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Use of Scutoid Inspired Geometry in Curved Surface of Revolution

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Abstract

Scutoid is a shape that is reminiscent to both a frustum and a prismatoid. These shapes are found in naturally occuring epithelial cells in animal bodies and are often exposed to variety of forces that mimic the loads experienced by a structure in its lifetime. This paper focuses on using bio-inspired scutoid shapes to use millenia of evolution to our advantage by creating geometry to place straight frame elements in a curved surface. Curved surfaces of revolution were generated using 3 points to form an arc . By varying the position of middle point, different curves of linearly varying curvature were generated. A number of random seed points were placed in the curve and was assymetrically scaled and transformed to facilitate the formation of scutoids by the use of voronoi diagrams. The models without scutoids contain fructums as the basis for position of straight frame elements. Three different load cases namely Self weight, Ramp time history load to simulate impact load and 100kN load distributed on all outside joints to simulate roof load were chosen for the analysis and was applied in linear range to both the scutoid and non-scutoid models. The results show varying degree of efficiency in deflection with minimal increase in material usage and models with smaller absolute values curvatures were shown to be effective for the chosen 3 load cases.

Keywords

Scutoid, Straight frame in curved surface, Bio-mimicry, Shells of revolution

1. Introduction

Scutoid is a shape that is reminiscent to both a frustum and a prismatoid. They all are a geometric solid bounded by two parallel planes on a polygon boundary. Similar to a frustum, the size of polygon formed between parallel surfaces of a scutoid are not constant. And similar to a prismatoid, the number of vertices in top and bottom face are different. To facilitate these two conditions, there lies atleast an additional vertex in between the top and bottom faces. A scutoid can be identified by the number of vertices in those faces. Given in the figure 1 is a pair of ordinary 5,6 scutoid that is commonly found in epithelial cells in a living being.

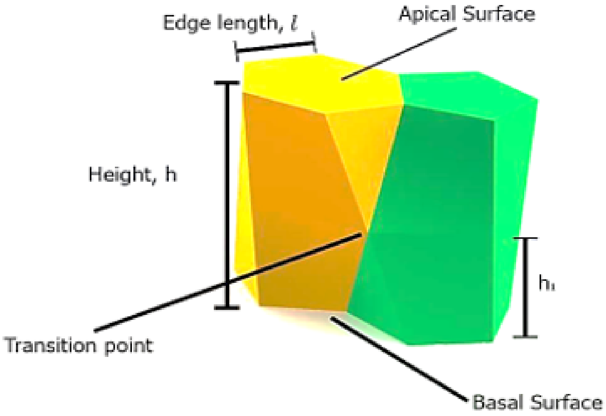

Figure 1: An ordinary 5,6 scutoid [1]

2. Objective

The objective of this paper is to use straight frame elements to model curved surface and compare stresses and deflection in curved surfaces of revolution of varying radius of curvature with and without scutoids and compare their performance under floolwing loads:

1. Self-weight
2. Ramp time history load
3. Gravity load on joints

3. History

Scutoidal shape emerged when modelling the epithelial cells, cells that line up the outer surface of organs and blood vessels, using a Voronoi diagram. Voronoi diagram is the method of dividing a plane with n points into regions such that every point under that region has exactly one generating point and is closer to that generating point than to any other[2]. Researchers found out that the Voronoi cells on apical and basal surfaces of curved model surface had different neighbors on each surface, which didn't align with the historical assumption that the cells were shaped like a frustum [3]. To facilitate this exchange of neighbors, they hypothesized a presence of another vertex between those planar faces, and named the resulting shape "scutoid". Their model predicted the presence of scutoid in curved epithelia, which they found shortly afterwards.

4. Voronoi diagram

A voronoi diagram, named after Georgy Voronoy, is a method to divide a plane into areas near to a specified collection of objects. The steps to draw voronoi diagram is[2]:

1. Distribute seed points in a plane
2. Join adjacent seed points with a straight line.
3. Draw their perpendicular bisectors.
4. Remove other unnecessary lines to result in a convex polygon.

This can be visualised by imagining a bee in it's beehive trying to deposit the honey. The neighboring bees compete for the space, and thus usually divide it half/half (thus the perpendicular bisector). This is also the case when our biological cells start to grow outward and compete for space with adjacent cells.

5. Formation of scutoid

Unlike a beehive that is planar, animal body is anything but planar. The induced curvature difference in top and bottom face of a curved surface will ensure formation of different voronoi diagram, and thus different polygon in top and bottom faces. Whenever there is assymetric scaling between 2 axes of a surface, then the resulting voronoi diagram will have different polygons, giving rise to the scutoid shape, as shown in figure 2. The figure 2 shows the transition of polygon shapes when y axis is unchanged, but x axis is stretched. First figure is a 3 units by 5 units rectangle with 5 random points selected insite it. Second, Third and fourth figure shows the voronoi cells whsn x is increased by 2 units tach time.

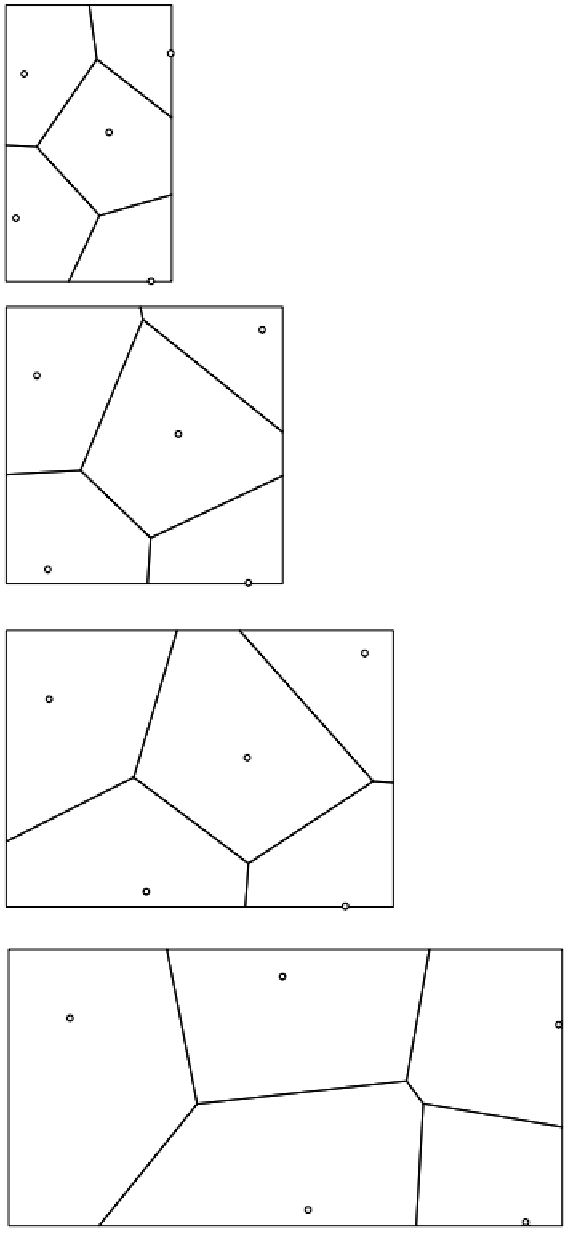

Figure 2: Formation of different polygon due to assymetric scaling in x and y axis

6. Geometric properties

Scutoid, as discussed above, has unequal number of vertices in its top and bottom planar surfaces. An ordinary m,n-Scutoid will consist of m-gon and n-gon (n¿m) on its top and bottom surfaces, with usually —n-m— vertices. Since it is a newly discovered shape, new set of rules have to be set up for proper identification and classification of a scutoid. Some of the general geometric properties of a scutoid are:

1. There exists atleast one point between two parallel faces named transition point[1].
2. The surface between those two surfaces may not be planar.
3. The surface between the transition point and n-gon is planar.
4. They can be tiled/tessellated to form a curved surface.

The 2nd and 3rd property can be hypothesized to arise from the process of minimization of surface area. But to simplify the calculations, the shape of the polygon may be chosen such that all the surfaces are planar. As evident from nature, scutoids can be tessellated to form large curved surfaces such as lining of various organs. A regular pentagon cannot tile a planar surface, but a hexagon can, and the combined use of hexagonal and pentagonal tiles in a football show that such type of tiling is possible for curved surfaces.

7. Use in structural Engineering

Scutoids, first modelled in 2018 as a geometrical solution to three-dimensional packing of epithelia, have been found abundantly in nature. Nature has always been very efficient in designing novel shapes that fit the purpose of what it is designed to do. Biomimicry is a very useful technique for structural engineers and architects since nature has been building things long before humans even existed. Thousands of years of evolution brought us honey-comb structure, eggshells, and turtle shells. Evolution also brought us easy solution to minimization of surface area through surface tension and soap bubbles, and now the question arises, why did nature use scutoids?

From lining of blood vessels to outer surface of heart, scutoidal shape is a byproduct of curvilinear shape of the organs. Understanding how stress is distributed in these cells will help us better understand human anatomy, as well as provide useful insight when developing artificial organs in the near future. Furthermore, blood vessels are analogous to thin walled vessels, and since our organs withstand different dynamic actions on daily basis, these things should be taken into account when using scutoids to model curved surfaces.

R. S. Dhari and N. P. Patel [1] had analysed crushing behavior of scutoid when compressed with different velocities and comparing it with honeycomb structure in ABACUS showed important role of transition point in providing additional stability.T teng[4] speculated that using scutoid bricks in masonry shell structure proved to be a easy construction due to the connection triangles preventing sliding failures as the brick elements were in contact with multiple elements and with curvature on both sides so as to act like multiple arches that are “perpendicularly mortising together”. The authors also claimed that the space grid made with scutoid bricks will have greater stability.and speculated that scutoids can distribute sudden change in stress more evenly than a non-masonry shell, i.e. ideal for use in blast proofing.

8. Methodology

Rhino 7 with Grasshopper plugin was used to select 3 points to form an arc that would be rotated around z axis to form surface of revolution. Of 2+1 points that were chosen in xz plane to make an arc: one was origin (0,0,0), 3rd point was (-10,0,-10) and 2nd point was taken to be (-5,0,-n) where n is the model number. The arc was then rotated a full turn to make surface of revolution. The arc was also transformed in both linearly and in size to form intermediate surface and inner surface. 2 new surfaces were generated. Then, 100, 200 and 300 points were chosen in random in outer surface and projected onto other surfaces to draw voronoi diagram from those points.Now, shortest nodes were searched and corresponding edges were constructed as shown in figure 3. The models were then exported to SAP2000 to analyse for dead load, 100kN roof load and Time history response when using Ramp load increasing to 1kN in 3 directions in 1sec, and drops to 0 from 4 seconds to 4.1 seconds. ISNB50M section was used to model line elements. Same analysis were done for model without scutoid.

Table 1: Average of 3 different number of seeds

n	2	3	4	5	6	7	8	Average
Radius of curvature	11.975	17.734	35.362	Infinity	-35.362	-17.734	-11.975	
Curvature	0.084	0.057	0.029	0	-0.029	-0.057	-0.084	
Fundamental Period %	**-11.297**	-10.418	**-12.241**	-5.1063	**-13.569**	**-13.927**	**-11.638**	-11.171
Weight %	24.372	24.453	24.676	18.285	**28.0217**	**28.6507**	**25.5887**	24.8638
Deflection %	-28.001	-17.64	-22.893	**-35.417**	**-39.663**	-21.114	-20.832	-26.509
Max member stress % for self wt	-20.537	-19.766	**-32.575**	-8.785	**-23.197**	**-27.102**	-18.143	-21.444
TH deflection %	**-41.915**	-32.589	-23.902	-26.921	**-51.035**	**-49.616**	-26.527	-36.072
Max member stress % for TH	-28.764	-24.515	**-34.255**	-21.714	**-33.845**	**-46.716**	-20.544	-30.05
100kN delfection %	**-43.801**	**-40.319**	**-37.949**	**-41.004**	-32.892	-27.204	-36.26	-37.061
Max member stress % for 100kN	-20.941	**-30.49**	**-44.101**	**-31.571**	**-30.785**	-25.841	-24.251	-29.711
DL+100kN deflection %	**-31.331**	-29.39	**-29.599**	**-34.74**	**-34.694**	-15.43	**-30.856**	-29.434
Max member stress % for DL+100kN	-24.452	-23.484	**-38.635**	-23.29	**-27.413**	**-31.601**	-22.956	-27.405

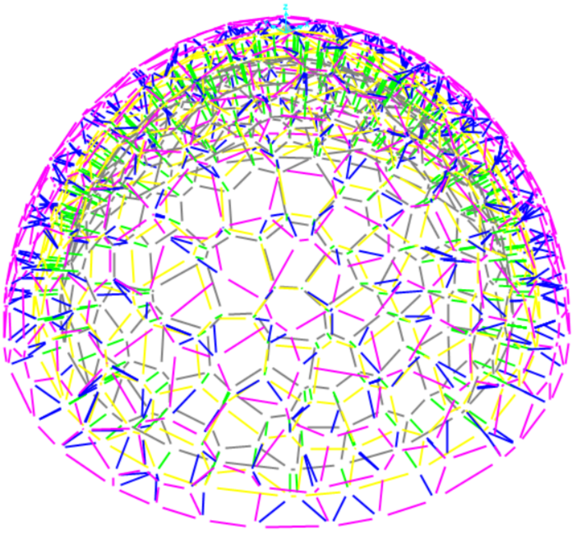

Figure 3: Sample model number 2 (curvature=0.084/m)

9. Results and Conclusion

A tool as explained in methodology has been developed to generate scutoids in a surface of revolution for any 3 given points. The models were chosen in such a way that the curvature increases uniformly, as seen in figure 4 showed 24.86% increase in weight with 26.51% decrease in deflection under self-weight and 21.44% reduction in stress for 11.17% increase in stiffness. The fundamental time period and stiffness are inversely related, so the change in time period can be used to compare stiffness. These results are expected since there are a greater number of frame elements in scutoids. Similar reduction in stresses and deflections is seen among all the load cased and combinations.

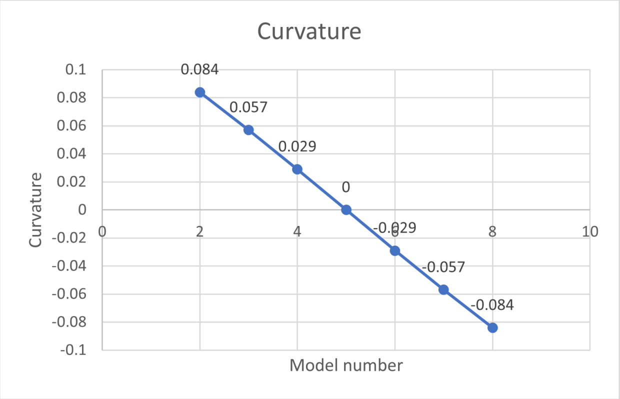

Figure 4: Curvature Vs. Model number

The Table **??** shows summary of results with desired results in bold. Sample variation of deflection and stresses for models with and without scutoids for 100 seed points are shown in table 2 and 3.

The results show that using scutoids in a curved structures show significant decrease in stresses in deflection without sacrificing much weight. The models with lower absolute curvatures (models 4, 5 and 6 with curvatures 0.029, 0 and -0.029) showed promising results when compared with scutoids. This may be due to the fact that the loads that were applied

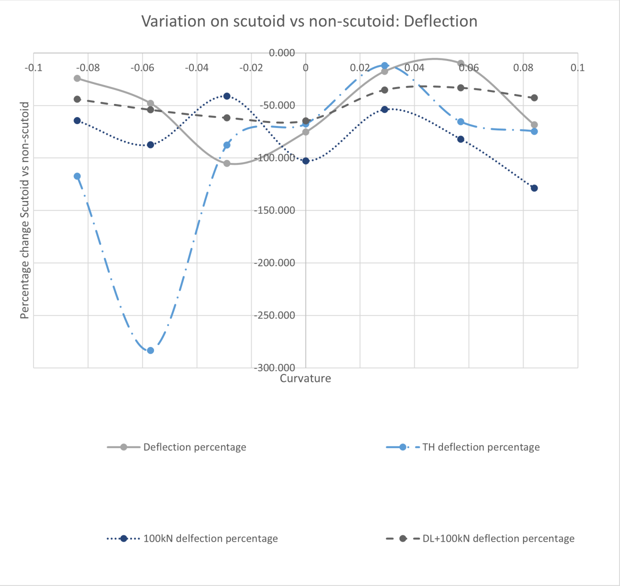

Figure 5: Deflection Vs. Curvature

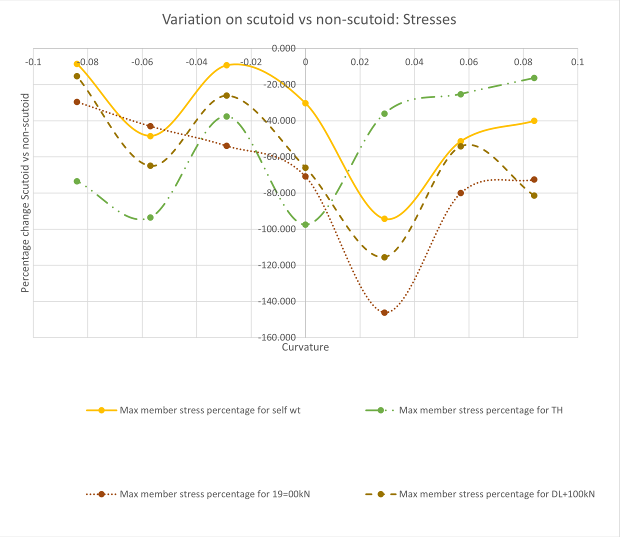

Figure 6: Stresses Vs. Curvature

were gravitational and higher curvatures require the frame elements to be connected at steep angles for scutoid models, instead of directed towards the center for frustums. So for the use case of roofing with impact protection, the models 4, 5 and 6 are most suitable.

Table 2: Variation of deflection in curvature

n	Deflection percentage	TH deflection percentage	100kN delfection percentage	DL+100kN deflection percentage
2	-40.642	-42.728	-56.291	-29.944
3	-9.077	-39.605	-45.089	-24.867
4	-14.916	-10.652	-35.007	-26.009
5	-42.958	-40.35	-50.675	-39.264
6	-51.23	-46.737	-29.176	-38.153
7	-32.44	-73.914	-46.637	-35.101
8	-19.416	-54	-39.219	-30.554
	-30.097	-43.998	-43.156	-31.985

Table 3: Variation of stresses in curvature

n	Max member stress percentage for self wt	Max member stress percentage for TH	Max member stress percentage for 19=00kN	Max member stress percentage for DL+100kN
2	-28.586	-14.053	-42.044	-44.85
3	-33.91	-20.121	-44.435	-35.111
4	-48.51	-26.45	-59.357	-53.566
5	-23.229	-49.363	-41.467	-39.736
6	-8.452	-27.237	-35.015	-20.699
7	-32.566	-48.309	-30.073	-39.374
8	-7.704	-42.319	-22.802	-13.276
	-26.137	-32.55	-39.313	-35.230

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