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Effect of Change in Rise/Span Ratio on Performance of Open Thin Circular Cylindrical Shells

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Abstract

The analytical solution of differential equations of thin cylindrical shell, although being very rare, is very useful in many ways, for instance to validate the results of approximate theory like membrane theory, semi-membrane theory, finite element method. Up until today the methods , based on classical moment theory, commonly applied to analyze open thin cylindrical shell are Schorer theory, D-K-J theory, Chandrashekhar theory where certain assumptions are made to simplify the solution procedure. The more accurate solution is presented in Timoshenko (1959) [1] also where small simplifications are made. In this paper we solve the eighth order differential equation as presented in Ventsel (2001) [2] without neglecting any terms therein.

The geometry is open thin circular cylindrical shell subjected to uniformly distributed load in gravity direction. The boundary condition is rigid diaphragm on curved edges and clamped on straight edges. The solution of differential equation is achieved by writing a potential function in Fourier series form. The numerical calculation is performed via MATLAB coding where the operations made are solving system of linear equations and performing partial derivatives. The results obtained are compared with results from CSI SAP2000. Finally parametric study is conducted to study the effect of change in ratio of rise to transverse span on force components. From the parametric study it is seen that for straight edge clamped and curve edge on rigid diaphragm, forces and moments are minimum at rise/transverse span at 0.1.

Keywords

Open thin circular cylindrical shell, Moment theory, Analytical solution, Rigid diaphragm, SAP2000, MATLAB, Rise / Transverse Span

1. Introduction

Nowadays there are several finite element based software to analyze the shell structures within few minutes or less. But the problem is, using software can sometimes be dangerous due to error in modeling which cannot be seen easily because it's like black box, all the calculations are performed internally. So the analytical solutions, although being rare, are of utmost importance.

Although analytical solutions of moment theory are seen in previous literature, there are assumptions made to simplify the theory, for instance many researchers have used Schorer theory, D-K-J theory, Chandrashekhar theory, etc. where Schorer theory is valid for long shells, D-K-J theory is valid for short shells. So this paper aims to present analytical solution which can be applied to obtain better results for any span, radius and thickness of cylindrical shell where thin shell theory can be applied. It is to be noted that the method adopted in this paper has been suggested in Timoshenko (1959) [1] but for approximate form of differential equations, i.e. by neglecting the effect of in-plane displacements on moments and transverse forces. That method is extended to achieve analytical solution of more accurate form of differential equations (i.e. by considering the effect of in-plane displacements on moments and transverse forces). The results obtained from analytical solution are compared with finite element analysis. It is well known that shell's efficiency is due to its curvature but excessive rise for given width of shell is obviously uneconomical. So the parametric study is conducted to study the effect of change in rise and transverse span (or width) on structural performance. The author intends to achieve at optimum rise for given width of shell.

2. Literature Review

(Abhishek Adhikari, 2017) analyzed open circular reinforced concrete shell roof with curved edge simply supported and rectilinear edge once free and then supported on edge beam. The structure was analysed using Chandrashekhara and Schorer theory and also was modeled in SAP2000. It was found that the results from analytical and numerical methods were acceptable within permissible limits.[3]

(Jose, Ramadass and Ramanujan, 2013) did parametric study on the structural forces and the moments of cylindrical shell roof using ANSYS. The parameters considered in this study were thickness to radius ratio and length to radius ratio of a single cylindrical shell roof. The boundary condition adopted was simply supported along rectilinear edge and free along curve edge. It was found that when the ratio of length of the cylindrical shell to the radius of shell is equal to 4.0, the structural forces and the moments developed on the surface of cylindrical shell roofs are minimum. [4]

(Angalekar and Kulkarni, 2011) studied linear elastic behavior of cylindrical shell. For a fixed cross section of shell, a parametric study involving various spans and thickness was conducted. Besides the self weight, loads considered were uniform surcharge over the shell surface and longitudinal thrust. For the finite element analysis type of elements employed were 4 noded Kirchoff flat plate elements. The conclusions drawn were for the same span length shell, as the thickness increases there is a decrement in the displacement profile. Stress and moment profile shows a similar profile pattern as that of the displacement profiles.[5]

(Lende and Talikoti, 2015) analyzed cylindrical shell structures with varying radius and thickness. The linear static analysis was performed via SAP2000. The conclusion drawn was moments and Stresses of cylindrical shell structure are significant when the semi central angle is 40 degree and thickness of shell is 80mm. [6]

3. Research Objectives

For open thin circular cylindrical shell:

- To obtain analytical solution of thin cylindrical shell with curved edges supported on rigid diaphragm and rectilinear edges clamped and compare with results from finite element method via SAP2000.
- To determine effect of change in rise / transverse span ratio on membrane and bending forces and obtain optimum ratio.

4. Methods of Analysis

The open cylindrical shell was analyzed using basically two methods.

4.1 Moment Theory

The equilibrium equations in terms of displacements (obtained after using constitutive relations) are solved using Fourier series method via two approaches.

1. Approximate Analytical Solution
   The method presented in this section has been suggested in Timoshenko (1959) [1]. In this method the effect of displacements u and v on moments and transverse shear forces is neglected so that equilibrium equations simplify. The homogeneous solution is sought by writing u, v & w in terms of some potential function F and so single governing differential equation (homogeneous form) in F is achieved as follows.

$$\nabla\nabla\nabla\nabla F + \frac{1-\nu^2}{c^2}\frac{\partial^4 F}{\partial \xi^4} = 0 \tag{1}$$

   F can be obtained by expressing it in Fourier series form

$$F = \sum_{m=1}^{\infty} f_m(\theta)\, sin\lambda_m\xi = \sum_{m=1}^{\infty} Ae^{\alpha\theta} sin\lambda_m\xi \tag{2}$$

   By substituting equation 2 in equation 1, we get α and by satisfying boundary condition on straight edge we get constants A. Also a particular solution is obtained by writing u, v & w in terms of Fourier series as follows.

$$u_0 = \sum_{m=1}^{\infty} A_{0m}\cos\theta\cos\lambda_m\xi$$

$$v_0 = \sum_{m=1}^{\infty} B_{0m}\sin\theta\sin\lambda_m\xi$$

$$w_0 = \sum_{m=1}^{\infty} C_{0m}\cos\theta\sin\lambda_m\xi \tag{3}$$

By substituting above expressions of displacements and load components in system of differential equations, we get system of three linear equations with three unknowns A_{0m}, B_{0m} and C_{0m} which are solved using MATLAB.

2. Exact Analytical Solution
The approximate method is extended to achieve the analytical solution of more accurate form of differential equations wherein the effect of in-plane displacements on moments and transverse shear are considered. The exact differential equations following Gol'denveizer have been solved. The single governing differential equation (homogenous form) in this method is

$$\begin{aligned}&\left(1+4c^2\right)\frac{\partial^8 F}{\partial \xi^8}+4\left(1+c^2\right)\frac{\partial^8 F}{\partial \xi^6 \partial \theta^2}\\&+\left[6+c^2\left(1-\nu^2\right)\right]\frac{\partial^8 F}{\partial \xi^4 \partial \theta^4}+4\frac{\partial^8 F}{\partial \xi^2 \partial \theta^6}\\&+\frac{\partial^8 F}{\partial \theta^8}+\left(8-2\nu^2\right)\frac{\partial^6 F}{\partial \xi^4 \partial \theta^2}+8\frac{\partial^6 F}{\partial \xi^2 \partial \theta^4}\\&+2\frac{\partial^6 F}{\partial \theta^6}+\left(1-\nu^2\right)\left(1/c^2+4\right)\frac{\partial^4 F}{\partial \xi^4}\\&+4\frac{\partial^4 F}{\partial \xi^2 \partial \theta^2}+\frac{\partial^4 F}{\partial \theta^4}=0 \quad (4)\end{aligned}$$

Now the procedure to obtain homogenous and particular solution is pretty much similar to approximate method.

4.2 Finite Element Method

SAP2000 is used in analysis of thin cylindrical shell where four noded flat shell elements with 6 DOF at each node are utilized. This method is adopted solely for comparison purpose.

5. Model Description

Altogether 25 models have been analyzed. Model 1 is for comparison of results from different methods. Models 2 to 13 are for parametric study with transverse span $= 5m$ and models 14 to 25 are for parametric study with transverse span $= 10m$.

Materials Properties Indian standard M25 grade of concrete is taken whose modulus of elasticity $(E) = 25000$ MPA and Poisson's ratio $(\nu) = 0.2$

Boundary Condition Curved edges on rigid diaphragm (rigid on its own plane only, i.e. $v = 0$, $w = 0$, $N_1 = 0$ and $M_1 = 0$) and straight edges clamped (i.e. $u = 0$, $v = 0$, $w = 0$ and $\frac{\partial w}{\partial \theta} = 0$)

Loading Condition Uniformly distributed load along gravity direction with magnitude 10 kN/m^2 is applied.

Geometry For all models thickness is kept constant as 50 mm and longitudinal span as 10 m. The other geometric details are as shown in table 1

Table 1: Geometry of Models

Model No	Radius (m)	Width(m)
1	3	$3\sqrt{2}$
2 to 13	varies from 3 to 500	5
14 to 25	varies from 6 to 1000	10

6. Results

6.1 Comparison of Analytical Solution with SAP2000 Results

So the analysis has been performed from three different methods, namely approximate analytical solution, exact analytical solution and finite element method via SAP2000. For the validation purpose, approximate and exact analytical results are compared with SAP2000 results. The following tables show the membrane forces and moments at three different angular locations from these three methods.

Table 2: Comparison of N1 at mid span: Model 1

Angle	Exact	Approx.	SAP2000
0	-10.167	-9.985	-10.188
22.5	-7.087	-7.028	-7.080
45	-7.176	-7.212	-7.080

Table 3: Comparison of N2 at mid span: Model 1

Angle	Exact	Approx.	SAP2000
0	-27.122	-27.183	-27.140
22.5	-29.796	-29.701	-29.820
45	-35.880	-36.059	-35.440

Table 4: Comparison of M1 at mid span: Model 1

Angle	Exact	Approx.	SAP2000
0.0	0.057	0.057	0.058
22.5	-0.040	-0.039	-0.039
45.0	0.121	0.117	0.118

Table 5: Comparison of M2 at mid span: Model 1

Angle	Exact	Approx.	SAP2000
0.0	0.269	0.269	0.270
22.5	-0.188	-0.182	-0.180
45.0	0.604	0.587	0.590

6.2 Parametric Results

The parametric study was performed to see the effect of change in ratio of rise to transverse span on force and moment components. Among the various methods of analysis, the exact analytical method of moment theory (as presented in this paper) is applied to conduct parametric study. Once the transverse span is kept at 5 m and rise varying from 1.342 m to 0.00625 m (total 12 models as presented in table 1) and the effect on forces are observed and plotted below. Then the transverse span is kept at 10 m and rise varying from 2.683 m to 0.0125 m (total 12 models) and again the effect on forces are observed and plotted below:

- For transverse span=5m

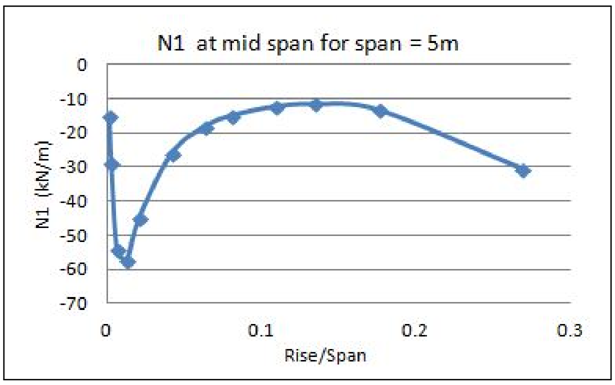

Figure 1: Plot of N1 vs Rise/span at mid span

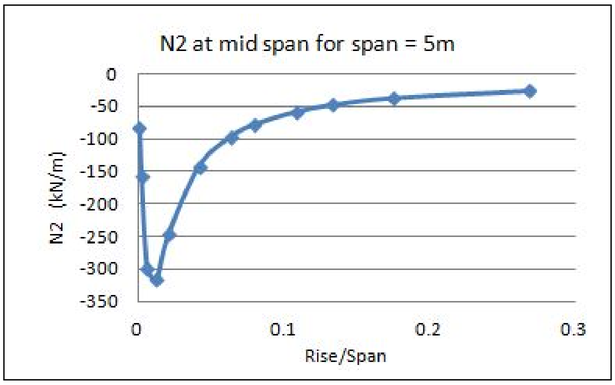

Figure 2: Plot of N2 vs Rise/span at mid span

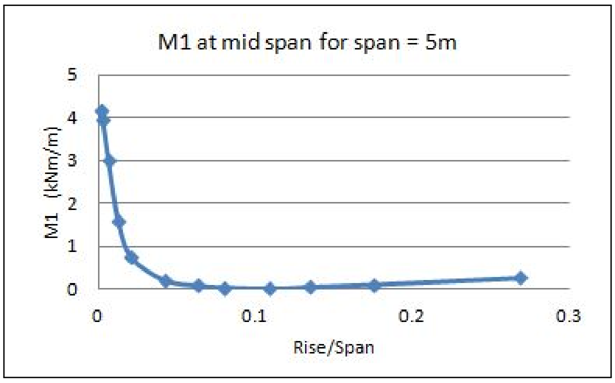

Figure 3: Plot of M1 vs Rise/span at mid span

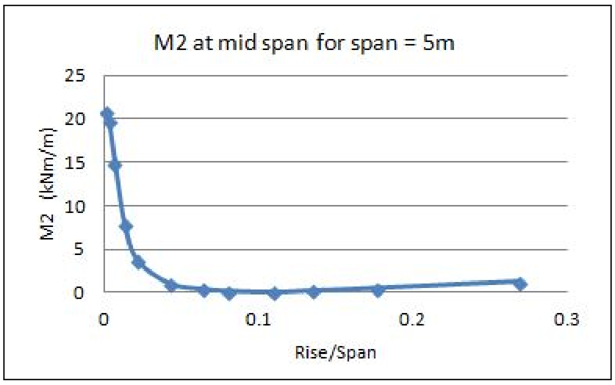

Figure 4: Plot of M2 vs Rise/span at mid span

- For transverse span=10m

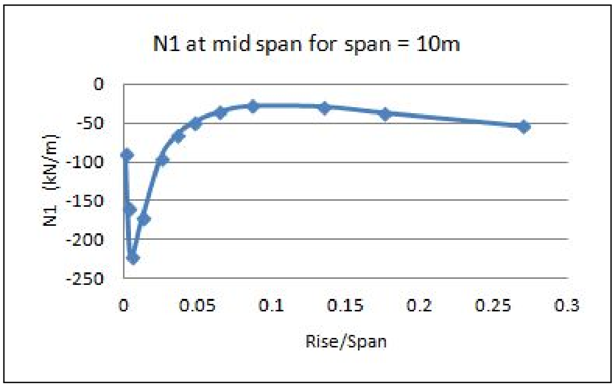

Figure 5: Plot of N1 vs Rise/span at mid span

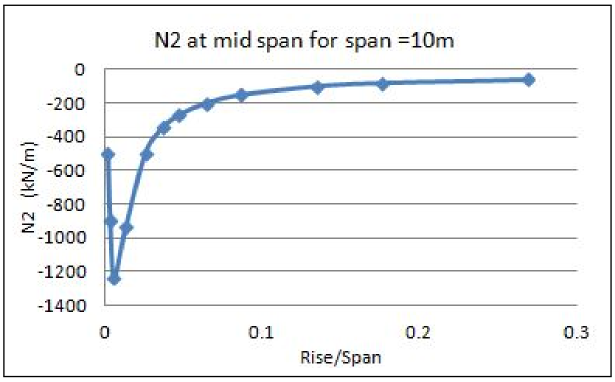

Figure 6: Plot of N2 vs Rise/span at mid span

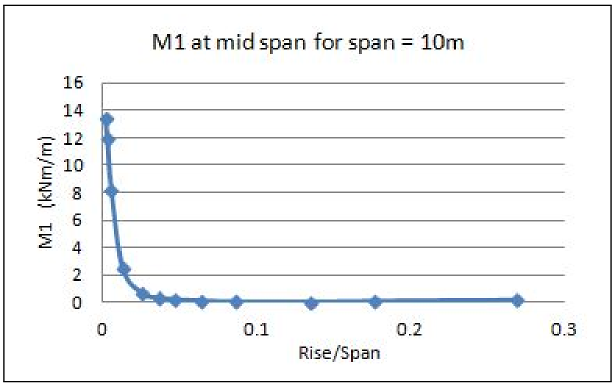

Figure 7: Plot of M1 vs Rise/span at mid span

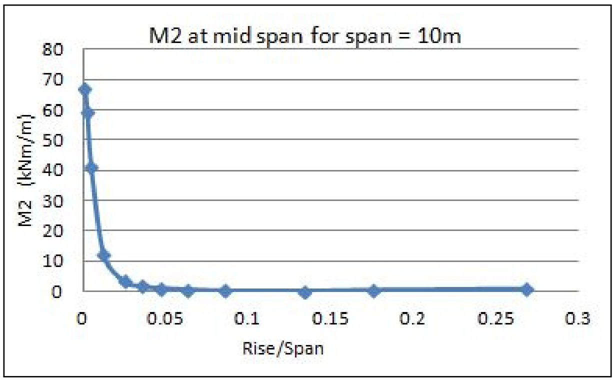

Figure 8: Plot of M2 vs Rise/span at mid span

7. Discussion

In this paper analytical solution, in most accurate way so far, has been achieved. First, solution is obtained as per method presented in Timoshenko (1959) [1] wherein certain terms are neglected (i.e. effect of displacements u and v on bending and twisting moments are neglected). Then exact solution for differential equations as presented in Ventsel and Krauthammer (2001) [2] is obtained without neglecting any terms therein. The results from two methods are compared and it is seen that the discrepancies between these two methods are less than 4 % in every case. Model 1 is also analyzed in SAP2000 where shell is modeled via four noded thin shell elements with 6 DOF at each node. The SAP2000 results are compared with exact analytical solution and it is found that discrepancies between these two methods are less than 4 % for forces and moments. Also to check the convergence of the method applied, the number of terms in the series are increased till the discrepancy is negligible to desired level.

After the validation of the method, parametric study is conducted. The parameters chosen are transverse span (i.e. distance between straight edges) and rise. First the transverse span is kept constant as 5 m and rise is varied from 0.00625 m to 1.342 m (total 12 models) and various forces are plotted against rise/span ratio. It is observed that moments are significantly reduced when rise/span ratio > 0.05. Also the force component N1 is minimum at rise/span ratio 0.1 and N2 is very small at that ratio of rise/span. And this is true for sections at both mid span and quarter span. In order to draw some conclusion the transverse span is changed to 10 m and rise is varied from 0.0125 m to 2.68 m. Again it is seen that moments are negligible when rise/span ratio > 0.05. Also N1 is minimum at

rise/span ratio 0.1 and N2 is very small at that ratio. So we can say that rise/span ratio of 0.1 is optimum for this particular boundary condition. Also to note that even with very small rise/span ratio moments are significantly reduced. And this can be very useful in practical design of slab structures because if we can provide even slight curvature on slabs then thinner slab sections can be achieved.
The effect of change in ratio of longitudinal span to transverse span on forces and moments can also be seen from figures 1 to 8. As the transverse span increases from 5m to 10m (and longitudinal span being constant as 10m), the graphs tend to shift left which indicates two way action. In other words, moments vanish quickly with rise for span=10m than for span=10m. Similarly membrane forces develop quickly. Finally it is to be noted that the method applied to achieve analytical solution here can be applied to any arbitrary boundary condition on straight edge.

8. Conclusion

From this research work, the following conclusions are drawn:

- The exact analytical solution has been achieved for rigid boundary on curve edge and clamped on straight edge and the results are verified by comparing with finite element method via SAP2000. But any arbitrary boundary condition on straight edge can easily be achieved following the same procedures.

- The parametric study shows that the optimum rise/span ratio is 0.1. At this ratio moments are negligible, also N1 is minimum. It is to be noted that N2 keeps on decreasing with increase in rise/span but at decreasing rate.

- From the graphs of moments vs rise/span ratio we see that moments are significantly reduced even at very small rise/span ratio of 0.05. This idea can be implemented in practical design of slabs as well.

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