Proceedings of IOE Graduate Conference, 2019-Summer
Peer Reviewed
Year: 2019 Month: May Volume: 6
ISSN: 2350-8914 (Online), 2350-8906 (Print)

Fundamental Time Period of RC Moment Resisting Frames

Jagadish Chandra Rimal [a], Prem Nath Maskey [b]

[a,b] *Department of Civil Engineering, Pulchowk Campus, IOE, Tribhuvan University, Nepal*
Corresponding Email: [a] 073msst108.jagadish@pcampus.edu.np

Abstract

The fundamental period of vibration is one of the most important parameters for seismic design of structures. Building codes generally specify the empirical formulas to estimate the fundamental period of vibration of buildings, most of which are the functions of height without taking into account other parameters. In this study, the fundamental period of vibration of regular RC framed buildings are examined using 3D finite element modelling and Rayleigh method. The influence of different parameters like the height of the building, number of storeys, number of bays and the length of the bays on the fundamental period of the buildings is presented. Various empirical expressions are proposed based on the regression analysis to relate fundamental time period as the function of different parameters: height, number of storeys, base dimension and the number of bays of the building.

Keywords

Fundamental Period, Moment Resisting Frames, Building Design Code, Reinforced Concrete Frame Buildings

1. Introduction

The determination of fundamental time period of vibration (T) of a structure, which is a function of its mass and stiffness, is an important parameter required for the seismic resistant design of structures. The design codes for seismic resistant building construction throughout the world recommend an empirical expression for calculation of fundamental period of building. Generally, these expressions are expressed as a function of building type, overall dimension of the building (especially height and in a few cases the number of storeys) as well as the building material(RCC, steel), etc. As has been pointed out by several researchers[1, 2, 3, 4, 5], there are several other parameters which affect the fundamental period such as the number and length of bays, storey height and storey number, irregularity in plan as well as elevation, soil-structure interation, etc. and thus there is room for improvement in these empirical expressions.

The objective of this paper is to investigate the effect of some of these parameters on the fundamental period of regular RC buildings and to propose empirical equations to relate the fundamental time period as the function of different selected parameters.

2. Existing expressions for the estimation of fundamental period

Most of the codes refer to the following expression which have been obtained by regression analysis on the periods of vibration measured during earthquakes:

$$T = C_t H^{0.75} \tag{1}$$

where, H is the height of the building and C_t is a numerical coefficient.

Equation (1) was first developed in the USA in 1975 as part of the ATC3-06 project(1978), based on the measurement of periods of buildings during the San Fernando earthquake(1971). This equation was derived using Rayleigh's method with the assumptions that the equivalent static lateral forces are distributed linearly over the height of the building, seismic base shear is proportional to $1/T^{2/3}$ and that the heightwise distribution of stiffness is in such a way that the interstorey drift is uniform over the height. The coefficient C_t was found out to be 0.025 for RC MRF buildings(H expressed in feet) which was later changed to be 0.030 according to the SEAOC-88 commentary and was subsequently used in codes like Uniform Building Code(UBC 1997) in USA[6].

Other codes around the world have since adopted the period-height equation (1) with slight changes in the value of C_t. For example, Eurocode 8(2004)[7] and Indian Code IS1893(Part 1):2016[8] use the same expression as given in equation (1) with the value of C_t transformed to 0.075 considering the height of the building to be in meters. The Building Code of Pakistan with Seismic Provision(2007)[9] recommends C_t to be 0.0731 for RC MRF. According to the New Zealand code NZS1170.5:2004[10], the value of T are obtained by multiplying the right side of equation (1) by 1.0 and 1.25 for serviceability and ultimate limit state respectively. Nepal National Building Code NBC 105:1994[11] suggests C_t to be 0.06 for concrete frames.

There are other forms of expressions given in building codes for determining the value of fundamental period as well. ASCE-7[12] recommends the equation given as follows:

$$T = C_t h^x \tag{2}$$

where, h is the height of building in feet,the value of C_t and x being respectively 0.016 and 0.9 for RC moment resisting frames. The Bangladesh National Building Code (BNBC)(2015)[13] uses effectively the same equation (2) with the value of C_t as 0.0466 and m as 0.9 taking the building height in meter.

Building Standard Law in Japan (BSLJ)(1981)[14] recommends the following expression:

$$T = H(0.02 + 0.01\alpha) \tag{3}$$

where, H is the height in m and α is the ratio of the total height of steel construction to the height of the building i.e. $\alpha = 0$ for concrete frames.

A graphical comparison of the code provisions for period-height expressions for RC moment resisting frames is shown in figure 1.

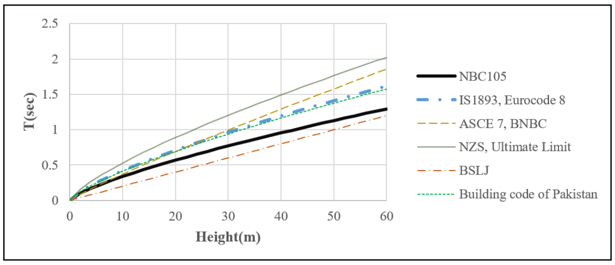

Figure 1: Comparison of code provisions for period-height expressions for RC moment resisting frames

Codes are found to suggest the expressions for fundamental period as a function of number of stories of the building above the base as well. For structures not exceeding 12 storeys above the base for a MRF and average storey height at least 10ft(3m), ASCE-7[12] suggests

$$T = 0.1N \tag{4}$$

where, N is the number of storeys above the base. The above expression was also adopted by the Canadian National Building Code (NBC 1995)[15]. IS1893-1984 [16] also prescribes the expression (4) for moment resisting frames without bracing or shear walls.

Code provisions generally also permit the use of techniques like Rayleigh method but under the condition that the obtained values do not exceed those obtained from empirical expressions by a certain factor ranging from 1.2 to 1.7.

Besides the code provisions, many researchers have proposed their own expressions for the determination of fundamental period.

Goel and Chopra[6] using regression analysis from the data obtained from motions of buildings during eight California earthquakes occurring over the period from 1971 to 1994, developed improved empirical relationships to estimate the fundamental vibration period of moment-resisting frames (MRF). The expressions

$$T_L = 0.016H^{0.90} \tag{5}$$

and

$$T_U = 0.023H^{0.90} \tag{6}$$

were proposed for RC MRF buildings with height H taken in feet, where T_L and T_U represent the lower limit and upper limit for fundamental period. The lower limit formula was later adopted by ASCE-7 as can be seen from equation (2). It was concluded that the period from rational analysis should not exceed the value of the recommended equation by a factor larger than 1.4 and also pointed out that since the analysis was carried out based on data from buildings in California, the developed equations may not hold good for other parts of the world with different seismicity and construction practices.

Hong and Hwang[17] presented an empirical formula through regression analysis to the fundamental vibration of RC MRF in Taiwan of the form

$$T = 0.0294H^{0.804} \tag{7}$$

It was also found that the identified fundamental vibration periods of RC MRF buildings in Taiwan was lower than those either measured in California or predicted in the code formula of the UBC-97 because of the differences in construction practices justifying the need of different equations for different parts of the world.

Crowley and Pinho[18], using analytical procedures such as eigenvalue, pushover and dynamic analyses, proposed a linear relationship as follows:

$$T = 0.1H \tag{8}$$

while Guler et al.[19], utilizing experimental and analytical methods proposed the expression

$$T = 0.026H^{0.90} \tag{9}$$

Combining the parameters height(H) and number of storeys(N) in a single equation, Salama[20] proposed the equation of the form:

$$T = 0.021N^{0.16}H^{0.75} \tag{10}$$

Besides the above described formulations which express the time period as a function of height (in a few cases, number of storeys) only, other researchers have taken into consideration more parameters to achieve better reliability for determination of fundamental period of buildings. Amanat and Hoque[1] identified the significance of span length, number of spans and infill amount while Kose[2] included even more parameters such as frame type, ratio of percentage of shear walls to total floor area to come forth with their own expressions. Other researches include those done by Hatzigeorgiou and Kanapitsas [3] and Asteris et al. [4, 5] that came up with their own lengthy expressions considering several different parameters.

As can be seen from the expressions provided by the codes as well as suggested by different researchers, the expressions for the estimation of fundamental period of vibration vary widely from simple height-related expressions to more complex relationships taking into consideration many different parameters. Evidently, the determination of fundamental period for a structure carried out by different relations leads to a large range of results. Also, as indicated by Goel and Chopra[6] and Hong and Hwang[17] it can be seen that the expressions developed are based on the analysis of structures located at specific regions or countries which may not give good estimation reliability for a different region with different design, working condition and construction practices. So, a deeper study of the fundamental period of vibration and the parameters affecting it is essential to develop a reliable equation in comparison to the existing ones especially in the context of Nepal where the seismic code NBC105:1994 has not been updated since its publication in 1994.

3. Methodology

In this study of the parameters affecting the fundamental period of RC moment resisting frames, 126 different fictitious buildings are selected for sensitivity analysis after which they are modelled and designed using finite element software. The buildings considered are regular, with equal number of bays in both horizontal directions. Analysis of each building model is then carried out by using Rayleigh method to find the fundamental time period of the building. After that, the influence of different parameters on the fundamental time period is analysed. Finally, approximate formulation of the fundamental time period based on the selected structural parameters is proposed.

Building Parameters and Material Properties

Buildings considered for this study are bare frame type buildings without considering the stiffness contribution of the infill walls. The number of storeys of the building models considered vary from 3 to 13 storeys. For each number of storeys considered, different storey heights were chosen which vary from 2.5m to 4m. Thus, the overall height of the building vary from 7.5m to 39m.

3, 4 and 5 number of bays are considered and the length of each bay vary from 3m to 5m. So, the total base dimension of the building vary from 9m to 25m.

In order to study the influence of a parameter on the fundamental period of the building, the parameter under study is varied keeping other parameters constant.

The size of the beam for all the building models was fixed to be 230mmx350mm while the size of the columns differ according to the number of storeys in the building model. However, all the columns in a building model are of same size. The column size is taken in such a way that the percentage of steel reinforcement remains low. The thickness of the slab is taken to be 150mm.

The building parameters and material properties used for the development of the models are listed in Table 1 and the size of columns adopted are listed in Table 2.

Table 1: Building parameters and material properties

Concrete strength	25MPa
Modulus of elasticity of concrete	25000MPa
Poisson's ratio of concrete	0.2
Unit weight of concrete	$25kN/m^3$
Steel tensile yield strength	500MPa
Modulus of elasticity of steel	200000MPa
Poisson's ratio of steel	0.3
Unit weight of steel	$7850kg/m^3$
Size of beams	230x350mm
Slab thickness	150mm
Number of storeys	3, 6, 9, 13
Storey height	2.5m, 3m, 3.5m, 4m
Number of bays(spans)	3, 4, 5
Span(bay) length	3m, 4m, 5m
Floor finish dead load	$1kN/m^2$
Live load	$3kN/m^2$
Unit weight of brick masonry	$19.05kN/m^3$

Table 2: Column size adopted for different buildings

Number of storeys	Column size (mm × mm)
3	350 × 350
6	500 × 500
9	600 × 600
13	700 × 700

Design and modelling of buildings

The frame buildings were designed using linear static method taking in reference the design codes IS456:2000 and IS1893:2016.

Dead load, live load and earthquake loads are assigned to the models. In addition to the self-weight of the members, floor finish dead load intensity of $1kN/m^2$ is applied on each floor slab. Live loads are taken to be $3kN/m^2$ for all floors. The loads due to brick infill walls are applied as uniformly distributed load on beams. The peripheral beams are considered to be loaded with 230mm thick walls while the inner beams are considered to be loaded with 115mm walls with 30% opening in both cases. The specific weight of brick masonry is taken as $19.05kN/m^3$. The design load combination was taken as given in seismic design code. The seismic weight of the building includes the total dead load of the structure and a fraction of the live load assigned to the structure. In this study, 25% of the live load is considered to be included in the seismic weight along with the dead load. The building models are considered to be special moment resisting frames founded on medium soil condition. The seismic zone was considered to be the zone with very severe seismic intensity.

For design and modelling of the buildings, finite element analysis software SAP2000v20 was used. Beams and columns were modeled with two-node frame elements having six degree of freedom per node. Floor slab was modeled using four-node thin shell element. All the floor levels were assumed to be rigid in their own plane by considering rigid floor diaphragm. Secondary effects such as temperature, shrinkage and creep were not considered in the modeling. Also, no soil-structure interaction was considered, hence the foundation of the models were assumed to be rigid foundation.

Rayleigh method

Rayleigh method is a universally accepted method of finding the fundamental period of structures based upon the principle of conservation of energy[21]. For linear elastic analysis, Rayleigh method is known to provide a satisfactory approximation of fundamental periods of structures[22] and is found to be used by different researches such as ATC 3-06 project[6]. Rayleigh method is prescribed by most of the codes including NBC105:1994[11], Eurocode 8[7] and NZS 1170.5:2004[10], etc. The expression for finding fundamental time period using Rayleigh method is as follows:

$$T = 2\pi\sqrt{\frac{\Sigma_{i=1}^{n}(W_i d_i^2)}{g\Sigma_{i=1}^{n}(F_i d_i)}} \tag{11}$$

where, d_i, F_i and W_i are the horizontal displacement of the center of mass, displacing force and seismic weight at level i respectively.

4. Results and Discussion

Influence of the height of the building on the fundamental time period

Height is a very important parameter which influences the fundamental period of a building. The influence of the building height on the fundamental time period of RC framed building was investigated for different span lengths and storey heights.

As study of a typical example, Figure 2 shows the plot of the fundamental period versus the height of the buildings with 3 bays having bay length of 3m and storey height of 3m. Also, comparison is made with three code-based equations:

$$T = 0.0466H^{0.90} \tag{12}$$
$$T = 0.075H^{0.75} \tag{13}$$
$$T = 0.06H^{0.75} \tag{14}$$

It is seen that the calculated fundamental period of the building increases with height. As the height of the building increases in the bare frame model, the mass of the building increases but not the stiffness. Thus, the flexibility of the building increases with increase in the height which causes the fundamental period to increase.

It is found in most cases that the values of the fundamental period given by the code-based empirical equations are lower than those calculated by using Rayleigh method. This clearly shows that the code-based period-height formulae tend to underestimate the fundamental periods of buildings which makes the design more conservative. The time period value provided by NBC105:1994 expression $T = 0.06H^{0.75}$ is even lower than those provided by other code formulas like IS1893:2016 and ASCE-7. This means that, the design of buildings using NBC105:1994 is even more conservative in comparison to other codes.

As can be seen from the figure 2, the difference between the values of the fundamental period obtained by Rayleigh method and the empirical equations is smaller for smaller building heights but when the height increases, the difference between them becomes larger. Thus, the code-based empirical equations are more accurate for low-rise buildings in comparison to the mid-rise and high-rise buildings.

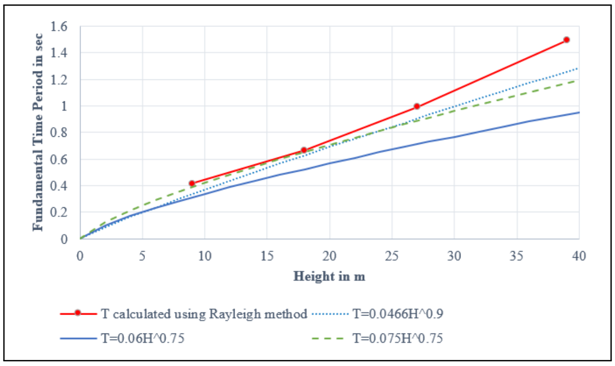

Figure 2: Height of the building versus fundamental time period of buildings with 3 bays of 3m bay length and 3m storey height

Influence of the storey number and storey height on the fundamental time period

In this study, the influence of storey number on the fundamental period of RC frame buildings is investigated for different span lengths and span numbers by varying the storey height from 2.5m to 4m.

As an example, figure 3 shows the variation of the fundamental time period with the number of storeys for 3 bay buildings with 3m bay length keeping the storey height as 3m. It is seen that with the increase in the storey number, the time period increases. This is expected as the number of storeys is directly related with the height of the building and as the height increases, the fundamental period value also increases.

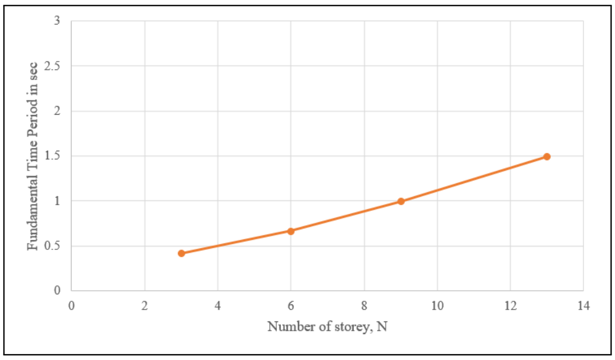

Figure 3: Number of storey versus fundamental time period of buildings with 3 bays of 3m bay length and 3m storey height

Influence of the bay(span) number on the fundamental time period

For finding the influence of the number of bays on the fundamental period, buildings having 3, 4 and 5 bays were studied by keeping other parameters constant.

Figure 4 and 5 show the variation of fundamental period with respect to the bay number keeping 3m bay length and 3m storey height fixed for a 13 storey building and 3 storey building respectively. As can be seen from the figure 4, as the number of bays increases from 3 to 5, there is a slight decrease in the fundamental period. However, the decrease is very small(about 0.06 in the considered case) and the time period can practically be considered to be the same.

The result is similar for the 3 storey building as shown in figure 5.

Thus, it can be considered that the number of bays or spans is not a significant parameter affecting the fundamental time period of a building if other parameters are kept constant.

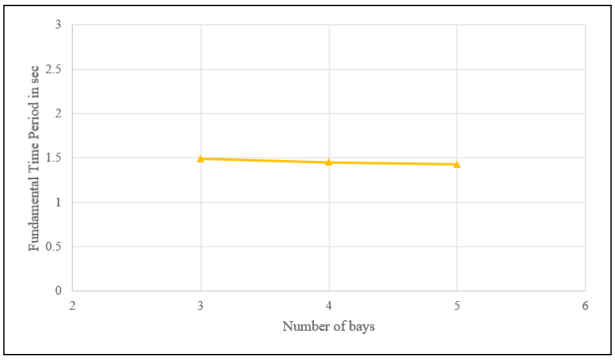

Figure 4: Number of bays versus fundamental time period of 13 storey buildings with 3m bay length (span) and 3m storey height

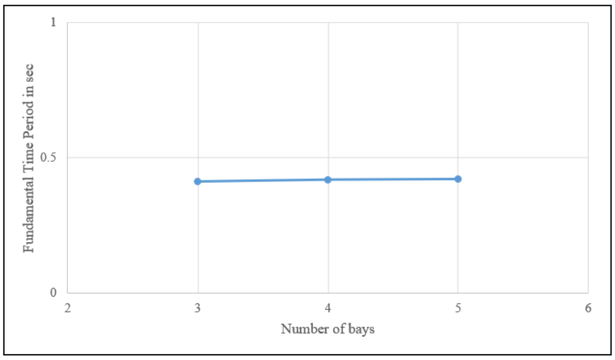

Figure 5: Number of bays versus fundamental time period of 3 storey buildings with 3m bay length (span) and 3m storey height

Influence of the base dimension on the fundamental time period

The influence of the base dimension on the fundamental period of RC framed buildings was investigated. Different base dimensions of length varying from 9m to 25m were chosen for the sensitivity study.

As a representative case, figure 6 shows the fundamental period for various base dimensions of a 6 storey building keeping 4 bays and 3m storey height fixed.It is observed that with the increase in the dimension of the base for fixed number of bays and fixed storey height, the fundamental period of the building increases.

Increasing the length of the building keeping fixed number of bays causes an increase in the mass of the building but not the stiffness of the columns against the lateral force. Because of this, the building becomes more flexible and hence the fundamental period of the building increases.

The code-based empirical equations don't have the provision to include the effect of overall base dimension of the building for RC frame buildings without infill panels. Therefore, the time periods predicted by these expressions for a certain height are same regardless of the base dimension of the building.

However, from figure 6, it is clear that the fundamental period is significantly affected by the base dimension of the building and cannot be ignored. Thus, it is necessary to include the effect of the base dimension in the empirical equation for finding the fundamental period of the building.

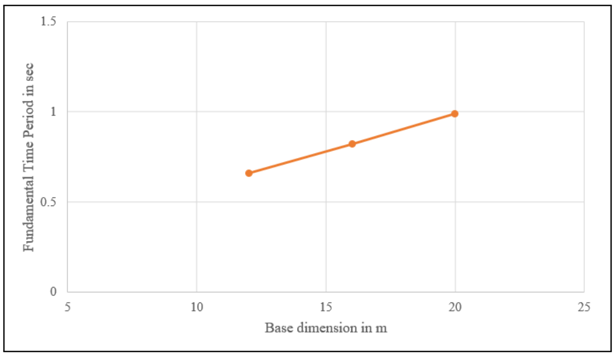

Figure 6: Base dimension versus fundamental time period of 6 storey buildings with 4 bays and 3m storey height

Regression Analysis

After the values of the fundamental time period of the considered building models are obtained from Rayleigh method and the effect of different parameters upon the fundamental period is investigated, regression analysis is performed to obtain an approximate equation for the fundamental time period.

The parameters considered for the regression analysis are height of the building(H), base dimension(D), storey number(N) and bay number(B).

For goodness of fit of the regression model, two statistical parameters, adjusted R^2 (also called as coefficient of determination) and standard error are checked.

First, considering the fundamental time period as the function of height of the building only, regression analysis is done. Based on the code formulae and recommended expressions by different researchers, the proposed expression adopted in this paper is of the form:

$$T = \alpha H^{\beta} \tag{15}$$

in which H is the height of the building in m while α and β are the regression coefficients to be determined. Regression analysis yielded the expression as follows:

$$T = 0.084H^{0.82} \tag{16}$$

with the value of R^2 as 0.819 and standard error of 0.1997.

Again, considering the fundamental time period as the function of storey number(N) only, another regression analysis was performed. The proposed equation is of the form:

$$T = \alpha H^{\beta} \tag{17}$$

in which α and β are the coefficients to be determined. The regression analysis yielded the equation:

$$T = 0.263N^{0.706} \tag{18}$$

with the standard error of 0.2925 and adjusted R^2 of 0.614.

It is noted that the values of standard errors for the period-height relationship as given by equation 16 is lower in comparison to the period-storey relationship as given by equation 18. Similarly, the value of R^2 for the period-height expressions given by equation 16 is larger in comparison to that of equation 18 . This shows that the period-storey relationship fit is not as good as the period-height relationship. Thus, period-height relationship is preferable in comparison to the period-storey relationship.

Regression analysis was again conducted considering fundamental time period as the function of height, base dimension and the number of bays. The proposed expression is of the form

$$T = \alpha H^{\beta} D^{\gamma} B^{\delta} \tag{19}$$

in which H is the height in m, D is the base dimension in m and B is the number of bays. α, β, γ and δ are the regression coefficients. The result of this regression analysis is the equation as given below.

$$T = 0.030H^{0.82} D^{0.766} B^{-0.784} \tag{20}$$

with the value of R^2 as 0.938 and the standard error as 0.1175 showing good fit of data.

5. Conclusion

Although several different parameters affect the fundamental period of a frame RC structure, design codes throughout the world including Nepal prescribe the use of empirical expressions which are mostly the function of building height and in a few cases, the number of storeys. In the present paper, the influence of the height, the bay length, the number of storey and the bay number on the fundamental time period was investigated. Rayleigh method was used for determination of the fundamental time period.

From the present study, the following conclusions can be drawn:

- Height is the most important parameter influencing the fundamental time period. But, besides height, other parameters like the base dimension also influence the fundamental period of the building.
- Keeping the length of the bay constant, the number of bays does not have significant influence on the fundamental period.
- The code-based empirical expressions generally underestimate the value of fundamental time period of moment-resisting frame buildings.

Different empirical equations were also proposed by conducting regression analysis on the selected data by considering time period as the function of different parameters: height, number of storeys, base dimension and the number of bays.

This study was related with the investigation of the influence of only a few parameters, viz. height, storey number, bay number and the bay length on the fundamental time period of a building. There are other several different parameters which affect the fundamental time period of the structure such as presence and position of infills, effect of soft storeys, soil-structure interaction, etc. which can be extensively studied on a larger data set to develop a more reliable equation for the estimation of the fundamental time period of a RC building.

References

[1] Khan Mahmud Amanat and Ekramul Hoque. A rationale for determining the natural period of rc building frames having infill. *Engineering structures*, 28(4):495–502, 2006.

[2] Mehmet Metin Kose. Parameters affecting the fundamental period of rc buildings with infill walls. *Engineering Structures*, 31(1):93–102, 2009.

[3] George D Hatzigeorgiou and George Kanapitsas. Evaluation of fundamental period of low-rise and mid-rise reinforced concrete buildings. *Earthquake Engineering & Structural Dynamics*, 42(11):1599–1616, 2013.

[4] Panagiotis G Asteris, Constantinos C Repapis, Athanasios K Tsaris, Fabio Di Trapani, and Liborio Cavaleri. Parameters affecting the fundamental period of infilled rc frame structures. *Earthquakes and Structures*, 9(5):999–1028, 2015.

[5] Panagiotis G Asteris, Constantinos C Repapis, Emmanouela V Repapi, and Liborio Cavaleri. Fundamental period of infilled reinforced concrete frame structures. *Structure and Infrastructure Engineering*, 13(7):929–941, 2017.

[6] Rakesh K Goel and Anil K Chopra. Period formulas for moment-resisting frame buildings. *Journal of Structural Engineering*, 123(11):1454–1461, 1997.

[7] *Eurocode 8: Design of structures for earthquake resistance*. EUROPEAN COMMITTEE FOR STANDARDIZATION, 2004.

[8] IS1893. *Indian Standard Criteria for Earthquake Resistant Design of Structures-Part I: General Provisions and Buildings(Sixth Revision)*. New Delhi: Bureau of Indian Standards, 2016.

[9] *Building Code of Pakistan Seismic Provisions-2007*. Government of Pakistan Ministry of Housing & Works, 2007.

[10] *Structural design actions Part 5: Earthquake actions-New Zealand NZS 1170.5:2004 Incorporating Amendment No. 1*. Standards New Zealand, PO Box 1473, Wellington 6140, 2016.

[11] NBC105:1994. *Nepal National Building Code. Seismic Design of Buildings in Nepal*. Department of Urban Development and Building Construction, 1994.

[12] ASCE. *Minimum Design Loads and Associated Criteria for Buildings and Other Structures,ASCE/SEI 7-16*. American Society of Civil Engineers, 2017.

[13] *Bangladesh National Building Code(BNBC)*. Housing Building Research Institute and Bangladesh Standards and Testing Institution, 2015.

[14] Marjan Faizian and Yuji Ishiyama. Comparison of seismic codes of 1981 japan (bslj), 2000 usa (ibc) and 1999 iran (ics). In *10th World Conference on Earthquake Engineering*, 2004.

[15] *The National Building Code(NBC) Canada*. National Research Council, 1995.

[16] IS1893:1984. *Indian Standard Criteria for Earthquake Resistant Design of Structures(Fourth Revision)*. New Delhi: Bureau of Indian Standards, 1984.

[17] Li-Ling Hong and Woei-Luen Hwang. Empirical formula for fundamental vibration periods of reinforced concrete buildings in taiwan. *Earthquake engineering & structural dynamics*, 29(3):327–337, 2000.

[18] H Crowley and R Pinho. Period-height relationship for existing european reinforced concrete buildings. *Journal of Earthquake Engineering*, 8(spec01):93–119, 2004.

[19] Kadir Guler, Ercan Yuksel, and Ali Kocak. Estimation of the fundamental vibration period of existing rc buildings in turkey utilizing ambient vibration records. *Journal of Earthquake Engineering*, 12(S2):140–150, 2008.

[20] Magdy I Salama. Estimation of period of vibration for concrete moment-resisting frame buildings. *HBRC Journal*, 11(1):16–21, 2015.

[21] Ray W Clough and Y Penzien. *Dynamics of Structures*. 2003.

[22] Anil K Chopra and Anil K Chopra. *Dynamics of structures: theory and applications to earthquake engineering*, volume 2. Prentice Hall Englewood Cliffs, NJ, 1995.