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Probabilistic FEM Analysis of Tunnel Lining Stability – A Comparison with Traditional Deterministic Approach

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Abstract

In routine design, the approach to tunnel support analysis is to conduct a deterministic analysis using a single mean value for all input parameters. Such an assessment of tunnel stability does not consider variation in the ground parameters which is inherent in geomaterials. While this approach may be sound when extensive investigations reveal that the mean design parameters represent the ground well, the typical ground encountered and the level of investigations conducted in the industry seldom justify these assumptions. Probabilistic analysis allows the incorporation of the variability in the input parameters and thus is better suited for the analysis of tunnel in typical design cases. The probabilistic approach to design is only recently gaining prominence around the world and in the context of Nepal, it is yet to be adopted. In this study, data collected from Phukot Karnali Hydroelectric Project have been analyzed probabilistically with Rocscience's Phase2 software to find the reliability index and subsequently the probability of failure for a tunnel section supported by patterned rock bolts and steel ribs with shotcrete. Sensitivity analysis was conducted to find which input parameters have the highest effect on tunnel performance. The probability of failure of tunnel is evaluated at the crown and sidewall. Probabilistic analysis has shown that the steel ribs perform better at the crown than the sidewall with probabilities of failure of 2.85% and 3.90% respectively, the shotcrete also performs better at crown than sidewall with probabilities of failure of 5.24% and 4.26% respectively. From the deterministic analysis, done using mean value of all input parameters, the tunnel is found to be stable with factor of safety greater than one in all region, indicating that the supported sections deemed stable in routine design may not be acceptable when considering the variability of the rock mass.

Keywords

Deterministic approach, point estimate method, probabilistic approach, reliability index

1. Introduction

In computational mechanics, the problems can be solved with two main approaches; deterministic approach and probabilistic approach. The traditional deterministic approach uses a mean value of input parameters and gives a single output [1]. The probabilistic analysis allows variation in the input parameters to be entered and can give multiple sets of output based on the best-case scenario, most likely scenario, and worst-case scenario [1]. In the case of tunnel analysis, the deterministic approach gives the output as to whether the tunnel is stable or not by checking if the factor of safety is greater than one or not. But, the likelihood of failure of the tunnel decreases with an increasing factor of safety. The deterministic approach doesn't accurately quantify what will be the change in the likelihood of failure as the factor of safety changes. On the other hand, the probabilistic approach gives the exact value of the probability of failure for every value of the factor of safety.

Tunnel construction is also a complex work as the geological parameters in the field are never certain and there will be variation and uncertainties concerning the properties of rock mass or soil where the tunnel is to be constructed. The variation in the geological parameters is caused by various kinds of uncertainties as shown in Figure 1 [2]. The natural variability is due to the inherent feature of geology or the natural processes [3]. Temporal variability deals with the variation at the same location with respect to time while spatial variability deals with variation with respect to place at the same time. Knowledge uncertainty relates to the understanding of the site and process which relates to availability and correct collection of data, ability to model the real world in software, and such. Model uncertainty relates to the degree to which the numerical model built-in computer represents the real world while parameter uncertainty is related to the degree to which the parameters can be measured accurately.The third type of decision model uncertainty is mainly related to the operational process. For the design stage, this uncertainty is not relevant. In such areas with high variation and uncertainty, the probabilistic approach is combined with FEM to better analyze the structure [4].

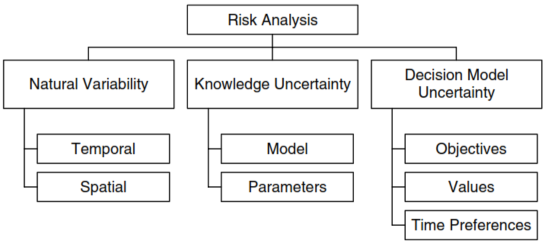

Figure 1: Various types of uncertainties in geological parameters [2]

The tunnel lining stability is analyzed deterministically using the core replacement technique, a method to simulate the 3D excavation of the tunnel in 2D [5]. This method is similar to convergence-confinement method[6] but instead of decreasing internal pressure, the material at the core is replaced by a new material with reduced stiffness and no initial element loading. Due to no internal loading, the internal pressure goes to zero when the material at core is replaced. So, the tunnel compresses the core material and the radial displacement will progressively increase as the core material is replaced by weaker material in each step simulating the 3D tunnel. Then the Vlachopoulos and Diederichs graph [7], is used to find the closure of tunnel prior to support installation, and this closure is used to find the equivalent value of core modulus of rock which, when incorporated into the material model, produces the same deformation. Thus, the properties of material before support installation is found and support can be applied after adding a new stage with the equivalent core modulus.

In probabilistic approach, the analysis is done using Rosenbluth's Point Estimate Method(PEM) [8]. In this method, the input parameters are assumed to be normally distributed and the analysis is done at two points: mean plus standard deviation and mean minus standard deviation. This method is simpler and uses less resources than Monte Carlo simulation but is not as reliable as Monte Carlo simulation[9]. PEM is useful in the case when it is required to get approximate results with as less resources as possible.

Probabilistic analysis also involves evaluation of reliability index, which is the likelihood that the structure will continue to work [9] i.e., it is the probability that the structure won't fail. So, the probability of failure and Reliability index are inverse of each other. Reliability index for the tunnel can be calculated by using the mean and standard deviation of the factor of safety of the tunnel area calculated for various conditions, from which probability of failure can be obtained. The obtained probability of failure can be compared to the values given by some authors to know if the obtained values are considered acceptable. [10] has given the value probability that a project should try to achieve or the qualitative explanation of the achieved probability of failure for further assessment. Also, [11] has proposed a table for the acceptable value of the probability of failure based on the Q-system [12] recommendation for ESR values.

In this study, the data are collected from the Phukot-Karnali Hydroelectric project, located at Kalikot district.

Figure 2: Location of Phukot Karnali Hydroelectric project

The purpose of this study is to analyze the stability of tunnel lining or support using probabilistic FEM approach and evaluate the probability of failure of the tunnel. The analysis is also done by traditional deterministic approach to compare the difference in results obtained from both methods.

2. Methodology

The steps involved to conduct this study are shown in Figure 3.

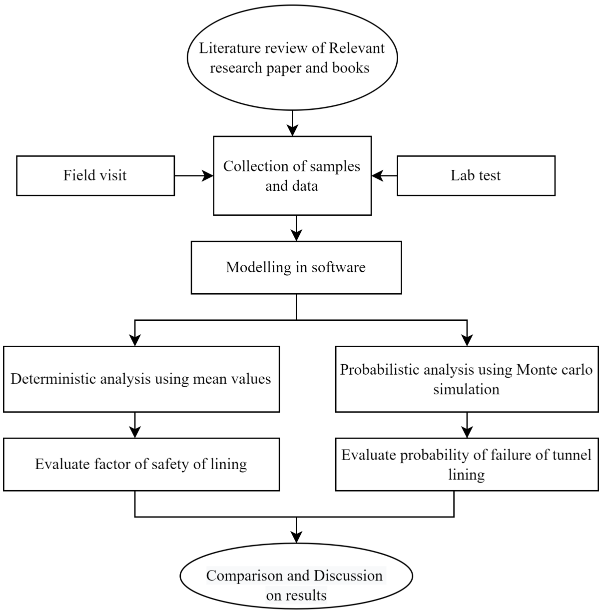

Figure 3: Flowchart showing the deterministic and probabilistic analysis steps.

2.1 Collection of input data

For phase2 software, there are multiple parameters required to fully analyze the tunnel. The general information on the site, geological maps, and lab test results are provided by the client of the Hydroelectric project, i.e., Vidhyut Utpadan Company Limited (VUCL). Then some other data was collected from the site visit. The collected data were data required for RMR and Q classification, Schmidt hammer rebound numbers, the information about the jointing in the area along with their attitudes, GSI classification, etc.

2.2 Modeling of site

Phase2 software version 8.0, developed by Rocscience, is used for the analysis of the tunnel of the Phukot Karnali Hydroelectric project. The modelling is initially done in nine stages: initial stage, which represents the in-situ condition of the site where the mean value of Young's modulus obtained from the lab test, i.e., 8324 MPa, is used. Then the value of Young's modulus of the material is gradually decreased until the final excavated stage which represents the excavation phase where the value of Young's modulus is zero. The tunnel is inverted D-shaped with dimensions 8mx8m. The tunnel profile is drawn and the expansion factor is taken as 3. The graded mesh with six nodded triangles element type is chosen for discretizing and meshing.

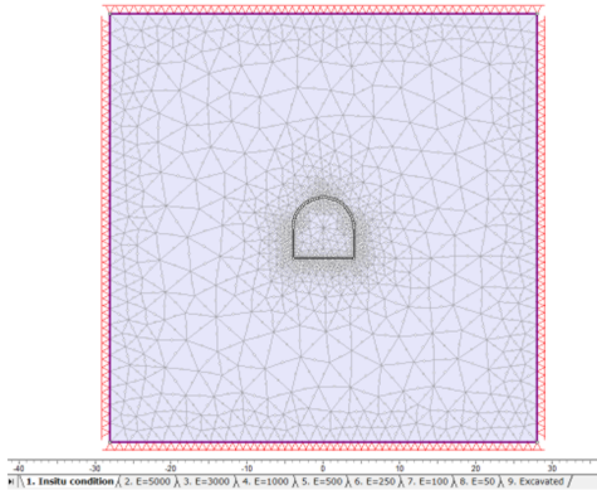

Figure 4: Phase2 model of Phukot-Karnali tunnel showing various stages

2.3 Material Properties

For the deterministic analysis, one data for each of input parameters is enough. To make analysis more reliable, we can take multiple data and take mean value of the data. But, for probabilistic analysis, we need at least one parameter with varying data which can be considered a variable parameter. In this study, the varying data is available for uniaxial compressive strength(UCS), Poisson's ratio, Young's modulus of elasticity for intact rock, and Geological Strength Index(GSI) as shown in Table 1. These parameters are assumed to follow normal distribution as the Phase2 software used for this study doesn't have capability to incorporate other distribution, and also previous research have shown that most geological parameters follow normal distribution [2].

Table 1: Input parameters with varying data

SN	Property	Mean	Std Dev
1	Young's Modulus	8324	2100
2	Poisson Ratio	0.25	0.02
3	Uniaxial Compressive strength	85.74	19.51
4	GSI	59	4.94

Since, the value of Young's Modulus provided by VUCL were for intact rock, these values were obtained for rock mass using equation 1.

$$E_{rm} = E_i \left(0.02 + \frac{1 - \frac{D}{2}}{1 + e^{\frac{60+15D-GSI}{11}}} \right) \quad (1)$$

where, E_{rm} is Young's modulus for rock mass, E_i is Young's modulus for intact rock, and D is disturbance factor.

Other General input parameters for the phase2 model are as shown in Table 2

Table 2: General Input parameters in Phase2

A. Rock Description		
1	Rock Type	Calcareous Gneiss
2	Initial Element loading	Field stress only
3	Elastic type	Isotropic
B. Strength Parameters		
1	Failure Criterion	Generalized Hoek-Brown
2	Material type	Plastic
3	Intact Rock Constant	28
4	Disturbance Factor	0.5

The field stress data was not available, so, the values are estimated by assuming only the gravity stress by using the equations 2 and 3.

$$\sigma_v = \gamma h \quad (2)$$

$$\sigma_x = \sigma_y = \frac{\nu}{1-\nu}\sigma_v \quad (3)$$

where, σ_V is vertical stress, γ is unit weight of rock, ν is Poisson's ratio, h is overburden, and σ_x and σ_y are horizontal stresses. The overburden was taken as 700m, so the value of vertical stress was obtained to be 18.9 MPa and horizontal stress as 6.3 Mpa for ν = 0.25.

2.4 Parameter Evaluation

The variation in input parameters with varying data may or may not have significant effect in the stability of tunnel lining. If the variation in certain parameters have negligible effect in tunnel stability, these parameters can be ignored and only the mean value can be used for further analysis. This will reduce the time and resource requirements without having any effect in final result. To find the most influencing varying input parameters a parametric sensitivity analysis is done. For this, the effect on the tunnel plastic radius and radial deformation while using the maximum and minimum value of a particular parameter is checked. For all other parameters the mean value is used. The value of radial displacement and plastic radius may either decrease or increase while increasing or decreasing the value of input parameters. So, if max is the maximum value of radial deformation or plastic radius and min is the minimum value of radial deformation or plastic radius obtained during the analysis, the percentage difference is obtained by using Equation 4.

$$\text{Percentage difference} = \frac{max - min}{max} * 100\% \quad (4)$$

The result of sensitivity analysis is shown in the Figure 5.

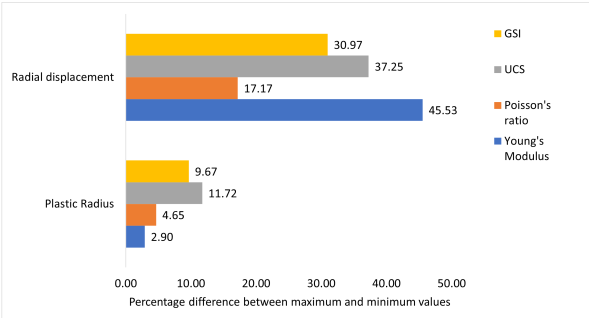

Figure 5: Graph showing the result of sensitivity analysis

In case of radial displacement, Young's Modulus seems to be the most influencing factor while Poisson's ratio seems to be the least influencing. But in case of plastic radius, Poisson's ratio has higher effect than Young's Modulus. Even in the radial displacement, Poisson's ratio has significant effect. So, all of these parameters are selected as variable input parameters.

For PEM, the total number of discrete points where analysis is to be done is 2^4, which is 16. So, there will be a total of 16 Phase2 models with values of varying input parameters as shown in Table 3

2.5 Reliability Index and Probability of failure

The reliability analysis is done by Point Estimate method over sixteen discrete points. There will be a value of factor of safety for each model. The reliability index can be calculated from the mean and

Table 3: All discrete points in PEM

Point	GSI	UCS	Poisson's Ratio	Young's Modulus
P1	64	105.25	0.27	10424
P2	64	105.25	0.27	6224
P3	64	105.25	0.23	10424
P4	64	105.25	0.23	6224
P5	64	66.23	0.27	10424
P6	64	66.23	0.27	6224
P7	64	66.23	0.23	10424
P8	64	66.23	0.23	6224
P9	54	105.25	0.27	10424
P10	54	105.25	0.27	6224
P11	54	105.25	0.23	10424
P12	54	105.25	0.23	6224
P13	54	66.23	0.27	10424
P14	54	66.23	0.27	6224
P15	54	66.23	0.23	10424
P16	54	66.23	0.23	6224

standard deviation of factor of safety as shown in Equation 5 [13].

$$\beta = \frac{\mu_{FS} - 1}{\sigma_{FS}} \tag{5}$$

where, β is reliability index, μ_{FS} is mean of factor of safety, and σ_{FS} is standard deviation of factor of safety.

Then the probability of failure(P_f) can be determined from the reliability index with standard normal cumulative function (NORMSDIST) [14].

$$P_f = NORMDIST(-\beta) \tag{6}$$

3. Result Analysis

3.1 Deterministic Analysis

By using the mean value of input parameters, the Phase2 model was run and the values obtained are: Maximum radial displacement = 0.02412m, plastic radius = 7.6m

The area of the tunnel = 4*4 + π * 4^2 = 57.13 m^2

The equivalent radius of tunnel that has area of 57.13 m^2, (Rt) = $\sqrt{\frac{57.15}{\pi}}$ = 4.26m

Distance from tunnel face from which support is applied (X) = 2m (Assumed)

Thus, X/Rt = 2/4.26 = 0.47,

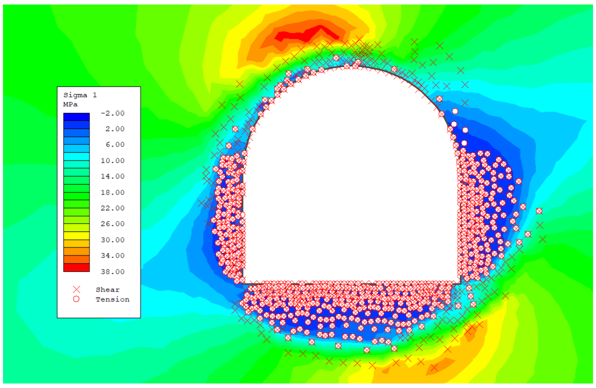

Figure 6: The plastic zone formed around the tunnel

Rpl/Rt = 7.6/4.26 = 1.78

For the above values of X/Rt and Rpl/Rt, from the graph shown in Figure 2.6, we get, Ur/Umax = 0.58

So, Ur (radial deformation before installation of support) = 0.58 * 0.02412 = 0.01392m

Now, to determine the equivalent value of young's modulus just before the installation of tunnel support, the maximum radial deformation in each stage of the Phase2 model are recorded and the graph of Radial displacement vs Young's modulus is drawn by plotting the value of radial deformation in each stage against Young's modulus assumed for that stage in Microsoft Excel. The graph showing the relation between radial deformation and Young's modulus is shown in Figure 7.

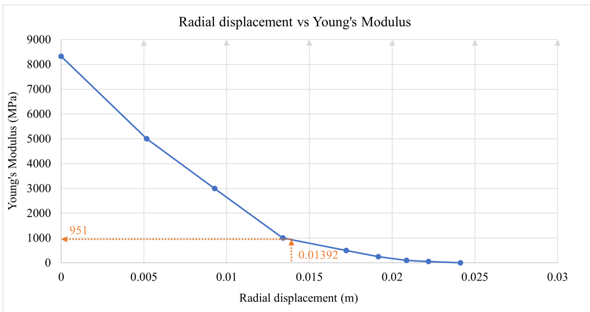

Figure 7: Radial displacement vs Young's Modulus curve

From Figure 7, the equivalent value of Young's Modulus just prior to support installation is found to be 951 MPa. This means the material around the tunnel behave as having Young's modulus of elasticity of 951 MPa just before support application. Since after support application the material softening no longer happens, the stage with Young's Modulus value lower than 951 MPa will not be reached. So, a stage is added with 951 MPa value of Young's modulus and all the stages with lower value of Young's modulus are removed and support is applied. The final tunnel stages in Phase2 model for deterministic analysis is as shown in Figure 8.

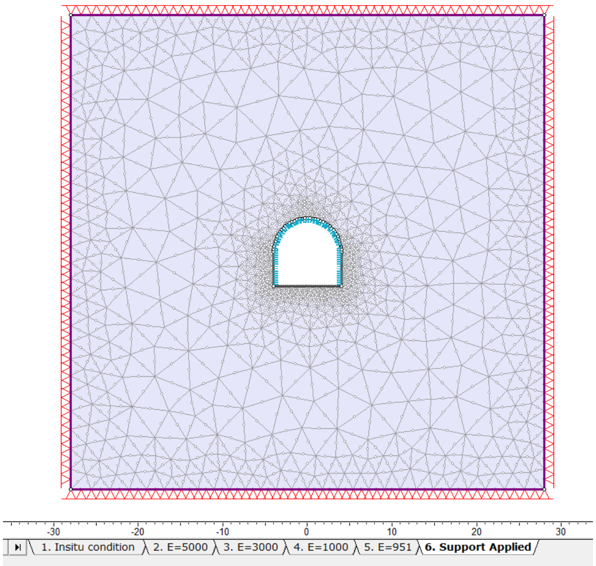

Figure 8: Final Phase2 model for deterministic analysis

The support used is steel ribs with shotcrete with properties as shown in Table 4.

Table 4: Properties of tunnel support system

SN	I. Steel Ribs Properties	
1	Shape	I-beam
2	Designation (metric)	W150x18
3	Area (m2)	0.00474
4	Young's modulus (MPa)	200000
5	Poisson Ratio	0.25
6	Compressive Strength (MPa)	400
7	Tensile Strength (MPa)	400
9	Spacing of Ribs(m)	2
	II. Shotcrete Properties	
1	Thickness (m)	0.1
2	Young's Modulus (MPa)	25000
3	Poisson's ratio	0.15
4	Compressive strength (MPa)	30
5	Tensile strength (MPa)	3

Then the model is run and the support capacity plots are obtained as shown in Figure 9.

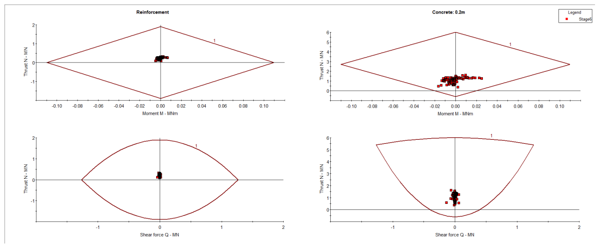

Figure 9: Support capacity plots obtained form Phase2

From the support capacity plots, it is observed that all the points lie within the factor of safety envelope of 1. Thus, the tunnel supports are stable.

3.2 Probabilistic Analysis

The model is run with "probabilistic analysis" enabled in project settings of Phase2 with mean and standard deviation of UCS, GSI, Poisson's ratio, and Young's Modulus and the plastic zone obtained from the analysis is shown in Figure 11.

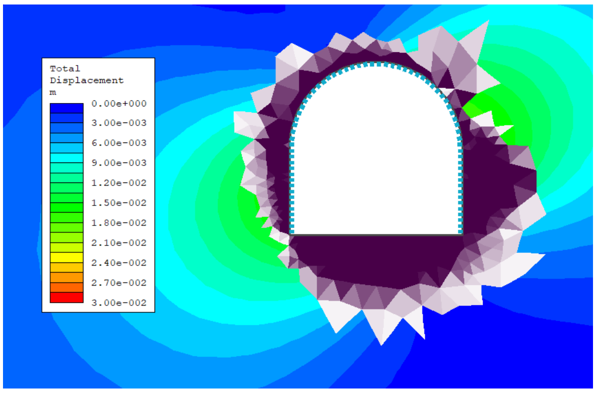

Figure 10: Plastic radius envelope obtained from probabilistic analysis

For this study, probability of failure is calculated one point on the crown and one point on the sidewall are selected, as shown in Figure 11. Point 1 corresponds to the crown while point 2 corresponds to the sidewall of the tunnel.

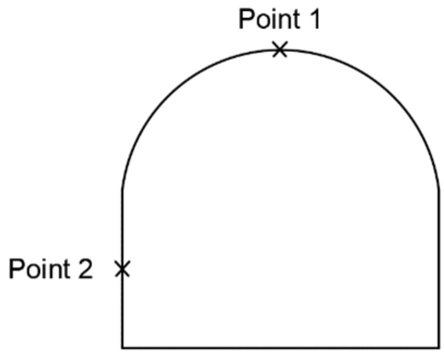

Figure 11: Point 1 and Point 2 considered for probability of failure analysis

For sixteen points of PEM, a total of sixteen models are built. From these sixteen models, sixteen factors of safety for both points 1 and 2. The values of factor of safety are obtained for both steel ribs and shotcrete. From these sixteen factor of safety, mean and standard deviation is calculated. Then, finally reliability index and probability of failure are evaluated as shown in Equation 5 and 6.

A summary of results thus obtained are shown in Table 5 and 6.

Table 5: Probability of failure calculation for point 1

Description	I Beam W150x18	20 cm shotcrete
Mean of FS(μ) Standard deviation of FS(σ) Reliability index(β)	7.21 3.26 1.90	4.46 2.01 1.72
Probability of Failure(Pf) in %	2.85	4.26

Table 6: Probability of failure calculation for point 2

Description	I Beam W150x18	20 cm shotcrete
Mean of FS(μ) Standard deviation of FS(σ) Reliability index(β)	8.28 4.13 1.76	5.16 2.56 1.62
Probability of Failure(Pf) in %	3.90	5.24

The acceptable probability of failure for water tunnel proposed by [11] is 2-3%. In this case. the probability of failure of support are all above this limit instead of steel ribs in crown.

Also, comparing to values proposed by [10], the probability of occurrence of these failures fall under "Unlikely" category.

4. Conclusions

In this study, the analysis of stability of tunnel support was done using deterministic and probabilistic approaches. The sensitivity analysis has shown that Uniaxial compressive strength is the overall most influential input parameter for this case. It has the highest effect on plastic radius and second-highest effect on radial deformation next to Young's Modulus of Elasticity. The deterministic analysis shows that the tunnel constructed at the Phukot-Karnali Hydroelectric project is stable with a factor of safety greater than one. The probabilistic analysis has shown that the shotcrete and steel ribs perform better at the crown than the sidewall. The probabilities of failure obtained from this study are higher than the proposed values in the available works of literature except for the steel ribs in crown. To bring the probability of failure down to an acceptable level, the tunnel construction team should focus on improving the drilling and blasting action to reduce the disturbance factor as poor-quality blasting was observed on site. The thickness of the shotcrete can be increased and the spacing of steel ribs can be decreased if reducing the disturbing factor doesn't decrease the probability of failure significantly.

Acknowledgments

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References

[1] Zdzislaw Pawlak, S. K. Michael Wong, Wojciech Ziarko, et al. Rough sets: probabilistic versus deterministic approach. *International Journal of Man-Machine Studies*, 29(1):81–95, 1988.

[2] Gregory B Baecher and John T Christian. *Reliability and statistics in geotechnical engineering*. John Wiley & Sons, 2005.

[3] B Valley, PK Kaiser, and D Duff. Consideration of uncertainty in modelling the behaviour of underground excavations. In *Proceedings Fifth*

International Seminar on Deep and High Stress Mining (Deep Mining 2010), M. Van Sint Jan and Y. Potvin (eds), pages 6–8, 2010.

[4] Wing Kam Liu, T Belytschko, and YJ Lua. Probabilistic finite element method. In *Probabilistic structural mechanics handbook*, pages 70–105. Springer, 1995.

[5] Rocscience. Rs2 tutorils — 3d tunnel simulation using core replacement. `https://www.rocscience.com/help/rs2/tutorials/tunneling/3d-tunnel-simulation`. Accessed: 2022-08-31.

[6] Carlos Carranza-Torres and C Fairhurst. Application of the convergence-confinement method of tunnel design to rock masses that satisfy the hoek-brown failure criterion. *Tunnelling and underground space technology*, 15(2):187–213, 2000.

[7] Nicholas Vlachopoulos and MS Diederichs. Improved longitudinal displacement profiles for convergence confinement analysis of deep tunnels. *Rock mechanics and rock engineering*, 42(2):131–146, 2009.

[8] Emilio Rosenblueth. Point estimates for probability moments. *Proceedings of the National Academy of Sciences*, 72(10):3812–3814, 1975.

[9] Andrzej S Nowak and Kevin R Collins. *Reliability of structures*. CRC press, 2012.

[10] Christopher RI Clayton. *Managing geotechnical risk: improving productivity in UK building and construction*. Thomas Telford, 2001.

[11] Tamara Jovanovska, Issa Togo, and Vasko Gachevski. One method for defining an acceptable level of risks in tunneling. *Facta Universitatis. Series: Architecture and Civil Engineering*, 17(3):315–325, 2019.

[12] Nicholas Barton, Reidar Lien, and J Lunde. Engineering classification of rock masses for the design of tunnel support. *Rock mechanics*, 6(4):189–236, 1974.

[13] E Bukaçi, Th Korini, E Periku, S Allkja, and P Sheperi. Reliability analysis for tunnel supports system by using finite element method. *American Journal of Engineering Research (AJER)*, 5(9):1–8, 2016.

[14] Richard J Bathurst, Tony M Allen, and Andrzej S Nowak. Calibration concepts for load and resistance factor design (lrfd) of reinforced soil walls. *Canadian Geotechnical Journal*, 45(10):1377–1392, 2008.