylindrical bluff body upstream at L=3D for inlet velocity of 4 m/s.

Then a splitter plate was added behind the square cylinder. The lift coefficient for the square cylinder only increased drastically from 0.98 to 2.3. CFD analysis was done on this setup for velocity ranging from 0.5 m/s to 10 m/s. As the velocity increases lift as well as the vortex shedding frequency increases. But at around 7.5 m/s due to shear layers reattachment between the two bluff bodies, no more oscillating lift is observed. A bistable region can be expected near this velocity range. It is the lock-out velocity region for vortex-induced vibration. Beyond it, periodic vortices cease to exist but galloping phenomenon may be observed that cannot be visualized by only CFD but requires multi-physics simulation between the fluid and solid domain i.e., Fluid-Structure Interaction (FSI).

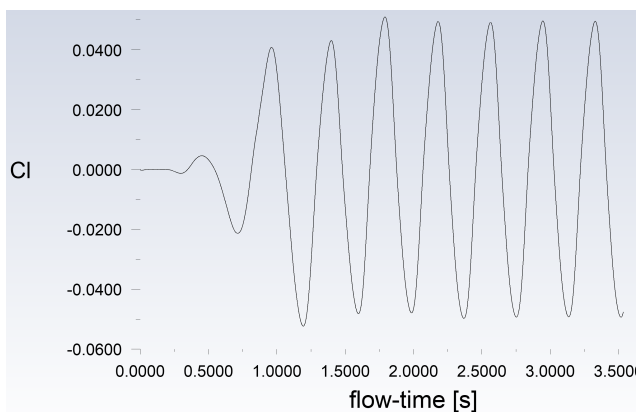


Figure 5: Coefficient of lift of square prism at 0.5 m/s.

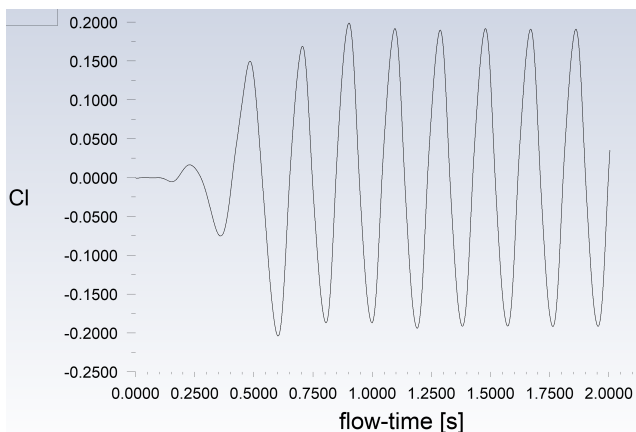


Figure 6: Coefficient of lift of square prism at 1 m/s.

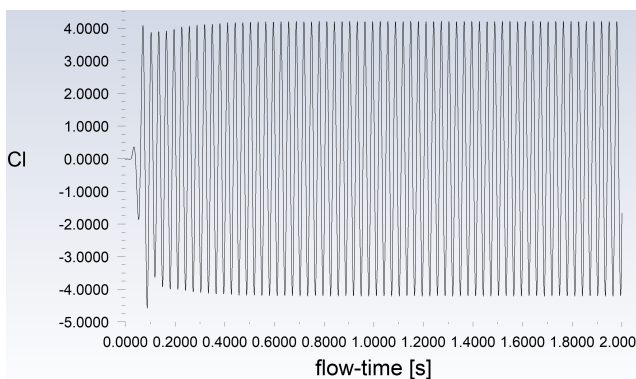


Figure 9: Coefficient of lift of square prism at 6 m/s.

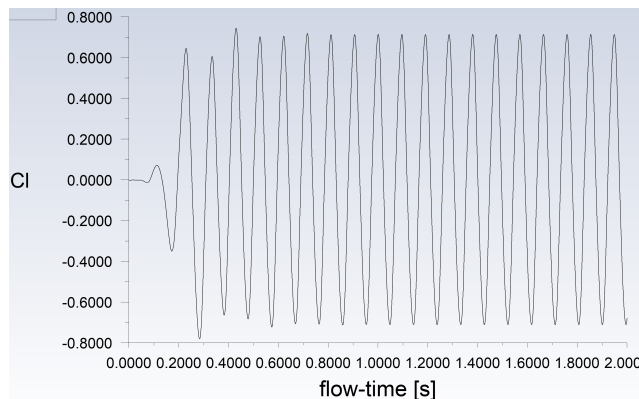


Figure 7: Coefficient of lift of square prism at 2 m/s.

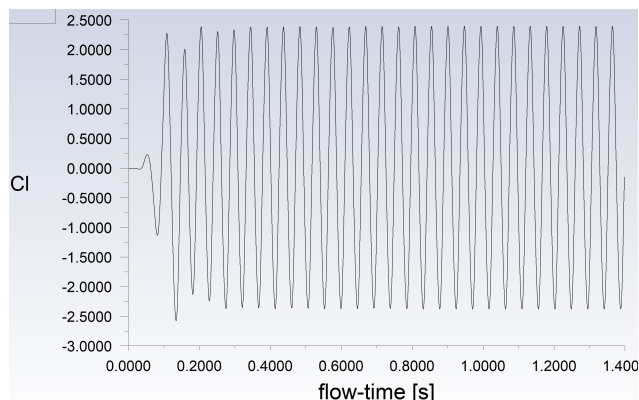


Figure 8: Coefficient of lift of square prism at 4 m/s.

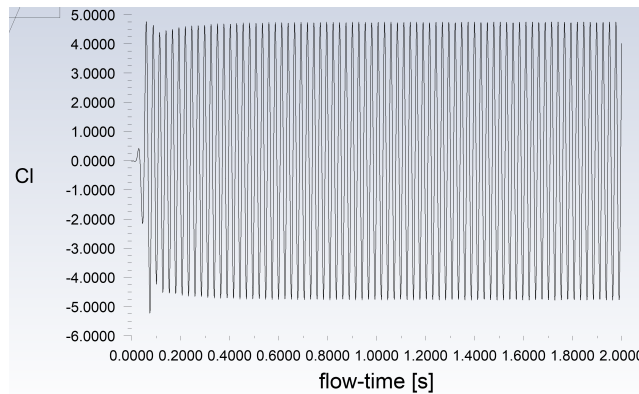


Figure 10: Coefficient of lift of square prism at 7 m/s.

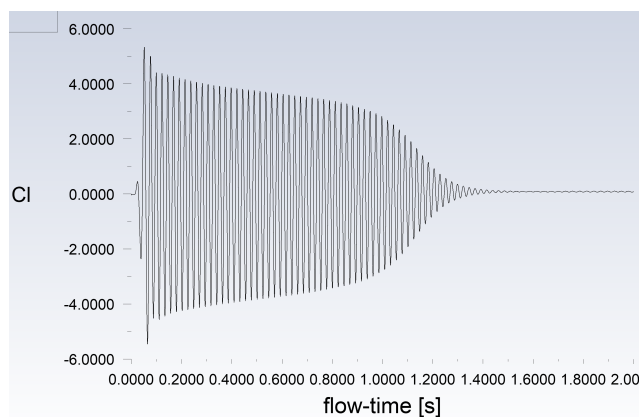


Figure 11: Coefficient of lift of square prism at 7.5m/s.

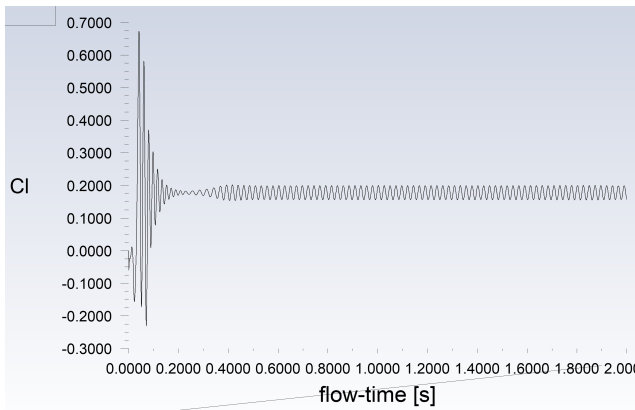


Figure 12: Coefficient of lift of square prism at 10 m/s.

As we plot the Frequency VS velocity graph, a linear relationship is found for velocity range of 1 m/s to up to near 8 m/s. Here, the frequency (f_{so}) is the nonvibrating vortex-shedding frequency. This follows the Strouhal law and the average Strouhal number can be obtained from the graph as 0.2132 which agrees with the existing experimental values. From the CFD analysis, we can conclude that the proposed energy harvester has the potential for piezoelectric energy harvesting from wake-induced vortices.

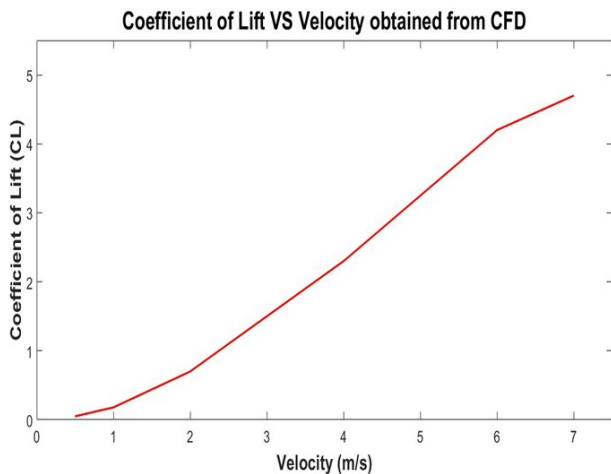


Figure 13: Coefficient of Lift at different inlet velocities.

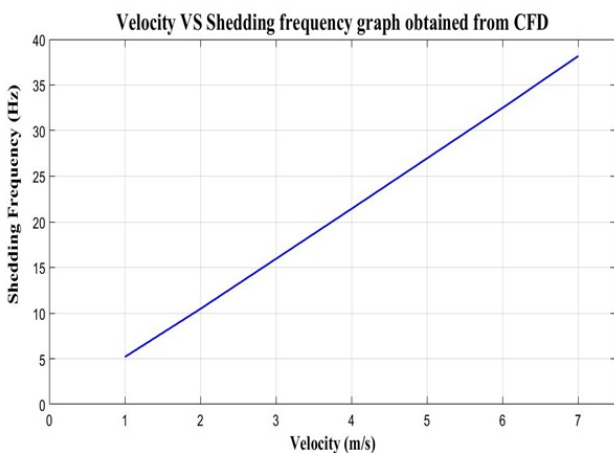


Figure 14: Non-vibrating vortex shedding frequency at different velocities.

4.3 Performance of Energy Harvester: FSI

The fluid-structure interaction simulation of the piezoelectric energy harvester was run for 4m/s inlet velocity. The piezoelectric patch of thickness 1mm and 0.3mm were tested that produced sinusoidal voltage with peak of 1.2 V and 1.59 V respectively. Similarly experimental study was also carried out at wind tunnel of the Pulchowk campus at 2.18 m/s velocity using piezoelectric sensor and Arduino programming. Circular piezoelectric sensor and 16mm wide square plate was used and length of splitter plate was decreased to obtained the same primary natural frequency as the numerical design. Experiment was carried out at this setup and simulation was also run mimicing the experimental setup.



Figure 15: Flow Visualization at $Re = 1354$ at wind tunnel in Pulchowk Campus.

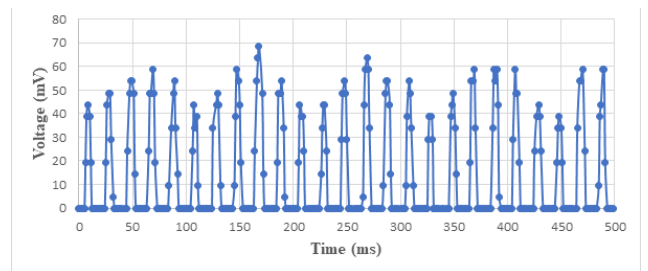


Figure 16: Positive voltage readings of PZT sensor through Arduino Interface during experiment

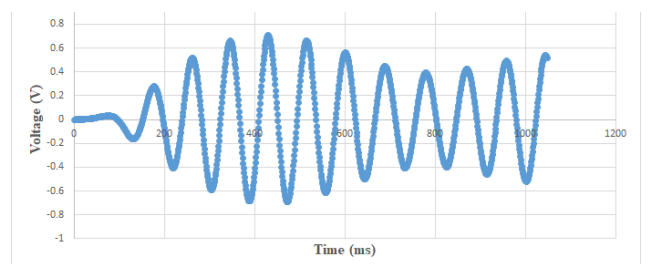


Figure 17: Average voltage obtained from FSI Simulation

During the experiment only positive cycle of the voltage was plotted due to directional limitation of the Arduino but it can be reproduced on both sides if needed. The sinusoidal AC-like Voltage was generated in the experiment but its amplitude was much lower than that obtained from simulation. This can mainly be attributed to limitations in the experiment from sensor quality, contact between PZT patch and splitter plate, irregularities, wind tunnel limitations, losses and other factors. The frequency of the volate generated was in coherence with

that of the simulation providing a greater legitimacy to this numerical study.

5. Conclusion

As per the motive of the study, the designed energy harvester is found effective at the chosen regime of low flow velocity and small structural dimensional. Upto 7 m/s, using upstream larger cylinder helped to generate fluctuating wake that produces sinusoidal lift on secondary cylinder. Beyond that, the direct interaction with the wind can create fluctuating lift force on secondary cylinder. The results of the numerical study were found to be valid when compared to standard values of dimensionless parameters at this experimental regime. The chosen tandem design broadened the velocity range at which the harvester can perform reducing the lower cut-off velocity upto 1 m/s. The arrangement of cylinder at 3D, use of splitter plate, frequency synchronization at 2 m/s, all worked to enhance the performance of the energy harvester. FSI Simulation was done for inlet velocity of 4 m/s which generated sinusoidal emf with peak voltage of 1.2V and 1.59 V for piezoelectric plate of 1mm and 0.3mm thickness. The experimental setup was created with circular piezoelectric patch whose result showed voltage generation similar to the simulation that was done mimicking the experiment. One of the experiment result to bolster the numerical study is that the frequency of the voltage generated from the experiment matched to that obtained from the simulation. From the study, we can conclude that the proposed energy harvester has the potential for piezoelectric energy harvesting at the studied low velocity and small dimensional regimes. It can also serve as a reference and guidelines for future studies. The performance of this piezoelectric energy harvester can be enhanced with further explorations and use of advanced manufacturing approaches.

6. Future Enhancements

Since electric load can act as damper reducing vibration of transient structure, a three-way system coupling between fluid, solid, and electric parameters can provide a better numerical study. The experiment and simulation can be carried out in a greater number of velocities and design iterations can be made along the study. Advanced manufacturing of the harvester and future optimizations may result in the harvester being

practically feasible for energy generation applications.

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