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Axisymmetric Analysis of Cylindrical Shells Considering Geometry and Lattice Variation

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Abstract
An analytical approach has been developed for the axisymmetric analysis of lattice cylindrical shells as a homogenization technique based on the equivalent continuum shell theory. This approach has been applied to the case of axisymmetric strained state of lattice cylindrical shells considering variation in geometry and lattice configurations. The major contrast between continuous shells and lattice shells is found to be in the constitutive equations. The geometry is lattice cylindrical shells subjected to uniformly distributed load in longitudinal and transverse directions. The boundary conditions are fixed edge at the origin/bottom end of the shell and loads applied at the z = L or the top edge of the shell. The analytical solution is obtained using the homogenization technique via MATLAB coding. The results are compared to that obtained from the FEM package ANSYS. Also, the parametric analysis has been done to study the effect of change in radius, length and grid length as well as various lattice configurations of the lattice cylindrical shell. Lattice configuration n = 4 has been found to have less deformations.

Keywords
Axisymmetric, Lattice, Cylindrical shells, Homogenisation, Equivalent continuum, Constitutive, MATLAB, ANSYS

1. Introduction

Shells are spatially curved surface structures that can withstand externally applied loads. They can be defined by their middle plane, thickness and material properties. As a result of their curvature, shells can support out-of-plane loads using in-plane membrane forces, which is one of the main reasons why they are such durable and affordable constructions. Among several shell types, surfaces of revolution are generated by the revolution of a meridional curve, about an axis of revolution. The meridional curve is a straight line segment in case of cylinder and conical surfaces.

Shells can be designed in the form of lattice shells, which derives its strength from its double curvature, but is constructed of lattices or grids. Considering this, when shell of revolution is constructed as a lattice shell, the resulting structure becomes relatively light and exceedingly efficient; making it simple and quick to erect. Shells constituting of lattices have been considered a fine replacement to solid shells.

A lattice structure is a network of connected ribs composed of continuous, incredibly strong, stiff, and durable metals or fibres. Various configurations are possible for the ribs that make up a structure. Typically, an axial load is applied to the ribs making up a grid construction. They can be constructed with variety of cross-sections, skin thickness, lattice's configuration.

The analysis of lattice cylindrical shells has been taken considering the static, geometric [1]and constitutive equations[2]. For axisymmetrical strained state of the shell, the variables of the shell along the circumferential direction vanish and the system of equations governing the statics of the shell can be rather simplified, which further helps in quick analysis of the shell structure.

The deformations of a structure are usually calculated from the stress in the lattices. The one layer reticulated shell theory based on continuum design model can been done [2], which gives the homogenization technique for the shell of revolution.

The models differ from the general shell theory in terms of the basic group of equations involved such that the statics, kinematics equations are the same as usual whereas, the constitutive equations, depending on the lattice structure and material gets more complicated. First, the constitutive equation for the structural members, i.e. its constituent grids of the reticulated shells need to be calculated. Then, those equations can be used for the determining the forces, moments, stresses in the lattices considering various configuration. Brief idea on loading conditions and lattice configuration has been provided.

The analysis of lattice shells of revolution for the structure loaded with tension and torque can be found in Slinchenko (2001) [3]. The numerical results were obtained for the equivalent homogeneous model of the cylindrical structure with three families of ribs. And, optimum design and criteria for failure of ribs can be found in Slinchenko (2000) [4].

2. Theoretical Background and Methodology

The proposed method treats the elastic shell as a continuous system, in which external stresses and the stress-strain state are represented by functions. This helps for the proper application of methods of solid mechanics in the analysis of lattice shells. The axes of the structural members form the families of ribs on the median surface of shell.

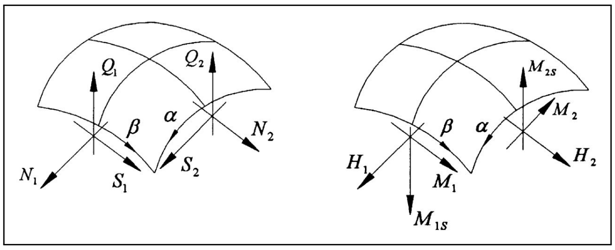

Figure 1: Directions of forces and moments acting in a general shell [3]

The positioning of points on the middle surface of the shell of revolution is done considering the cylindrical coordinates z, θ.

The surface meridian of the shell has the equation $r = r\ (z)$. We put the primary curvature radii R_1, R_2 of the middle surface and coefficients A, B of the first quadratic form as:

$$A = \left(1 + r'^2\right)^{1/2}, B = r \tag{1}$$

$$r'' R_1 = -\left(1 + r'^2\right)^{3/2}, R_2 = r\left(1 + r'^2\right)^{1/2} \tag{2}$$

In the case of axisymmetrical strained state of the shell of rotation, all the desired functions depend only on coordinate z. Here,

$$v = \omega = \tau = S = Q_2 = H = Y = 0 \tag{3}$$

Now, the system of the equations above can be reduced to:

Static equations:

$$\begin{gathered}\frac{\partial}{\partial z}(BN_1) - N_2\frac{\partial B}{\partial z} - Q_1\frac{AB}{R_1} + ABX = 0\\ \frac{\partial}{\partial \mathrm{z}}(\mathrm{BQ_1}) + \mathrm{AB}\left(\frac{\mathrm{N_1}}{\mathrm{R_1}} + \frac{\mathrm{N_2}}{\mathrm{R_2}}\right) + \mathrm{ABZ} = 0\\ \frac{\partial}{\partial \mathrm{z}}(\mathrm{BM_1}) - \mathrm{M_2}\frac{\partial \mathrm{B}}{\partial \mathrm{z}} - \mathrm{ABQ_1} = 0\end{gathered} \tag{4}$$

Geometric equations:

$$\begin{gathered}\varepsilon_1 = \frac{1}{A}\frac{\partial u}{\partial z} - K_1 w\\ \varepsilon_2 = \frac{1}{AB}\frac{\partial B}{\partial z}u - K_2 w\\ \chi_1 = -\frac{1}{A}\frac{\partial}{\partial z}\left(\frac{u}{R_1} + \frac{1}{A}\frac{\partial w}{\partial z}\right),\\ \chi_2 = -\frac{1}{AB}\frac{\partial B}{\partial z}\left(\frac{u}{R_1} + \frac{1}{A}\frac{\partial w}{\partial z}\right),\\ \gamma_1 = K_1 u + \frac{1}{A}\frac{\partial w}{\partial z}\end{gathered} \tag{5}$$

Constitutive equations:

$$\begin{aligned}N_1 &= \alpha_{11}\varepsilon_1 + \alpha_{12}\varepsilon_2\\ N_2 &= \alpha_{12}\varepsilon_1 + \alpha_{22}\varepsilon_2\\ M_1 &= \gamma_{11}\chi_1 + \gamma_{12}\chi_2\\ M_2 &= \gamma_{12}\chi_1 + \gamma_{22}\chi_2\end{aligned} \tag{6}$$

The above equations can be reduced into the form:

$$y'(z) = P(z)y(z) + f(z), \tag{7}$$

where,

$$y = [u; w; \gamma_1; N_1; M_1; Q_1] \tag{8}$$

$$f = [f_1 = 0; f_2 = 0; f_3 = 0; f_4 = -AX; f_5 = 0; f_6 = -AZ] \tag{9}$$

And, P is 6*6 matrix obtained from the above equation.

For lattice cylinder containing n = 4 family of ribs:

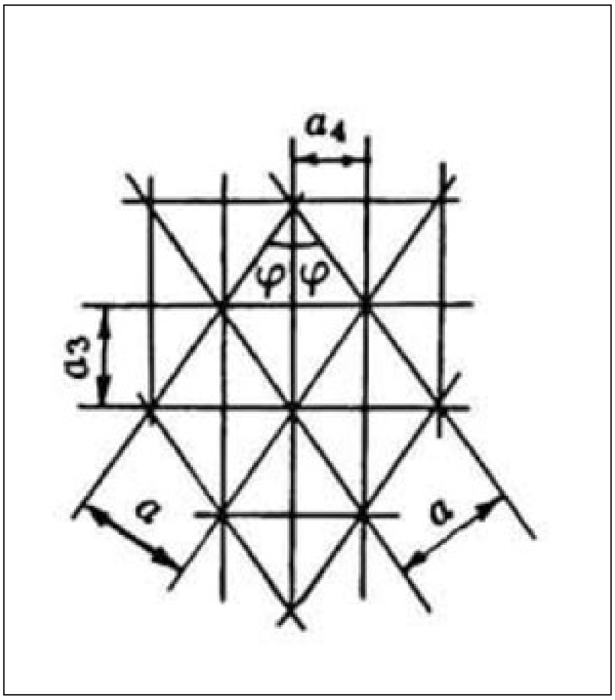

Figure 2: Lattice configuration

$$\mathrm{a}_1 = \mathrm{a}_2 = \mathrm{a}, \phi_1 = -\phi_2 = \phi, a = 2a_3 s = 2a_4 s$$

$$A = 1, B = 1, R_1 = \infty, R_2 = r$$

$$\alpha_{11} = 2Kc^4 + K_4, \alpha_{12} = 2Kc^2s^2, \alpha_{22} = 2Ks^4 + K_3,$$

$$\gamma_{11} = 2Ic^4 + I_4 + 2Cc^2s^2, \gamma_{12} = 2(I - C)c^2s^2,$$

$$\gamma_{22} = 2Is^4 + I_3 + 2Cc^2s^2$$

where,

$$K_i = \frac{E_iF_i}{a_i}, I_i = \frac{E_iJ_{1i}}{a_i}, C_i = \frac{G_iJ_{3i}}{a_i}.$$

For lattice cylinder containing n = 3 family of ribs:

$$A = 1, B = 1, R_1 = \infty, R_2 = r,$$

$$\alpha_{11} = \frac{9EF}{a}, \alpha_{12} = \frac{3EF}{a}, \alpha_{22} = \frac{9EF}{a},$$

$$\gamma_{11} = \frac{3EJ_1(3+m)}{8a}, \gamma_{12} = \frac{3EJ_1(3+m)}{8a}\frac{(1-m)}{(3+m)},$$

$$\gamma_{22} = \frac{3EJ_1(3+m)}{8a}$$

where,

$$m = \frac{GJ_3}{EJ_1}$$

3. Results and Discussions

Analytical solution has been done using the homogenization method in computation package MATLAB. Deformations have been calculated for the grid cylindrical structure with number of grids n = 4 as in figure 3 and n = 3 families of ribs and geometric parameters with a unit lattice as shown in the figure 2. Two load cases have been considered: uniaxial compression and radial compression. The results shown are for a model that ignores the impact of boundary effects on the general state of stress. The obtained results are compared to the solution of the same problem modeled using the Finite Element Method (FEM) package ANSYS.

The responses for the lattice structures has been plotted for the longitudinal and radial transverse deformations with uniform load of 10^8 N/m applied at the top edge of the cylinder with longitudinal and transverse forces separately to study the effect of compressive load with varying directions at top of structure, i.e. as for a tower.

Radius of shell (R) : Varying from 2.5 m to 20 m

Length of shell (L) : Varying from 5 m to 20 m

Grid length (a) : 500 mm, 1000 mm

Cross-section of the ribs: 200 mm * 200 mm

ANSYS: Element type - BEAM188, Mesh element size - 0.1 m

Boundary conditions:

Fixed end at the coordinate z = 0 m

Longitudinal force (N_1) = 10^8 N/m at z = L m

Transverse force (Q_1) = 10^8 N/m at z = L m

N_1 and Q_1 applied separately.

Material properties: Steel, E = $2*10^{11}$ N/m^2, Isotropic property.

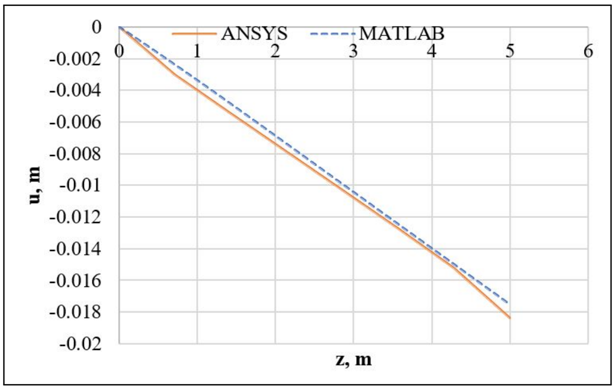

Figure 4: Deformation, u due to N1 R5m L5m a500mm for n = 4 family

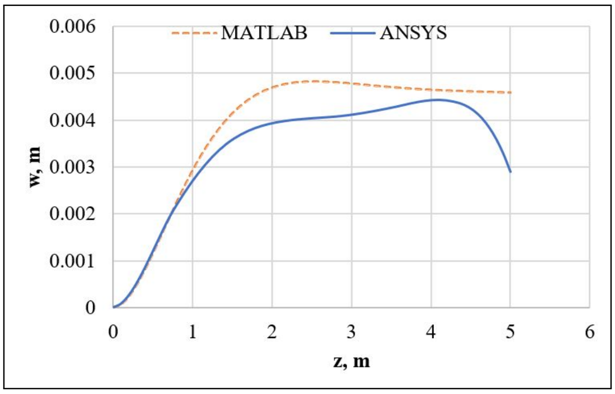

Figure 5: Deformation, w due to N1 R5m L5m a500mm for n = 4 family

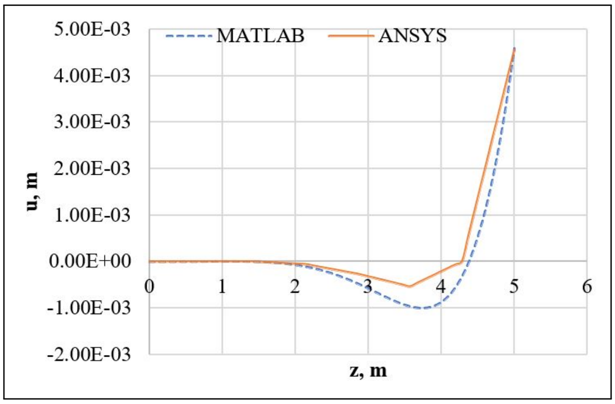

Figure 6: Deformation, u due to Q1 R5m L5m a500mm for n = 4 family

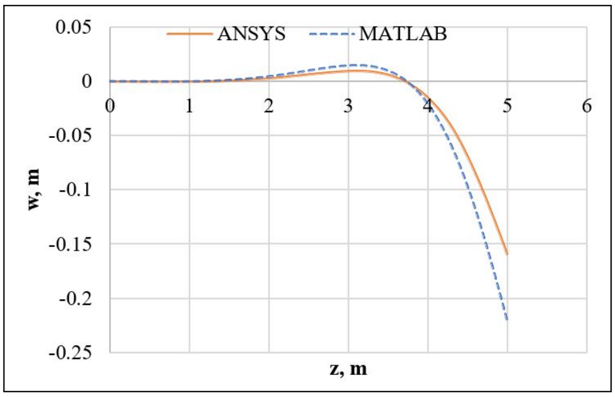

Figure 7: Deformation, w due to Q1 R5m L5m a500mm for n = 4 family

The graphs in figures 4, 5, 6, 7 clearly show that the response of the structure corresponding to the longitudinal and transverse deformations obtained using analytical solution through homogenized approach done in MATLAB and the FEM software ANSYS is within close proximity of one another, with the exception of the regions situated in the immediate vicinity to the loaded edge, especially for the transverse deformations.

The case of the lattice cylinder has been subjected to N_1 and Q_1 with various geometry and lattice configurations, with a case being shown of R5m, L5m and a500mm for n = 4 family of ribs, and the results show good correspondence for the lattice structure excluding loaded boundary edge. Similarly, the maximum deformation results have been generated for R10m, L10m and a1000mm, which show that the longitudinal and transverse deformations due to N_1 and Q_1 have been found to be four times the corresponding deformations for R5m L5m a500mm as can be seen from tables 1 and 2 , except higher

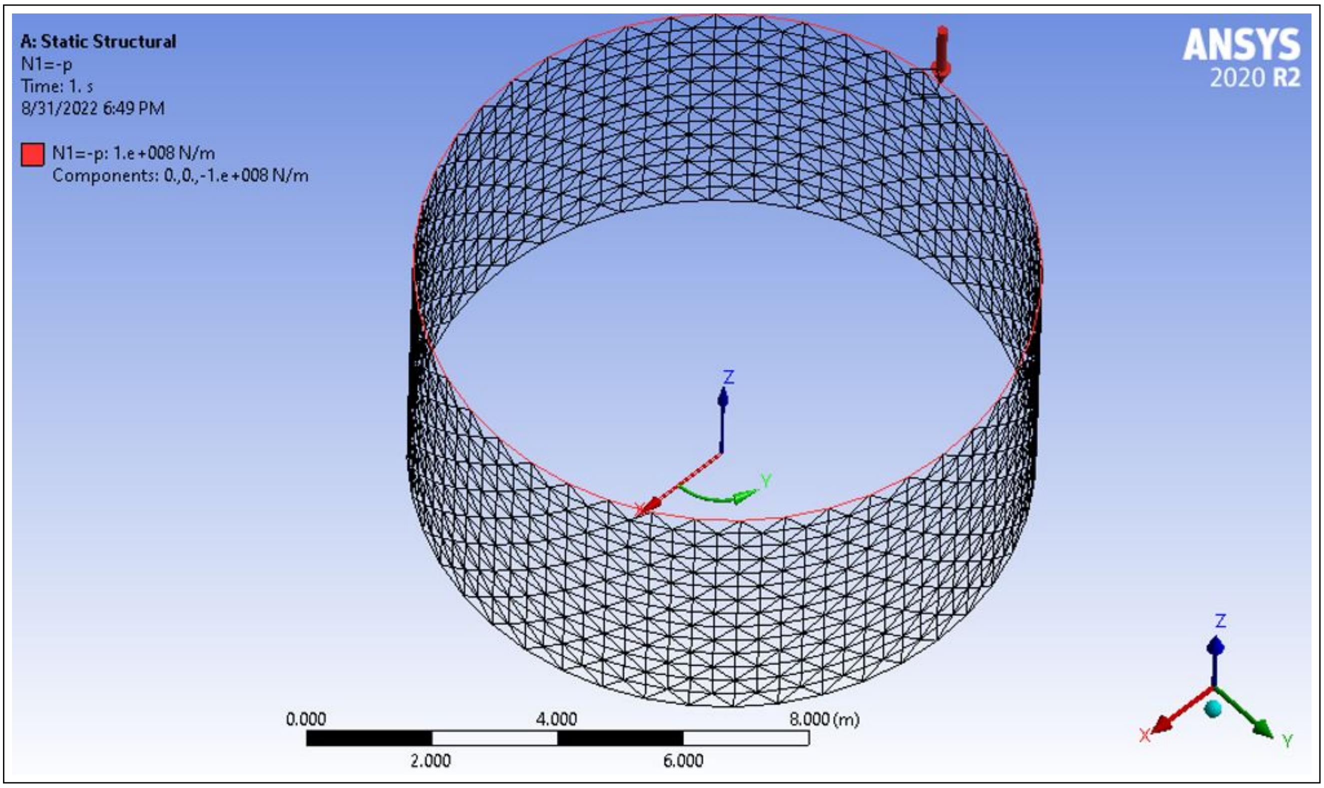

Figure 3: Lattice Cylindrical Shell, n = 4 family of ribs with N_1 loading

Deformation, m	due to N1		Error	due to Q1		Error
	MATLAB	ANSYS		MATLAB	ANSYS	
u	-0.0175	-0.01839	4.84895219	0.004585	0.004547	0.82672295
w	0.004585	0.004434	3.39383321	-0.22001	-0.15887	High due to boundary effect

Table 1: Max. deformation for R5m L5m a500mm for n = 4 family

Deformation, m	due to N1		Error	due to Q1		Error
	MATLAB	ANSYS		MATLAB	ANSYS	
u	-0.07022	-0.07614	7.77487265	0.018495	0.021017	11.999745
w	0.018495	0.022679	18.4487252	-1.24457	-0.81815	High due to boundary effect

Table 2: Max. deformation for R10m L10m a1000mm for n = 4 family

Deformation, m	due to N1		due to Q1	
	n=4	n=3	n=4	n=3
u	-0.0175	-0.03068	0.004585	0.010324
w	0.004585	0.008329	-0.22001	-0.38023

Table 3: Max. deformation for R5m L5m a500mm for n = 4 and n = 3 family

Deformation, m	due to N1		due to Q1	
	n=4	n=3	n=4	n=3
u	-0.07022	-0.12339	0.018495	0.041687
w	0.018495	0.041687	-1.24457	-2.15098

Table 4: Max. deformation for R10m L10m a1000mm for n = 4 and n = 3 family

discrepancy is observed for the transverse deformation 'w' due to Q_1 near the boundary region.

As can be seen from the tables 3 and 4, the deformation results for R5m, L5m, a500mm and R10m, L10m, a1000mm for n = 3 family of ribs, the max. deformations show that 'u' due to N_1 and 'w' due to Q_1 are 1.75 times more than that for n = 4 family of ribs. And, max. deformations 'w' due to N_1 and 'u' due to Q_1 are 2.25 times the case for n = 4 family of ribs. This show that deformations for n = 3 family of ribs are almost twice than that for n = 4 family of ribs.

4. Conclusions

In this paper, the axisymmetric analysis of lattice shell of revolution is done using the homogenized approach. The static analysis is performed for the structure loaded with uniaxial compression and transverse radial compression. The numerical results obtained for the lattice cylindrical shell with n = 4 and n = 3 family of ribs have been compared to those obtained from the FEM package ANSYS. The homogenized model predicts the deformations with high accuracy (the discrepancy not exceeding 20% for most cases). However, the high discrepancy generally results near the regions subjected to loaded boundary edges. The deformations for R, L, 'a' can be scaled up to four times the half dimensions R/2, L/2, 'a/2', and helps in shortening of analysis time. Deformations for n = 3 family of ribs are almost twice than that for n = 4 family of ribs.

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