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Form finding and Optimization of Grid Shells using Force Density Method

Nishan Thapa [a], Bharat Mandal [b]

[a, b] *Department of Civil Engineering, Pulchowk Campus, IOE, Tribhuvan University, Nepal*
Corresponding Email: [a] nishan.thapa977@gmail.com, [b] bharat@ioe.edu.np

Abstract
Force Density Method is widely used method for form finding of tensile cable nets and grid shells. A design tool that utilizes force density method along with Genetic Algorithm have been formulated in Rhino-Grasshopper, a parametric environment, for design and optimization of grid shells. Variation of structural weight and height of grid shell is studied for various topologies, subdivisions and force density values. Genetic Algorithm has been used for optimization of grid-shell to get minimum weight for prescribed grid shell heights.

Keywords
Force Density Method, Genetic Algorithm , Optimization, Grid Shell

1. Introduction

Grid shells are discrete form of thin shells, following a continuous surface, often built of steel and wooden members as structural elements. Shell geometry largely determine the characteristics and magnitude of forces arising in the members. Shell geometries for design can be distinguished mainly in three types, namely Freeform shells where shapes are taken arbitrarily, Mathematical shells taken for their convenience in fabrication and analytical calculations and Form-found shells, mainly natural hanging shapes associated with funicular structures (Adriaenssens, Block, Veenendaal, & Williams, 2014). Form finding is a forward process in which parameters are explicitly/directly controlled to find an 'optimal' geometry of a structure which is in static equilibrium with a design loading (Adriaenssens et al., 2014). In general, form found shells are structurally robust and optimized for design requirements, when compared to other shell geometries.

In this study, we employ Force Density Method (FDM) as a method of form-finding, and genetic algorithm (GA) for optimization of grid-shells. The force density method (Schek, 1974) was developed by H. J. Schek for finding equilibrium state of given pin-joint network consisting of cables or bars subjected to pre-stressing or any external loading with low computation costs. Preassigning force density, which is the force in member divided by its length, linearizes the geometrically non-linear equilibrium problem, thus finding the equilibrium shape in a direct way. Force density variation with systematic manual input or assignment through results of plane faced airy stress polyhedron as in (Konstantatou, 2020) as well as assignments of various geometrical and boundary constraints can be done to obtain various funicular shapes, also tension and compression structures. However, in this study, linear form of FDM and uniform force density values have been used in this formulation. Mathematical description on how force density linearizes the equilibrium equation is well explained in (Gidak & Fresl, 2012).

Here, a design tool is developed in widely used software- Rhino, that assist architects and engineers at early stages of design from form finding as well as discretization and optimization of grid shell, that would prove to be beneficial, minimizing the structural costs while satisfying architectural and structural demands. Case study of a 12m x 12m rectangular grid shell is done varying mainly parameters like force density, grid density and grid shapes regarding its effect on the structural height and structural weight. Furthermore, optimization is done for the same shell using GA to determine shell configuration with minimum weight regardless of consideration for height and separately for shell to have a height equal to 1.5m.

The rest of the paper is arranged in four sections.

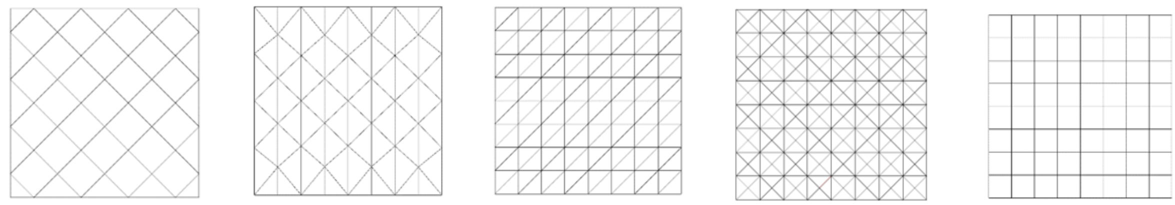

Figure 1: Grid shell typologies considered in study.

Section 2 details the FDM formulation. Section 3 discusses the use and basics of genetic algorithm in this study. Section 4 details the results of optimization and parametric study of grid shell taken. Section 5 concludes the finding of this study.

2. Force Density Method Formulation

FDM formulation has been done in Rhino using Grasshopper and Python programming language for calculation and solving linear equations, while Galapagos plugin within grasshopper has been used for optimization by genetic algorithm. Linear formulation of FDM as per Schek (Schek, 1974) without any constraints are used for form finding.

Validation of FDM formulation is done in two ways. Form finding of a pin connected line like bar network with constant force density is done. The resulting shape from FDM is that of a parabolic arch, same as a funicular form -catenary. Furthermore, a quadrilateral grid-shell of base dimension 4m *4m, with grid spacing 0.8m with quadrilateral panels has been analyzed for a constant force density and vertical point loading at grid joints. The resulting form of grid shell is analyzed in commercial design software ETABS v19, the resulting values of forces obtained from FDM and ETABS vary with 0.8% error, well within the acceptable limit.

Various studies for evaluation of grid shell performance and structural weight varying similar parameters have been done by various researchers, to name few, (Malek, 2012) using FEM for analysis, (Olsson, 2012)– using SMART form, (Green & Lauri, 2017) using Dynamic relaxation. For the study, buckling has not been considered, connections are assumed to be pin jointed, connection costs been not taken for optimization and only statically uniformly distributed load case has been done. The shell requires further analysis and detailed design check for lateral loading cases, buckling and nodal connection detailing. This paper considers form finding for static gravity loading for which optimization has been carried out considering it to be the most prominent load case. Final design will need to be checked for other load cases as well.

2.1 Structural weight and Total load path

Grid shell members are considered to be fully stressed, thus it is possible for representation of total structural weight by total load path of the structure. Load path for a member is given by its member length multiplied by force in the member. Suppose W represents the total weight of the structure. Then,

$$W = \sum A_i L_i \rho \tag{1}$$

where, n is the numbers of members, L and A length and area of each member, ρ density of the material used. Considering each member to be fully stressed to its stress capacity σ, σ= F/A, Thus,

$$W = \sum F_i L_i \rho / \sigma \tag{2}$$

In the study, material grade is considered constant, thus ρ/σ = constant. i.e.

$$W \propto \sum F_i L_i \tag{3}$$

Hence, in the study, structural weight is represented by total load path of the structure.

2.2 Force density

Shape analysis of tensile structures is a geometrically non–linear problem, the FDM linearizes the form–fitting equations analytically by using the force density ratio for each cable element, q = F/L, where F and L are the force and length of a cable element respectively (Southern, 2011) . Same principle holds for compression - tension and compression only structures like grid-shells. Force density of a member during FDM is assigned as per its axial strength.

For form finding of a grid shell, force density values input in FDM, are directly dependent on the choices of materials and section sizes available. For similar sectional sizes available, for material like wood, force density values assigned are lower, given their lower compressive strength, whereas higher values of force densities can be assigned for steel, given its higher material strength. Here, effect of variation of force density on shell weight and height is studied for force density ranging from 5KNm^{-1} to 40KNm^{-1}.

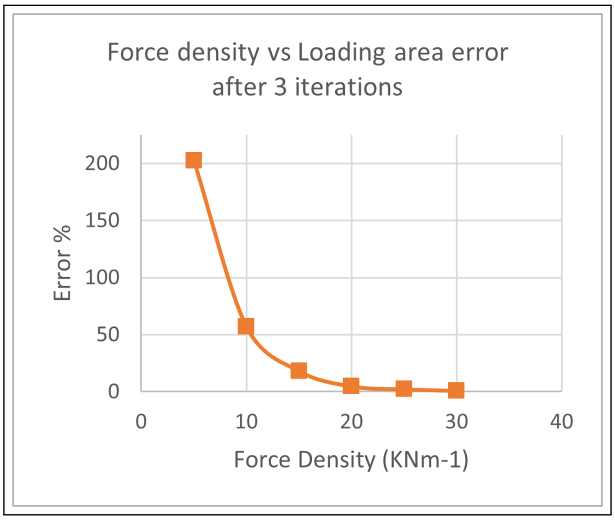

Figure 2: Loading error for quadrangular grid shell for various force density and 6 subdivisions.

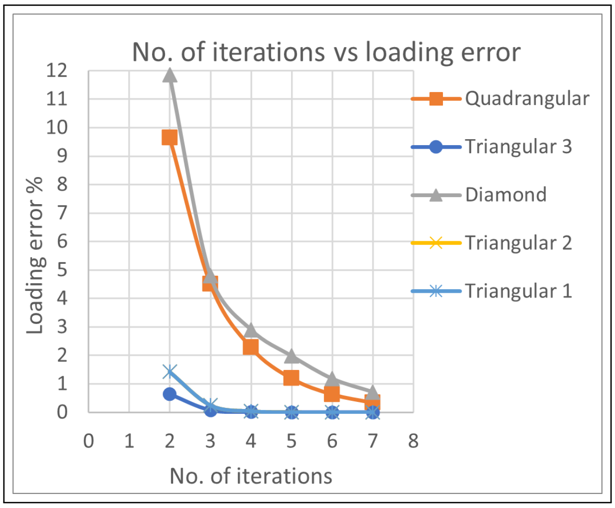

Figure 3: Loading error vs no. of iterations for various grid shell topologies at force density 20KNm^{-1} and 6 subdivisions.

2.3 Grid shell loading

Loads are applied as uniformly distributed static loads of value 5KNm^{-2} throughout the shell area, which is applied to the nodes as per their tributary area. It is observed that for lower values of force densities in all topologies of grid shells considered, surface area of grid shell after form finding is significantly greater than that of original projected plan. Thus, multiple iterations are required in form finding to minimize errors in loading values as a input for FDM.

Figure 2 shows, loading error for quadrangular grid shell for various force density and 6 numbers of subdivisions. Figure 3 shows loading error versus no. of iterations for various grid shell topologies at force density 20KNm^{-1} and 6 subdivisions. It has been observed that for all grid topologies considered maximum of three iterations suffice to get loading values within error of 5% for force density values higher than 20 KNm^{-1}. Thus, three iterations of FDM are done to get the loading on nodal points.

3. Genetic Algorithm

Genetic Algorithms (GAs), based on the principle of evolution is a stochastic method. In this method, best individuals, as per their fitness (here minimum structural weight) are chosen from a random population of individuals (here grid shells) with various gene sets (here topology, force density and subdivisions) are chosen for reproduction and with specific crossing techniques, solutions are combined to bring new offspring and in that way for a new generation (Dimcic & Knippers, 2011). Crossing methods are programmed to ensure conservation of good genes and addition of mutation algorithms enable random alteration of genes thus enabling convergence towards best fit solution. New generations are produced until a satisfactory result is found. In our case, the satisfactory result is statically stable grid-shells with minimum weight for various height configurations. More on application of GAs can be found in (Goldberg & Holland, 1988). GA has been utilized from the plugin, Galapagos Evolutionary Solver, available in Grasshopper with initial population size of 50, inbreeding 75%, maintaining 5%.

Two cases are examined for optimization, one being minimum grid shell weight without any regards for height and second minimum grid shell weight for height equal to 1.5m. For weight minimization with limit on height of 1.5m, the objective function for minimization is set as: Minimize W, where

$$W = \sum_{1}^{n} F_i L_i * (h - 1.5)^2 \quad (4)$$

The equation taken for weight minimization is considered arbitrarily to converge the genetic algorithm solver. Note the square of height difference between required and form found structure, amplify penalization of structures that are far off than the required height.

4. Results and Discussion

Exploration in the effect of variation of input parameters of FDM in basic property of grid shell, structural weight and height has been done taking a case study of a rectangular grid shell. Form-finding processes are scale independent i.e., geometries of structures in equilibrium obtained can be scaled to any values without the change in nature and ratio of forces acting on the structure. However, effects like buckling, global stability etc. become more prominent with increase in size of structure, which are out of scope of this study. As a representative size case study of 12m * 12m rectangular plan grid shell with pin support on all the edges is taken in consideration. Variation of structural weight, represented by total load path and corresponding height of grid shell as per variation in parameters listed below are studied:

1. Topology – quadrilateral grids (quadrangular and diamond shaped) and triangular grid of 3 types as shown in figure 1.

2. Grid density or number of subdivisions of span ranging from 6 to 24.

3. Force density ranging from 20 KNm^{-1} to 50 KNm^{-1}.

The shell topology or member network in 2D was created using meshing tools available in Grasshopper. Uniform force density values, loading and support condition were input of FDM, which results in the grid shell geometry corresponding to given input. Table 1 shows variation of total structural weight and grid shell height for five different grid topology types, keeping no. of subdivisions equal to 8 and constant force density of 25KNm^{-1}. From the table among considered grid types, quadrangular has lowest structural weight (57%) of highest which is triangular type 1 and 3. At the same time, triangular type 3 has the lowest grid shell height (37 %) of the diamond type. From table 1,it is seen that among other grid types, quadrangular and diamond types have lower structural weight, because of lower value of load path, as fewer members transfer the forces to the support.

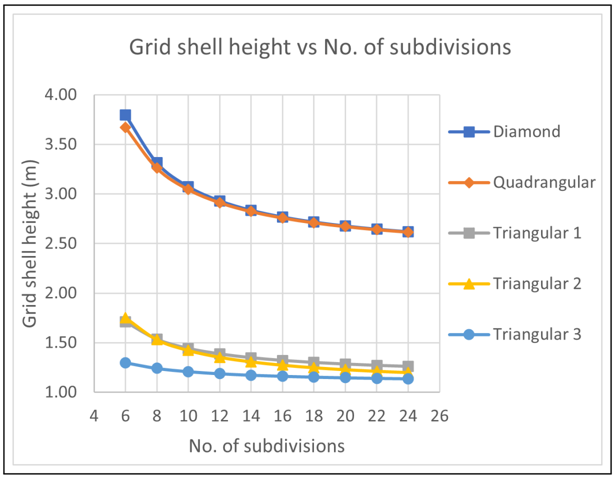

Figure 4: Variation of grid shell height as per grid density for force density of 20KNm^{-1}.

From figure 4 it is observed that keeping force densities constant for grid shell topologies, increase in grid density i.e. no. of subdivisions reduces the shell height considerably at lower values of subdivisions whereas, the changes are less pronounced at higher values. From figure 4 it is seen that, both of quadrilateral type grid topologies have similar variation with grid density for constant force density. Same is the case for triangular grid topologies.

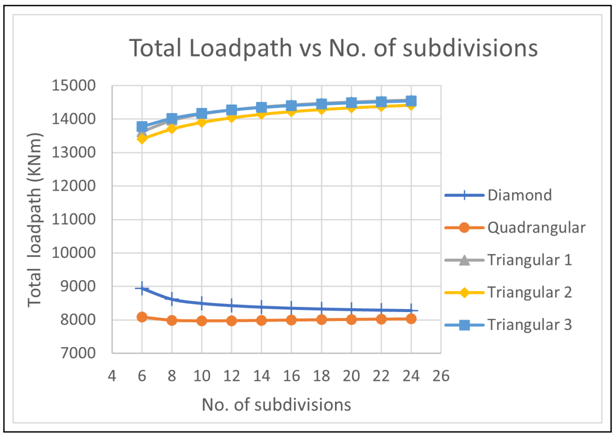

Figure 5: Variation of grid shell structural weight as per grid density for force density of 20KNm^{-1}.

Figure 5 shows that increasing sub-divisions increase

Table 1: Variation in Structural weight and height as per variation in grid topology

Grid Type	No. of divisions	Load path (KNm)	Normalised Load path	Gridshell height (m)	Normalised shell height
Diamond	8	8617.82	0.61	3.31	1.00
Triangular 1	8	13962.97	1.00	1.54	0.46
Triangular 2	8	13706.90	0.98	1.53	0.46
Triangular 3	8	14017.73	1.00	1.24	0.37
Quadrilateral	8	7984.91	0.57	3.26	0.98

grid shell structural weight for triangular type grid shells. In quadrangular type, grid shell weight decreases at lower values, however, with rise in subdivisions shell weight assume almost constant value. In diamond type grid shell, it is seen that increasing subdivision decreases shell weight considerably at lower values and assumes almost constant value at higher subdivisions alike quadrangular type. For all grid shells, variations are more pronounced at lower values of subdivisions but at higher values the grid shell weight variation with subdivisions is subdued.

This decreased variation in height and weight of grid shell as grid density is increased can be explained by the fact that as we approach higher grid density the grid shell assumes almost continuous surface with regular curvature. In lower subdivisions adjacent panels are highly irregular and even the small increment in subdivision makes more impact in making shell more regular. However, at higher subdivisions grid shell is almost regular and changes in subdivision have little or no impact in overall structural characteristic.

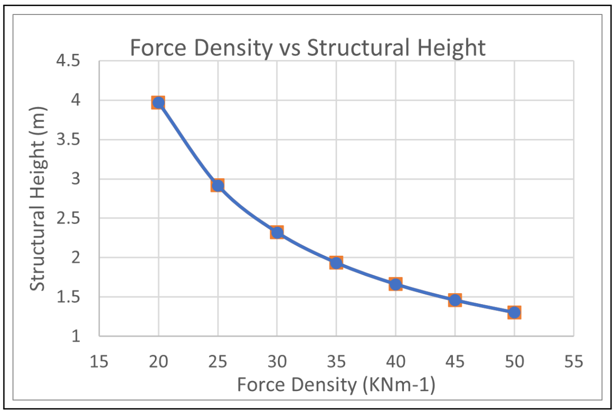

Figure 6: Variation of grid shell height as per force density for 12 subdivisions and quadrangular grid shell.

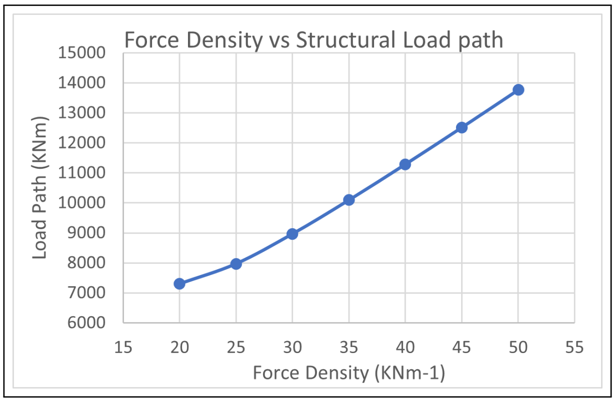

Figure 7: Variation of grid shell structural weight as per force density for 12 subdivisions and quadrangular grid shell.

In next case, for a 12m x 12m quadrangular grid shell with 12 numbers of subdivisions at support, variation of grid shell height and structural weight is determined with respect to force density. Figure 6 shows that at lower values of force densities structural height reduce highly with increase in force density whereas for higher values of force densities, the rate of change of structural height is decreased highly assuming almost linear variation. Further, figure 7 shows linear variation of grid shell structural weight with force density at higher values of force density with slight kink at the lower value.

The results of the case study show opposite variation of structural weight and height, for each of the parametric variation. Thus, optimization seems logical for the determination of case that satisfy both parameters. Results from genetic algorithm shows the value of optimum value of grid shell height and structural weight for given parameters as seen in table 2.

From optimization of the grid shell for minimum values of structural weight and height it is observed that quadrangular type grid shell has lowest structural weight when no height limitation is imposed. Furthermore, grid type topology of triangular 3 has

Table 2: Variation in Structural weight and height as per variation in grid topology

S.no.	Cases	Grid type	No. of subdivisions	Total loadpath (KNm)	Structural height (m)	Force density (KN/m)
1	Minimize structural weight	Quadrangular	24	7136.3	3.51	20
2	Minimize structural weight for height =1.5m	Triangular 3	14	11700.76	1.49	20

lowest structural weight with total load path of 11700.76 KNm^{-1} when height limitation of 1.5m is imposed. The results of optimization are presented in table 2.

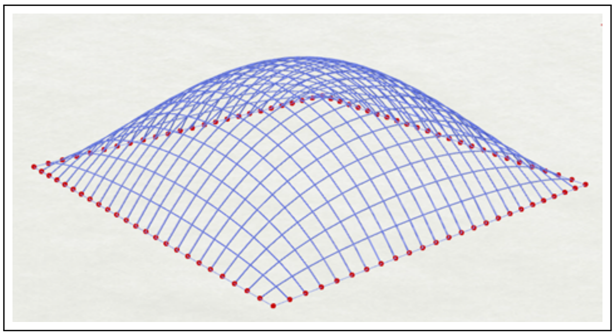

Figure 8: Grid shell form obtained for minimum structural weight as per case 1.

It has been observed that quadrilateral type grid shells have lower structural weight when no height limitations are imposed. However, given the non-planarity of panels, the fabrication costs are higher. Thus, further studies can be done, to obtain planar faced grid shells and then compare the results with triangular grid types in which planarity is inherent.

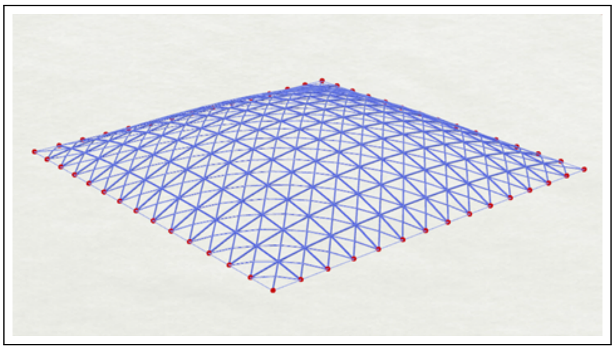

Figure 9: Grid shell form obtained for grid shell height of 1.5m as per case 2.

5. Conclusion

In this study, FDM has been utilized for design and optimization of grid-shells. A design tool for form finding and optimization at early design stage is created and using the same case study of a 12m x 12m rectangular plan grid shell has been done for various topologies, grid densities and force density values.

The FDM formulation has been validated against FEM model with errors within acceptable limits. Furthermore, GA has been employed to determine the optimum structural weight and height within the given limits. It has been observed, that for cases with no height limitation of grid shells, quadrangular followed by diamond shape grid type among other grid types have lowest structural weight whereas, triangular grid shapes have lowest height, although having higher structural weight.

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