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Flood Wave Propagation

Bharat Raj Pandey [a]

[a] *Chief Engineer, Hydro-Consult Engineering Ltd (HCE)*
[a] bharat.spbrp@gmail.com

Abstract
The hydraulic structures including dams pose an extreme threat to the people living in the downstream area during the event of failure in those structures and create a huge inundation in the downstream area and in some cases, the cultivated land as well as the inhabitance of the downstream communities will sweep away by the enormous flooding in the river stretches. This article intends to discuss the various methods for solving unsteady surge wave problems using Shallow Water Equations. The open-source solver HEC-RAS has been implemented to a rectangular channel to produce a diffusive surge wave propagating downstream with respect to time and space. The travel time of the peaks has been evaluated from upstream to downstream for the implication of early warning flood forecasting.

Keywords
Dam, Flood Attenuation, Unsteady Flow, Shallow Water Equation

1. Introduction

The Hydraulic Structures such as dams and spillway always pose a devastating threat to the people living in the downstream of the dams where those structures are used for various purpose such as reservoir impoundment for generating hydroelectricity, fulfilling the demand of water supply for the hugely populated inhabitances of Urban cities and supplying the demand of irrigation supply, for the navigation purpose and the also for the flood control to protect the cultivated land of the communities. In order to estimate the extreme flood propagation a simulation of flood events cause by these manmade structures for Hydropower projects is needed to be carried out for its assessment. An in-house computer model has been used to verify the result of the analysis in this study and has been will also be modified in case-to-case basis for several instances. The study will also involve a field investigation to collect data for the analysis. A numerical modelling tool is to set up for evaluating flood hazards caused by real world dam breaks and to understand its effects of the flood propagation thereby benefiting the people living to the downstream communities of the dam. There will be mostly two kinds of flow characteristic occurs in those dam failure events basically A) gradually varied unsteady flow and B) the rapidly varied unsteady flow.

2. Materials and Methods

While there is a failure of Dams, the nature and characteristic of the flood wave is unsteady flow hence it is discussed in the following sub heading to illustrate its basic characteristic of the flood propagation:

2.1 Gradually Varied Unsteady Flow

2.1.1 Continuity of Unsteady flow

The low of continuity for unsteady flow may be established by considering the conservation of mass in an infinitesimal space between two channel sections: "Since water is incompressible, the net change in discharge plus the change in storage should be zero"; Hence, the following equation governs:

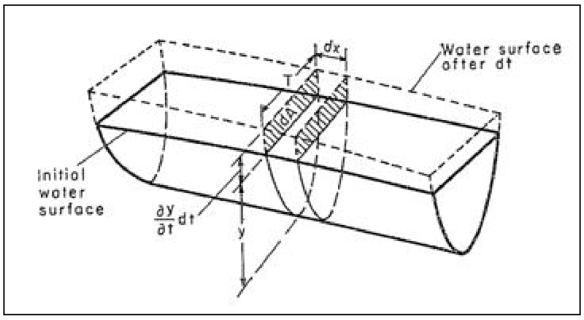

Figure 1: Continuity of Unsteady Flow (Chow, V.T, 1959, p. 525)

$$\frac{\partial Q}{\partial x}dxdt + Tdx\frac{\partial y}{\partial t}dt = \frac{\partial Q}{\partial x}dxdt + dx\frac{\partial A}{\partial t}dt = 0 \quad (1)$$

By simplifying,

$$y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} + \frac{\partial y}{\partial t} = 0 \quad (2)$$

The above equations are all forms of the continuity equation for unsteady flow in open channels. For rectangular channel of infinite width, the above original equation may be written as

$$\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} = 0 \quad (3)$$

where q is the discharge per unit width. This expression was first introduced by Saint-Venant. When the channel is to feed laterally with a supplementary discharge of q' per unit length, for instance into an area that is being flooded over a dike, then the above equation may be rewritten as,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + q' = 0 \quad (4)$$

OR,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + \frac{\partial A'}{\partial t} = 0 \quad (5)$$

2.1.2 Dynamic Equation of the Unsteady Flow

The unsteady flow will be considered in two-dimensional steady flow except that an additional variable for the time variable will be used. This time variable takes into account the variation in velocity of flow and accordingly brings to the fore the acceleration, which produces force and causes additional energy losses in the flow. Hence the following dynamic equation of gradually varied flow governs:

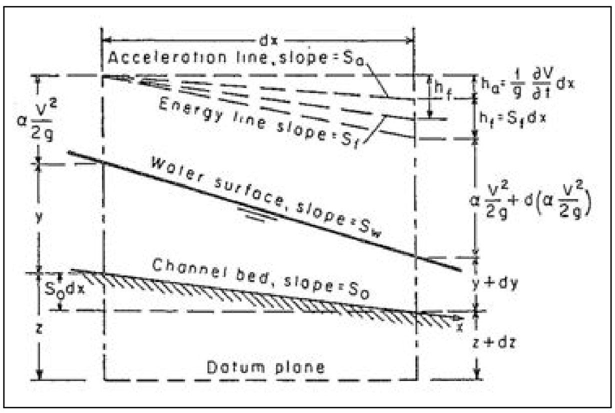

Figure 2: Dynamic Equation of Unsteady flow(Chow, V.T, 1959, p. 527)

$$g\frac{\partial y}{\partial x} + V\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} - g(S_o - S_f) = 0 \quad (6)$$

The continuity and dynamic equations for gradually varied unsteady flow were first published by Saint-Venant. However, owing to their mathematical complexity, exact integration of the equations is practically impossible. For practical applications, a solution of the equations may be may be obtained by finite difference scheme.

2.1.3 Basic numerical techniques for gradually varied unsteady flow

The equation governing gradually varied unsteady flow are the equation of continuity and conservation of momentum equation. The application of these equations in a rectangular channel for the case of subcritical, unsteady flow are considered in the following paragraphs. For rectangular channel, the equation of continuity is:

$$\frac{\partial y}{\partial t} + y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = 0 \quad (7)$$

where, y= depth of flow; t=time; V=average velocity of flow; x=longitudinal distance The conservation of momentum equation is:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0 \quad (8)$$

The above continuity and momentum equation are the set of simultaneous equations which can be solve for two unknowns V and y, given appropriate boundary conditions. The solution of this set of equations is virtually impossible without the aid of a high-speed digital computer.

2.1.4 Method of Characteristics

A numerical technique which has often used to solve unsteady flow problems involves the solution of governing equations by the use of Characteristic method. Again, assume a wide rectangular channel and by rearranging of continuity and momentum equations then we get the following,

$$H_1 = \frac{\partial y}{\partial t} + y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = 0 \quad (9)$$

(Richard H French 1987, p. 555) and

$$H_2 = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0 \quad (10)$$

H1 and H2 with an unknown multiplier λ can be combine in a linear combination to form a new function H.

$$H = \lambda H_1 + H_2 \tag{11}$$

where, for any two real, distinct values of λ will produce two equations in V and y that will retain all the attributes of continuity and momentum equations according to the above equation. the above equation can be written as:

$$H = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) + \lambda(\frac{\partial y}{\partial t} + y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x}) \tag{12}$$

or,

$$H = \frac{\partial V}{\partial x}(V + \lambda y) + \frac{\partial V}{\partial t} + \lambda[\frac{\partial V}{\partial x}(V + \frac{g}{\lambda}) + \frac{\partial y}{\partial t}] - g(S_o - S_f) \tag{13}$$

In the above equation the first and the second terms are the total derivatives of the velocity of flow and the depth of flow, or

$$\frac{dV}{dt} = \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial t} \qquad if\frac{dx}{dt} = V + \lambda y \tag{14}$$

and

$$\frac{dy}{dt} = \frac{\partial y}{\partial x}\frac{dx}{dt} + \frac{\partial y}{\partial t} \qquad if\frac{dx}{dt} = y + \lambda g/\lambda \tag{15}$$

Then,written as:

$$H = \frac{dV}{dt}\frac{dx}{dt} + \lambda\frac{dy}{dt} - g(S_o - S_f) \tag{16}$$

equating the expressions for $\frac{dx}{dt}$ from above equation which yields,

$$V + \lambda y = V + g/\lambda; \qquad andsolvingfor\lambda, or; \lambda = \pm\sqrt{\frac{g}{y}} \tag{17}$$

The two real, distinct roots for λ can be used to transform equation of continuity and momentum equation in to a pair of ordinary differential equations subject to the restrictions regarding $\frac{dx}{dt}$ specified in the forgoing equations,

$$dV + dy\sqrt{\frac{g}{y}} + g(S_o - S_f)dt = 0 \tag{18}$$

$$dx = (V + \sqrt{gy})dt \tag{19}$$

$$dV - dy\sqrt{\frac{g}{y}} + g(S_o - S_f)dt = 0 \tag{20}$$

$$dx = (V - \sqrt{gy})dt \tag{21}$$

At this point it is noted that the curve defined by above equation is term as positive characteristic (C^+) which is the curve in the LP in the following figure. The curve define by the above equation is termed as negative characteristic (C^-) which is curve RP as shown in figure 3.

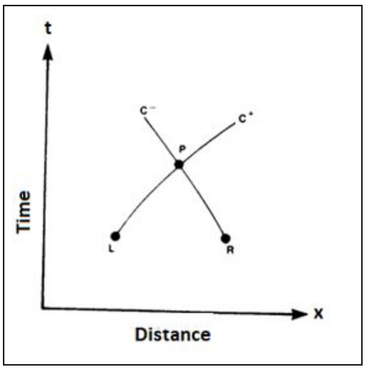

Figure 3: Definition of Characteristic curves(Richard H French 1987, p. 559))

The solution of above equations must be accomplished by numerical methods. Using the first order, explicit finite difference technique, the above equations become,

$$V_P - V_L + \sqrt{\frac{g}{y_L}}(y_P - y_L) + (t_P - t_L)(S_{fL} - S_o) = 0; \qquad x_P - x_L = (V_L + \sqrt{gy_L}(t_P - t_L) \tag{22}$$

$$V_P - V_R + \sqrt{\frac{g}{y_R}}(y_P - y_R) + (t_P - t_R)(S_{fR} - S_o) = 0; \qquad x_P - x_R = (V_L + \sqrt{gy_L}(t_P - t_R) \tag{23}$$

Unlike the Explicit finite difference technique with a fixed time step which was previously discussed, in this technique the length of the time step is determined by subtracting the equation of (C^-) from the equation of positive (C^+) hence for the time step

$$t_P = \frac{x_L - x_R + t_R(V_R - \sqrt{gy_R}) - t_L(V_L - \sqrt{gy_L})}{V_R - V_L - \sqrt{gy_L} - \sqrt{gy_R}} \quad (24)$$

The distance step in the longitudinal direction is determined from equation

$$x_P = x_L + (V_L + \sqrt{gy_L})(t_P - t_L) \quad (25)$$

The depth of flow at node P is determined by subtracting equation of negative characteristic C^- from positive characteristic C^+ equation and solving for y_P

$$y_P = \frac{V_L - V_R + y_L\sqrt{\frac{g}{y_L}} + y_R\sqrt{\frac{g}{y_R}}}{\sqrt{\frac{g}{y_L}} + \sqrt{\frac{g}{y_R}}} + \frac{(t_P - t_R)(S_{fR} - S_o) - (t_P - t_L)(S_{fL} - S_o)}{\sqrt{\frac{g}{y_L}} + \sqrt{\frac{g}{y_R}}} \quad (26)$$

From the above positive curve C^+ equation,

$$V_P = V_L - \sqrt{\frac{g}{y_L}}(y_P - y_L) - (t_P - t_L)(S_{fL} - S_o) = 0 \quad (27)$$

The primary difficulty with explicit finite difference techniques is the problem of numerically unstable solutions. unstable solution usually results if δt is large relative to δx. The concurrent stability condition requires, $\delta t \leq \frac{\delta x}{V+c}$
where c= wave celerity. However, it has been found that for the type of the explicit difference schemes discussed in this section δt given by equation should be approximately 20 percent value of the above equation. Viessman et al. (1972) noted that more stable solution can be obtained if a diffusing difference approximation is used; i.e., in the forgoing solution, substitute,

$$(\frac{\partial V}{\partial t})_M = \frac{V_P - 0.5(V_L + V_R)}{\delta t}; \quad (\frac{\partial y}{\partial t})_M = \frac{y_P - 0.5(y_L + y_R)}{\delta t}; \quad S_f = \frac{S_{fL} + S_{fR}}{2} \quad (28)$$

This formulation of derivatives allows the size of the time increment to be significantly increased; but concurrent stability condition as mention in the above stated equation must still be met. in addition, diffusing scheme also imposes a friction criterion or $\delta t \leq \frac{\phi^2 y_P^{\frac{4}{3}}}{gn^2|V|}$ For the stability, the lesser value of δt between this above equation is used.

2.1.5 Explicit Method

The most direct method of simultaneously solving the above continuity and momentum equation is an explicit finite difference scheme with a fix time step. Given the schematic definition of a rectangular finite difference scheme in the figure shown below, the derivatives appearing in the governing equation can be approximated by:

$$(\frac{\partial V}{\partial x})_M = \frac{(V_R - V_L)}{2\delta x}; (\frac{\partial V}{\partial t})_P = \frac{(V_P - V_M)}{\delta t}; \quad (\frac{\partial y}{\partial x})_M = \frac{(y_R - y_L)}{2\delta x}; (\frac{\partial y}{\partial t})_P = \frac{(y_P - y_M)}{\delta t} \quad (29)$$

where x= longitudinal distance between nodes, t=distance in time between nodes, and the subscript designates the nodes at which the variable is being evaluated.

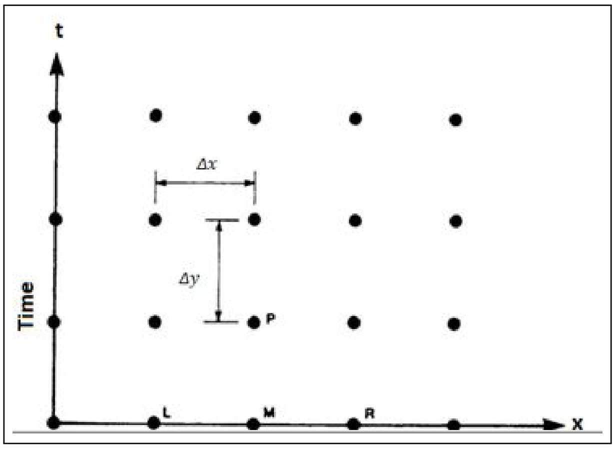

Figure 4: Finite Difference network(Richarad H French 1987, p. 555)

Substitution of the above expressions for the derivatives in continuity equation yields,

$$(\frac{y_P - y_M}{\delta t})_M + y_M(\frac{(V_R - V_L)}{2\delta x}) + V_M(\frac{y_R - y_L}{2\delta x}) = 0 \quad (30)$$

and solving for y_P

$$y_P = y_M + \frac{\delta t}{2\delta x}[V_M(y_L - y_R) + y_M(V_L - V_R)] \quad (31)$$

Substituting the finite difference approximations for the derivatives of conservation of momentum equation which yields,

$$\left(\frac{V_P - V_M}{\delta t}\right)_M + V_M\left(\frac{(V_R - V_L)}{2\delta x}\right) + g\left(\frac{y_R - y_L}{2\delta x}\right) - g(S_o - S_f) = 0 \tag{32}$$

In the computation of unsteady flow, it is usually assumed that the friction slope S_f can be estimated either the Mannings or Chezy resistance equations. Use of Mannings equation yields, $S_f = \frac{V|V|n^2}{\phi^2 R^{\frac{4}{3}}}$
where R=Hydraulic radius; n= Mannings resistance coefficient; ϕ= Coefficient depending on system used (1.49 for English system and 1 for SI system) It is noted that the absolute value of the velocity of flow is used in combination with a signed value of the velocity to ensure that the frictional resistance always opposes the motion. If the rectangular channel is wide, then R$\approx$ y, and the frictional slope can be represented in finite difference form as $S_f = \frac{V|V_P|n^2}{\phi^2 y_P^{\frac{4}{3}}}$
;Substituting the above equation to finite difference momentum equation which yields

$$\left(\frac{V_P - V_M}{\delta t}\right)_M + V_M\left(\frac{(V_R - V_L)}{2\delta x}\right) + g\left(\frac{y_R - y_L}{2\delta x}\right) = g\left(S_o - \frac{V|V_P|n^2}{\phi^2 y_P^{\frac{4}{3}}}\right. \tag{33}$$

For notational convenience define;
$f = \frac{\phi^2 y_P^{\frac{4}{3}}}{g\delta t n^2}$
Simplifying and re-arranging the momentum equation, we get the following,

$$V_P|V_P| + fV_P + f\left[\frac{V_M\delta t}{2\delta x}(V_R - V_L) + \frac{g\delta t}{2\delta x}(y_R - y_L) - g\delta t S_o\right] = 0 \tag{34}$$

The above equation is a quadratic equation in V_P and can be solved by the quadratic formula or

$$V_P = \frac{-f + (f^2 - 4\beta)^{0.5}}{2} \tag{35}$$

where $\beta = f[\frac{V_M\delta t}{2\delta x}(V_R - V_L) + \frac{g\delta t}{2\delta x}(y_R - y_L) - g\delta t S_o]$ with y_P is given previous equation; thus the solution of an unsteady flow problem by an explicit difference technique with a fixed time step in a wide, rectangular channel proceeds by determining yp an advanced time step from the previous equation then by the use of this value of new y_P, we can get the new value of y_P .

2.1.6 Implicit four-point difference scheme-channels of arbitrary shape

As indicated in the forgoing sections, the primary difficulty in analyzing gradually varied, unsteady flow is identifying a numerical scheme which is accurate, fast and efficient. The explicit finite difference solution of St. Venant equations discussed in the previous section has the advantage of simplicity, but

$$V\frac{\partial A}{\partial x} + A\frac{\partial V}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \tag{36}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} = g(S_o - S_f) - \frac{qV}{A} \tag{37}$$

The above shown continuity and momentum are nonlinear, partial differential equation of the hyperbolic type. The numerical solution of this set of governing equation is accomplished in two steps. First, the governing equations are replaced by a set of algebraic finite difference equations. second, method of solving the difference equations must be found. The numerical solution of the above-mentioned continuity and momentum equation will be expressed in the (x,t) plane at a discrete number of points arrange to form a rectangular grid as shown in figure 5, with regards to this network, the following points are noted:

1. The nodes of the network occur at intersections of straight lines drawn parallel to x and t axes.
2. The lines parallel to the t axis represent location along the channel while those drawn parallel to the x axis represent times.
3. The locations lines are drawn with a spacing δx while the time lines are drawn with a spacing δt .Although for convenience in this development δx and δt are assumed constant, in practices δx and δt may vary in space and time as required.
4. The t axis may be considered the upstream channel boundary location, and the last line drawn parallel to t axis, termed the Nth line, may be used to represent the downstream channel boundary.
5. Each node in the network is identified by two indices: A subscript designates the x position of the node while the superscript designates the location of the node in time.
6. At time t=t_0, it is assumed that the values of the velocity and depth of flow are available at all locations.

7. For convenience, it is assumed that at the upstream channel boundary either a stage or discharge hydrograph is available. The downstream channel boundary is assumed to be a control section so that either a stage discharge or velocity-depth relationship is available.

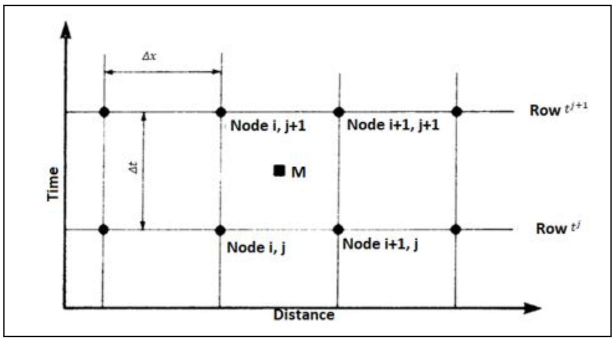

Figure 5: Grid definition for implicit difference scheme(Richarad H French 1987, p. 562)

For the development of the necessary set of algebraic finite difference equations, assumed that all variables are defined on the rowt^j and that is desired to advance the solution to the row t^{j+1} where $t^{j+1} = t^j + \delta t$. The equations governing unsteady flow are applied, in a finite difference form, to a point M entirely within a four-point grid as shown in the above figure. At the point M, the partial derivatives of an arbitrary function are represented as,

$$f(M) = \frac{1}{4}(f_i{}^j + f_i^{j+1} + f_i^{j+1} + f_i + 1^{j+1}) \quad (38)$$

$$\frac{\partial f(M)}{\partial x} = \frac{1}{2\delta x}[f_{i+1}{}^j - f_i^{j+1} - f_i^{j+1} + f_i + 1^{j+1} \quad (39)$$

$$\frac{\partial f(M)}{\partial t} = \frac{1}{2\delta t}[f_{i+1}{}^j - f_i^{j+1} - f_i^{j+1} + f_i + 1^{j+1} \quad (40)$$

Since the Eqs. 38 to 40 f is an arbitrary function, these equations define the variables which appear in the equations governing gradually varied unsteady flow. When y, V, $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial t}$ in equation simplified their analogies defined in equation 38 to 40 and two finite difference equations are obtained, the results are:

$$\begin{aligned} &\frac{1}{2\delta t}[-y_{i+1}{}^j - y_i{}^j + y_i{}^{j+1} + y_{i+1}^{j+1}] + \\ &V_{i+\frac{1}{2}}{}^{j+\frac{1}{2}}\frac{1}{2\delta x}[y_{i+1}{}^j - y_i{}^{j+1} - y_i{}^j + y_{i+1}^{j+1}] \\ &+(\frac{A}{T})_{i+\frac{1}{2}}^{j+\frac{1}{2}}\frac{1}{2\delta x}[V_{i+1}{}^j - V_i^{j+1} - V_i^j + V_{i+1}^{j+1}] \\ &-(\frac{q}{T})_{i+\frac{1}{2}}^{j+\frac{1}{2}} = 0 \end{aligned} \quad (41)$$

and

$$\begin{aligned} &\frac{g}{2\delta x}[y_{i+1}{}^j - y_i{}^{j+1} - y_i{}^j + y_{i+1}^{j+1}] + \\ &\frac{1}{2\delta t}[-V_{i+1}{}^j - V_i^{j+1} + V_i^j + V_{i+1}^{j+1}] \\ &+(V_{i+\frac{1}{2}}{}^{j+\frac{1}{2}})\frac{1}{2\delta x}[V_{i+1}{}^j - V_i^j - V_i^{j+1} + V_{i+1}^{j+1}] + \\ &\frac{g}{4}((S_{fi}{}^j + S_{fi+1}^{j+1}) + (S_{fi}^{j+1} + S_{fi+1}{}^{j+1})) + \\ &\frac{g}{\delta x}(z_i{}^j - z_{i+1}{}^j) + q(\frac{V}{A})_{i+\frac{1}{2}}^{j+\frac{1}{2}} = 0 \end{aligned} \quad (42)$$

where,

$$V_{i+\frac{1}{2}}^{j+\frac{1}{2}} = \frac{1}{4}(V_{i+1}{}^j + V_i^{j+1} + V_i^j + V_{i+1}^{j+1}) \quad (43)$$

$$(\frac{V}{A})_{i+\frac{1}{2}}^{j+\frac{1}{2}} = \frac{1}{4}(\frac{V_i^j}{A_i{}^j} + \frac{V_{i+1}{}^j}{A_{i+1}{}^j} + \frac{V_i^{j+1}}{A_i{}^{j+1}} + \frac{V_{i+1}{}^{j+1}}{A_{i+1}{}^{j+1}}) \quad (44)$$

$$(\frac{q}{T})_{i+\frac{1}{2}}^{j+\frac{1}{2}} = \frac{1}{4}g(\frac{1}{T_i^j} + \frac{1}{T_{i+1}^j} + \frac{1}{T_i^{j+1}} + \frac{1}{T_{i+1}{}^{j+1}}) \quad (45)$$

$$(\frac{A}{T})_{i+\frac{1}{2}}^{j+\frac{1}{2}} = \frac{1}{4}(\frac{A_i{}^j}{T_i^j} + \frac{A_{i+1}{}^j}{T_{i+1}{}^j} + \frac{A_i{}^{j+1}}{T_i^{j+1}} + \frac{A_{i+1}{}^{j+1}}{T_{i+1}{}^{j+1}}) \quad (46)$$

$$S_{fi}^j = \frac{V_i^j|V_i^j|n_i^{j^2}}{\phi^2 R_i^{j^{\frac{4}{3}}}} or, S_{fi}^j = \frac{V_i^j|V_i^j|n_i^{j^2} P_i^{j^{\frac{4}{3}}}}{\phi^2 A_i^{j^{\frac{4}{3}}}} \quad (47)$$

From these above equation by the numerical solution the flood wave propagation can tbe computed.

2.1.7 Flood wave computation by diffusion analogy (most simplified method)

An approximate hydraulic approach to the problem of flood computation in natural channels has been developed by using the classical statistical theory of flow diffusion. According to this theory, a differential equation could be written for the diffusion of an unsteady flow of particles:

$$\frac{\partial N}{\partial t} = G\frac{\partial^2 N}{\partial x^2} \quad (48)$$

where N is the number of particles, t is the time, x is the distance, and G is a coefficient known as diffusivity. When the particles are flowing in a direction along the axis, this equation gives the particle distribution in a direction along the x axis, this equation gives the particle distribution in the direction of flow as a function of time and position. this theory is commonly applied to problems of heat transfer; the above equation represents Fourier's general law of heat conduction, in which N designates temperatures and G is known as thermal diffusivity.

In natural streams, the disturbances of flow caused by local channel irregularities have definite magnitude at any time and position. They are mixed, dissipated, and diffused as the flow moves along the channel. In applying the theory of flow diffusion to the flow of water, it may be assumed that the diffusion of the disturbances is analogous to the diffusion of the particles. if the overall effect of the disturbances on flow is represented by variation in the flow depth y the above equation could be written as,

$$\frac{\partial y}{\partial t} = G\frac{\partial^2 y}{\partial x^2} \tag{49}$$

In natural streams the local irregularities provide irregular storage, and the above equations reflect the rate of the change in channel storages due to irregularities. Including this item, the continuity of flow in prismatic channels, the continuity equation for flow in natural channels could be written as when there is no lateral flow:

$$\frac{\partial N}{\partial t} + 0 = G\frac{\partial^2 N}{\partial x^2} \tag{50}$$

For determination of the flood wave propagation the following equation governs:

$$y = y_o \exp(-k^2 G_t)\sin[k(x - c_k t)] \tag{51}$$

Where: k is the conveyance of the channel $\frac{1}{n}R^{\frac{2}{3}}A$ from Manning equation G diffusion coefficient is equal to approximately, $\frac{c_k y}{3S_f}$

$c_k = \frac{3}{2}C\sqrt{(y(S_f - \frac{\partial y}{\partial x}))}$ Where C is the Chezy coefficient

2.1.8 Rapidly Varied Unsteady Flow

If there is abrupt change in curvature of flow or a sudden change in the depth of flow then it is called a rapidly varied flow in the open channel. This effect may be produced when a sudden increase in sluice gate opening at the channel entrance, as shown in fig below.

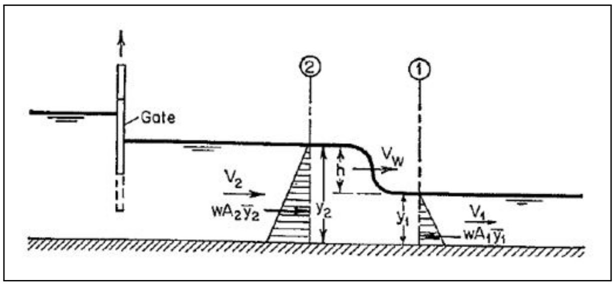

Figure 6: Rapidly varied uniformly progressive flow(Chow, V.T, 1959, p. 554)

The velocity of the mass of water between the gate and the wave front are increase from V_1 to V_2 and the momentum is increased accordingly. By Newton's second low of motion, the unbalance force required to change the momentum per unit time is the product of mass and the change in velocity per unit time, or

$$F = \frac{1}{g}(V_\omega - V_2)A_2\omega(V_2 - V_1) \tag{52}$$

where ω is the unit weight of water. the unbalanced force is equal to the different between the hydrostatic pressure on the area A_2 and A_1 at section 2 and 1 respectively; that is,

$$F = \omega A_2\bar{y}_2 - \omega A_1\bar{y}_1 \tag{53}$$

where $\bar{y}_2$ and $\bar{y}_1$ are the centroidal depths of the areas. Equating the above values of F and simplifying,

$$(V_\omega - V_2)A_2\omega(V_2 - V_1) = (\bar{y}_2 - \frac{A_1}{A_2}\bar{y}_1)g \tag{54}$$

Solving $V_\omega = \frac{V_1A_1 - V_2A_2}{A_1 - A_2}$ for

$$V_2 = \frac{V_1A_1 + V_\omega A_2 - V_\omega A_1}{A_2} \tag{55}$$

substituting the above expression for V_2 in Eq (3) and reducing;

$$(V_\omega - V_1)^2 = \frac{(A_2\bar{y}_2 - A_1\bar{y}_1)g}{A_1(1 - \frac{A_1}{A_2})} \tag{56}$$

$$(V_\omega - V_1) = \sqrt{\frac{(A_2\bar{y}_2 - A_1\bar{y}_1)g}{A_1(1 - \frac{A_1}{A_2})}} \tag{57}$$

$$V_\omega = \sqrt{\frac{(A_2\bar{y}_2 - A_1\bar{y}_1)g}{A_1(1 - \frac{A_1}{A_2})}} + V_1 \tag{58}$$

This is a general equation expressing the absolute wave velocity of the flow as shown, Mathematically speaking, the sign in front of the square- root term in the above equations may also be negative. However, since the two wave is moving downstream in the direction of the initial flow, its velocity must be greater than the velocity of the initial flow. In other word, $V_\omega - V_1$ should be positive. Therefore, only the plus sign is considered practical.

If the initial velocity $v_1 = 0$, that is, if the wave travels in the still water, then the square root term in the above equations is equal to the absolute velocity of the wave. In any case, this term, being equal to V. $V_\omega - V_1$, represents the velocity of wave with respect to the velocity of initial flow. It is, therefore, the celerity; that is,

$$c = \sqrt{\frac{(A_2\bar{y}_2 - A_1\bar{y}_1)g}{A_1(1 - \frac{A_1}{A_2})}} \tag{59}$$

For rectangular channels,$\bar{y}_1 = y_1/2, \bar{y}_2 = y_2/2, A_1 = by_1, A_2 = by_2$. Thus, the equation becomes

$$c = \sqrt{\frac{gy_2}{2y_1}(y_1 + y_2)} \tag{60}$$

It can be shown that, for waves of moderate height, Eq. (60) becomes C= $\sqrt{gy(1+3h/4y)}$. For very small waves, Eq. (60) becomes$\sqrt{gy}$. In this cases, Eq. (58) may be written

$$V_\omega = c + V_1 \tag{61}$$

Theoretically, there are four types of rapidly varied unsteady flow: type A, having an advancing wave front moving downstream; type B, having an advancing wave front moving upstream; type C, having a retreating wave front moving downstream; and type D, having a retreating wave front moving upstream. Type A has just been described. For type B, it can be shown that

$$V_\omega = c - V_1 \tag{62}$$

It can also be shown that Eq. (61) applies to type C and Eq. (62) to type D. Fig.2. Four types of rapidly varied uniformly progressive flow. (Top) unsteady flows;(bottom) the corresponding flows that appears steady to an observer following the wave front.

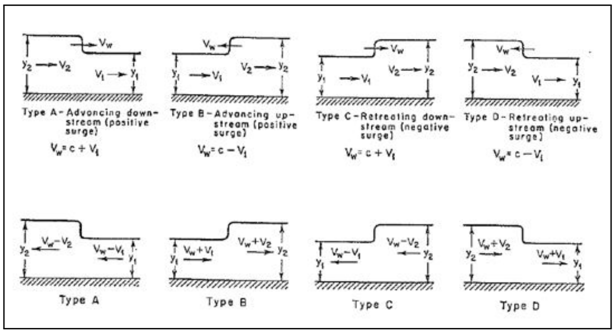

Figure 7: Four types of rapidly varied uniformly progressive flow. (Top) Unsteady floes; (bottom) the corresponding flows that appear steady to an observer following the wavefront (Chow, V.T, 1959, p. 556)

Possible for those velocities V_1 and V_2 to be opposite in direction. In that case, velocity is considered negative if its direction is opposite as shown above. The absolute velocity V_ω of the wave may also be expressed by $V_\omega = \frac{(V_1A_1 - V_2A_2)}{(A_1 - A_2)}$. Since this equation is derived on the basis of the principle of continuity, it applies to both gradually and rapidly varied monoclinal rising waves.

It should be noted that the wave profile is stable for an advising front and unstable for retreating front. The wave front can be assumed to be made up of a large number of very small waves placed one on top of the other. Since the wave on top has greater depth than the one below it, it has greater velocity, according to $\sqrt{gy}$, and moves faster. In case of the advancing front, the top waves will therefore, overtake the bottom waves in the forward direction. The tendency is for the waves to combine and to form eventually a single large wave front which is steep and stable. In case of the retreating front the top waves will retreat faster than the bottom ones. The result is that the wave front becomes sloping and eventually flattens out.

Positive Surges:
Positive surges have an advancing front with the profile of a moving hydraulic jump. When the height of the surge is small, the surge appears undular like an undular jump. When the height is increasing, the undulation will eventually disappear and the surge will have a sharp and steep front. Consider the positive surge type A (fig.7); the absolute wave velocity can be expressed by, for rectangular channel:

$$V_\omega = \frac{(V_1y_1 - V_2y_2)}{(y_1 - y_2)} \tag{63}$$

Eliminating from eqs.(63) and (64) simplifying,

$$(V_1 - V_2)^2 = (y_1 - y_2)^2 \frac{(y_1 - y_2)g}{2y_1y_2} \tag{64}$$

This equation represents the relationship among the initial and final velocities and the depth of the surge. Similarly, it can be shown that eq. (64) applies also to surge type B (fig. 7). Note that, if eq. eq. (64) used to determine y_1 ory_2, it must be solved by trial and error. Multiplying eq. (64) by the square of eq. (60) and simplifying, it can be shown that

$$(V_1 - V_2) = \pm \frac{h}{c}(\frac{y_1 + y_2}{2y_1})g \tag{65}$$

Where $h = y_2 - y_1$, or the height of the surge, and where c is the celerity. On the right-hand side of the equation, the positive sign applies to type B surges and negative sign to type A surges. When the height of surges is small compared with the depth of flow, $y_1 \approx y_2$. Thus, eq. (65) may be written

$$(V_1 - V_2) = \pm \frac{hg}{c} \tag{66}$$

In order to avoid the confusion of the sign convention, the use of the eq. (65) and (66) may be simplified by ignoring the negative sign on their right sides and remembering that and are always assumed to be positive quantities. Accordingly, $V_1 - V_2$ should always be positive. If $V_2 < V_1$ then $V_1 - V_2$ must be replaced by $V_2 - V_1$. If V_1 and V_2 are in oppositive directions, then their sum $V_1 + V_2$ must be used instead of their difference. Several typical cases of the analysis of positive surges are given as follows:

Negative surges:
Negative surges are not stable in form, because the upper portions of the wave travel faster than the lower portions. If the initial profile to the surge is assumed to have a steep front, it will soon flatten out as the surge moves through the channel. If the height of the surge is moderate or small compared with the depth of flow, the equations derived for a positive surge can be applies to determine approximately the propagation of the negative surge. If the height of the surge is relatively large, a more elaborate analysis is necessary as follows:

Type D surge (fig.7) of relatively large height, retreating in an upstream direction. The surge is caused by the sudden lifting of a sluice gate. The wave velocity of the surge actually varies from point to point. For example, V_ω is the wave velocity at a point on the surface of the wave where the depth is y and the velocity of flow through the section is V. During a time, interval, dt the change in y is dy. The value of dy is positive for an increases of y and the negative for a decrease of y.by the momentum principle, the corresponding change in hydrostatic pressure should be equal to the force required to the change the momentum of the vertical element between y and y+dy. Considering a unit width of the channel and assuming $\beta_1 = \beta_2 = 1$,

$$\frac{\omega}{2}y^2 - \frac{\omega}{2}(y + dy)^2 = \frac{\omega}{g}(y + \frac{1}{2}dy)(V + V_\omega)dV \tag{67}$$

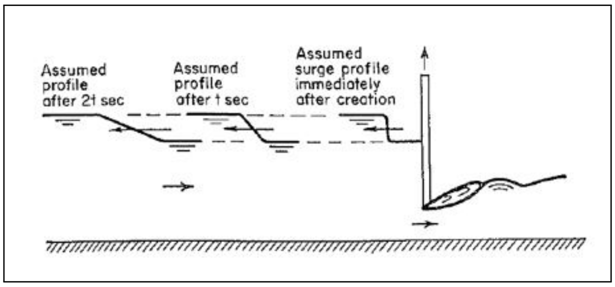

Figure 8: Propagation of a negative surge due to sudden lift of a sluice gate

Simplifying the above equation and neglecting the differential terms of higher order,

$$dy = \frac{-V + V_\omega}{g}dV \tag{68}$$

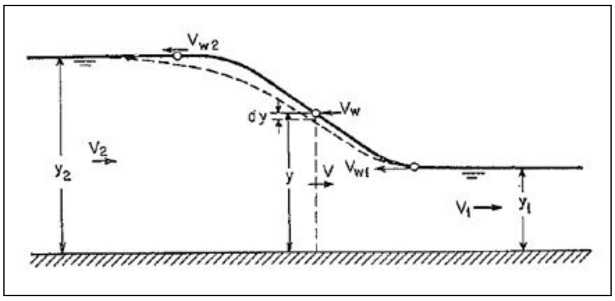

Figure 9: Analysis of a negative surge

As described previously, the whole wavefront can be assumed to be made up of a large number of very small wave placed one on top of the other. The velocity of small wave at the point under consideration may be expressed as in above eq.as:

$$V_\omega = \sqrt{gy} - V \tag{69}$$

Similarly, the velocity at the wave crest is:

$$V_\omega 2 = \sqrt{gy_2} - V_2 \tag{70}$$

And, at the wave trough:

$$V_\omega 1 = \sqrt{gy_1} - V_1 \tag{71}$$

When the surge is not too high, a straight-line relation between $V_\omega 1$ and $V_\omega 2$ may be assumed. Thus, the mean velocity of the wave may be considered to be

$$V_m = \frac{V_\omega 1 + V_\omega 2}{2} \tag{72}$$

Now, eliminating V_ω between Eqs.(68) and (69),

$$\frac{dy}{\sqrt{y}} = -\frac{dV}{\sqrt{g}} \tag{73}$$

Integrating this equation from y_2 to y and from V_2 to V, and solving for V,

$$V = V_2 + 2\sqrt{gy_2} - 2\sqrt{gy} \tag{74}$$

From Eq. (69),

$$V_\omega = 3\sqrt{gy} - 2\sqrt{gy_2} - V_2 \tag{75}$$

Thus, the wave velocity at the trough of the wave is

$$V_\omega 1 = 3\sqrt{gy_1} - 2\sqrt{gy_2} - V_2 \tag{76}$$

Let t be the time elapsed since the surge was created, or in this case, Since the sluice gate was opened. At t=0, the wavelength $\lambda = 0$. After t sec, the wave length is equal to

$$\lambda = (V_\omega 1 - V_\omega 2)t \tag{77}$$

The above analysis can be applied similarly to a negative surge of type C.

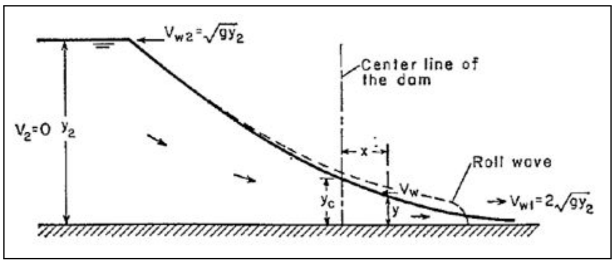

Figure 10: Wave profile due to dam failure

Establishment of the Dam Break Profile:
The wave profile, resulting from the failure of a dam is in the form of

$$x = 2t\sqrt{gy_2} - 3t\sqrt{gy} \tag{78}$$

where x is the distance measured from the dam site, y is the depth of the wave profile, y_2 is the depth of the impounding water, and t is the time after the dam broke. Since the impounding water has zero velocity, or V_2 = 0, the wave velocity is $V_\omega = 3\sqrt{gy} - 2\sqrt{gy_2}$. Since V_ω is in the negative direction of x, x = - (V_ω)t , which gives Eq. (78). Equation (78) represents a parabola with vertical axis and vertex on the channel bottom, as shown in Fig. 10. At the site of the dam, x = 0 and the depth $y_c = \frac{4y_2}{9}$. Owing to the channel friction, the actual profile takes the form indicated by the dashed line. This profile has a rounded front at the downstream end, forming a bore. At the upstream end, the theoretical profile thus developed has been checked satisfactorily with experiments by Schoklitsch.

2.2 Implementation of Shallow Water Equations using open-source solver

2.2.1 Flood modelling using HEC-RAS

HEC-RAS (Hydrological Engineering Centre's River Analysis System) is an open-source mathematical solver based upon the finite volume method for solving full dynamic Shallow Water Equations, which has been widely used for water related issues in different field of River Engineering, Hydropower with the implementation of Hydraulic Structure in the model its self. A hypothetical, five different flood events have been tested in the 10 km long rectangular channel to evaluate the time of attenuation of the peak from upstream to the downstream. The time difference evaluation of the peaks indicates the during of time to reach the flood events from upstream to downstream, which can be implemented as a flood forecasting for different flood events to aware the people living nearby to evacuate their places.

2.2.2 Model Setup

The HECRAS model has been setup for the geometry as shown in figure 11 and parameters shown in table 1.

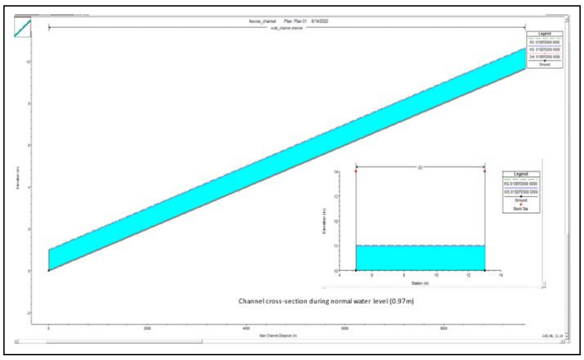

Figure 11: Longitudinal Channel section during normal water level

The upstream unsteady boundary condition was tested with different flood events hydrograph in the provided geometry to visualize the flood wave propagation in the channel during the flood events.

Table 1: Model Set up Data and Boundary Condition

Unsteady flow boundary condition	
Initial condition	Discharge = 7 m^3/s
Up-stream station	Flow hydrograph of different even for different case as per the hydrograph provided hydrograph (5 hrs)
Downstream	Normal depth (slope (s)=0.001)
Un-steady simulation boundary condition	
Simulation duration	5 hrs
Computational interval the Courant number <1 with provided interval)	10 sec (as per
Hydrograph output interval	1 minute
Detail output interval	1 minute

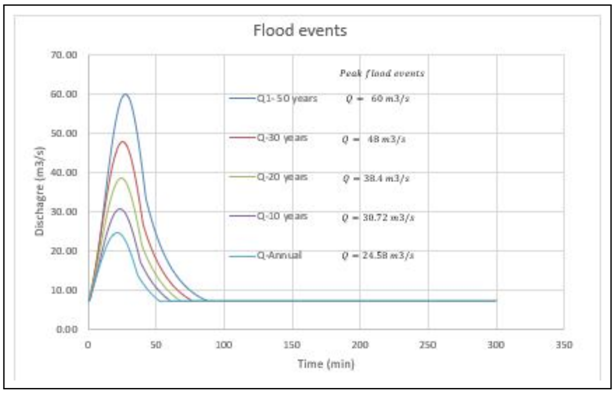

Figure 12: The hydrograph showing different flood events (50, 30, 20, 10 and Annual events) which were tested in HECRAS.

3. Result and Discussion

The water level -stage(m), discharge (m^3/s) verses duration of the time (min) were plotted after post processing of the data from the HECRAS after the simulation as shown in the graphs.

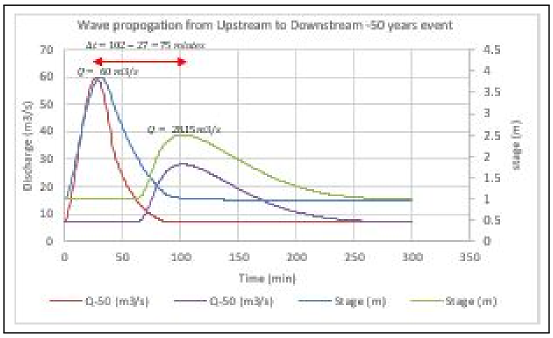

Figure 13: Flood Level Stage discharge verses time duration

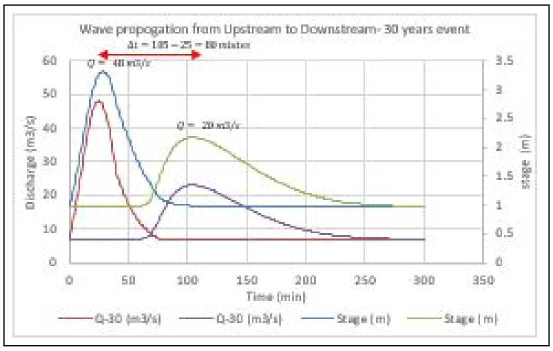

Figure 14: Flood Level Stage discharge verses time duration

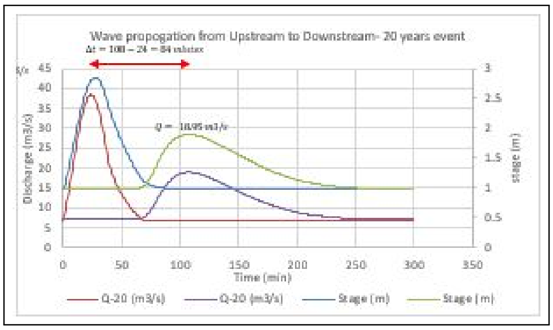

Figure 15: Flood Level Stage discharge verses time duration

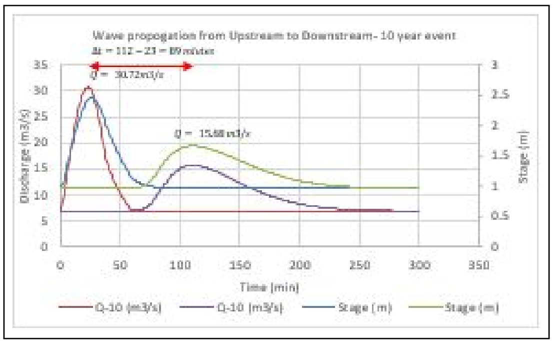

Figure 16: Flood Level Stage discharge verses time duration

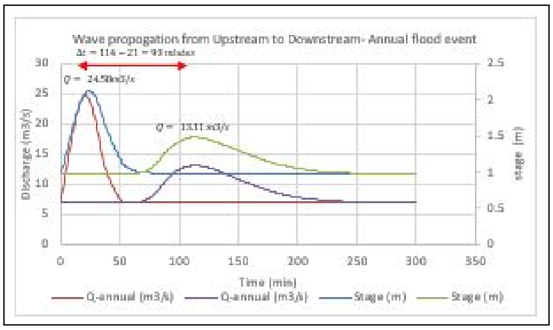

Figure 17: Flood Level Stage discharge verses time duration

The plotted graphs indicate the attenuation of the flood wave peaks for different hydrograph with respect to time and space, the data was extracted from the upstream and the downstream satiations of the HECRAS. The graph represents the water level upstream and downstream which has been completed matched after the flood wave propagates until it reaches to the normal water level of 0.97m in the channel with the design flow of $7m^3/s$. The difference in duration of the peak wave has been evaluated to estimate the flood wave time travel for a particular station from upstream to downstream. Thus, the above results clearly indicate the high peaks flood events reaches the downstream station more quickly than the flood events with the lower peaks, which helps to forecast the time travel of the different flood event to reach the downstream station. Furthermore, we can also evaluate the total distance of attenuation of the peaks until it diffuses and reaches to the design discharge and normal water level in the channel, this helps to understand and evaluate the impact length of the channel reach for different flood events.

4. Conclusion and Recommendations

From a hypothetical illustration; the flood wave has been successfully produced and propagated downstream in the channel for different flood events in the model. The diffusive peaks were studied in different hypothetical stations of the channel as it was propagating downstream to estimate a travel time. Implication of this type of models in the natural rivers cross-section helps to evaluate flood area through a flood plain mapping using 2D Shallow Water Equation. Further studies, like calibration of the model with manning's roughness and validation with respect to the water level through observed data considering natural river channel cross-section from the filed measurement is essential in order to be established a well tested result. The results can also be tested with different discretization methods for solving the partial differential Shallow Water Equations for verifying the model's results.

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