Performance Analysis of V-BLAST MIMO-OFDM using Transmit and Receive Beamforming

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Abstract: Multiple input Multiple Output (MIMO) systems in conjunction with orthogonal frequency division multiplexing (OFDM) are extensively used in modern communication systems in order to improve throughput and robustness in multipath fading environments. In this paper to enhance the order of spatial diversity in presence of deteriorative fading correlations we proposed Least Mean Square (LMS) and Recursive Least Square (RLS) adaptive Algorithm. This paper uses V-BLAST-ZF detection for MIMO-OFDM receiver. The performance measures used in this paper is BER and SNR. This study shows that the BER performance of MIMO-OFDM system with Beamforming is better than system without using Beamforming. This paper also shows the performance for the adaptive algorithm LMS and RLS, and concludes that RLS algorithm significantly outperforms over the LMS.

Keywords: Beamforming, BER, V-BLAST, LMS, MIMO-OFDM, MMSE, RLS, SNR, ZF

1. Introduction

The new advanced mobile communication systems are expected to provide high quality and high data rate multimedia packet transmission rather than voice-oriented legacy wireless systems. To achieve these innovative features into reality, capability of high spectral efficiency over limited frequency resource with limited transmission power is strongly required. As a challenging technique, the MIMO (Multiple Input Multiple Output) system equipped with sophisticated space-time signal processing algorithms such MMSE (Minimum Mean Squared Error) Zero Forcing (ZF) receiver combined with BLAST architecture [1].

Further in order to increase spectral efficiency and to effectively overcome deterioting effects of ISI (Inter Symbol Interference) and frequency Selective Fading by multipath, MIMO can be combined with OFDM (Orthogonal Frequency Division Multiplexing) system. OFDM is a multicarrier transmission technique that has been recently recognized as an excellent method for high speed bi-directional wireless data communication and its signal offers an advantage in a channel that has a frequency selective fading.

Recently Beamformers have drawn much attraction because of its capability of de-correlating spatially distinct channels. It has been already revealed that the Beamformers results in plausible performance under correlated MIMO channels provided that the average channel state information is available on the transmitter. Here in this paper we analyze the performance of Beamforming based MIMO-OFDM system with V-BLAST detection algorithm which uses Zero Forcing (ZF) and Minimum Mean Squared Error (MMSE) nulling technique to measure BER Versus SNR for Least Mean Square (LMS) and Recursive Least Square (RLS) adaptive Beamforming algorithms. Finally we compare the result with the system that does not exploit the Beamforming in MIMO-OFDM [1, 2].

2. Theoretical Background

a) *Multiple Input Multiple Output (MIMO)*

MIMO can be categorized into three different areas: precoding, spatial multiplexing, and diversity coding. Precoding refers to the spatial processing that is performed at the transmitter (also known as Beamforming). The goal of spatial processing is to reduce the effect of multipath fading from constructive interference of the signals being transmitted. Multipath is a propagation phenomenon that is characterized by the arrival of multiple versions of the same signal from different locations shifted in time due to having taken different transmission paths of varying lengths Precoding is used when a receiver consists of more than one antenna and Beamforming cannot maximize the signal level across all receiving antennas. In addition, precoding requires that the transmitter has knowledge of the CSI. Spatial multiplexing gain refers to using the degrees of freedom in a communication system by sending independent symbols in parallel over multiple spatial channels. This technique is a powerful one for increasing channel capacity for higher SNRs and can be used in either a closed loop system or an open loop system. Diversity coding requires a single stream of data to be encoded with a space-time code prior to transmission. Space-time codes create orthogonality among the data being sent to the receiver. No CSI is required to be known at the transmitter to take advantage of this mechanism of MIMO. The diversity order of a MIMO transmission system over an independent and identically distributed (i.i.d.) Rayleigh channel with n_t transmit antennas and n_r receive antennas at high SNR is given by:

$$n_r - n_t + 1$$

Thus, the diversity order can be improved by increasing the number of receive antennas and can be degraded by increasing the number of transmit antennas. However, increasing the number of transmit antennas increases spatial multiplexing gain so there is generally a trade of made in the design process. There are different architectures used for MIMO communications that can be used when trying to take advantage of this technology. Bell Labs Layered Space Time Architecture (BLAST), Per Antenna Rate Control (PARC), and Selective per Antenna Rate Control (SPARC) are among the most notable. Spatial multiplexing causes the complexity of receiver to drastically increase and as a result techniques such as OFDM are used to handle the multipath channel. The basic MIMO channel model is shown in below [3].

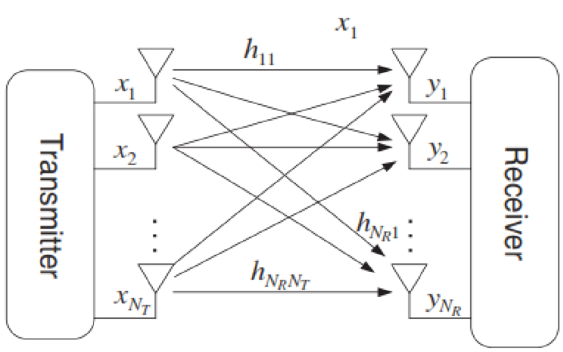

Figure 1: MIMO Channel Model

The transmitter sends several streams of data from multiple antennas. The number of possible paths that the channel will allow is n_t n_r where n is the number of transmit antennas and n r tis the number of receiver antennas. It is the receiver's job to decode the information received and output the original data set. A MIMO system can be modeled

$$y = \boldsymbol{H}x + n \quad (1)$$

Where y is the received vector, x is the transmitted vector, n is the noise vector, and H is the channel matrix.

b) *Orthogonal Frequency Division Multiplexing (OFDM)*

Frequency Division Multiplexing (FDM) uses multiple subcarriers within the same channel. The total data rate sent across the channel is divided among these subcarriers. The main advantage of FDM over single carrier modulation is that narrowband frequency interference will only affect particular subcarriers rather than the entire carrier. Due to the lowered information rate on each subcarrier, the symbol periods will be larger and this will add further protection against noise and reflections. FDM usually requires guard intervals between subcarriers to combat Inter Carrier Interference (ICI). Orthogonal Frequency Division Multiplexing (OFDM) requires all subcarrier frequencies to be orthogonal to one another. In other words, from tone to tone there is no cross over, or ICI and they can be considered uncorrelated from one another. This creates a higher level of spectral efficiency as there is no need for a guard interval. Each of these subcarriers will contain data that is being sent from the transmitter. If the transmitter and receiver are not frequency synchronized, the received tones will no longer be orthogonal to one another and ICI will result. In the case of a multipath environment, this synchronization is even more of a concern as reflections will appear at different frequency offsets. A Frequency selective channel is considered to be constant over each OFDM subcarrier if each tone is sufficiently narrow banded. Effectively, OFDM converts a frequency selective channel into a series of at fading sub-channels. Fig. illustrates the separation of one carrier into multiple subcarriers.

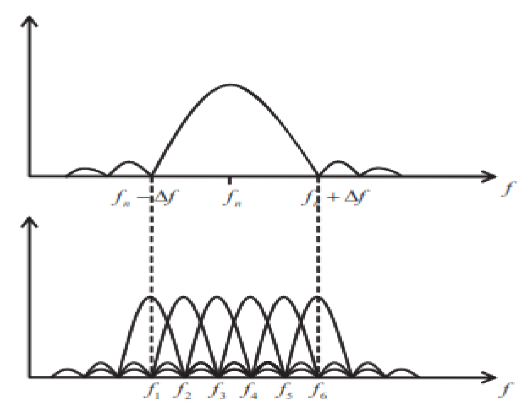

Figure 2: OFDM Subcarrier Splice

The interference of subcarriers on one another is zero if the subcarrier frequencies are perfectly orthogonal. As shown in Fig, the side lobes of each subcarrier cancel to zero at the peaks of all other subcarriers and do not cause ICI. Fig illustrates a simple block diagram of an OFDM system. An OFDM symbol is the sum of a number of orthogonal subcarriers. Each sub-carrier contains its own baseband data. In general, the transmitter and receiver architectures of a system utilizing OFDM use an Inverse Fast Fourier Transform (IFFT) and Fast Fourier Transform (FFT) block, respectively, to convert the frequency tones into a time domain signal to be sent over the air or vice versa. At the transmitter, this transform maps the input signal to a set of orthogonal subcarriers, or basis functions, of the FFT. At the receiver, the signals are then combined to form an estimate of the originally transmitted signal. Since the basic functions of the FFT block are uncorrelated, the correlation calculation performed in the FFT block will only see the energy for that particular subcarrier. The input to the IFFT block maps each bit of data to a set of orthogonal sinusoidal signals and these sinusoidal signals are summed together to be sent over the channel. The FFT block accepts this sum of sinusoidal and separates the signal into its orthogonal subset of frequency domain bits. Each of these bits corresponds to a particular modulated signal mapping such as BPSK, QPSK, 16-QAM or 64-QAM [4].

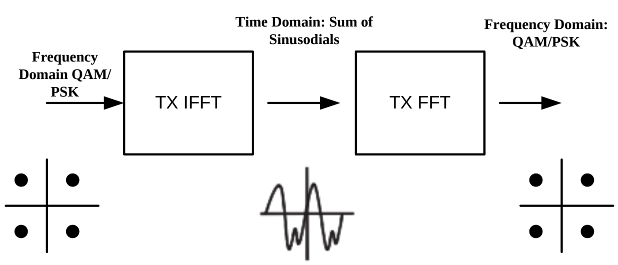

Figure 3:OFDM System Model

Since it has been observed that wideband wireless channels are frequency selective in nature, OFDM can be used in a MIMO system to take advantage of this. MIMO takes advantage of the spatial and temporal dimensions of transmission and adding OFDM into this framework includes the frequency dimension. The general structure for a MIMO OFDM transmitter and receiver is outlined in. As previously explained, a multipath channel is one in which multiple versions of the same transmitted signal arrive at the receiver delayed in time from one another. This type of channel will pose two main problems for a system using OFDM, Inter Symbol Interference (ISI) and ICI. In single carrier systems the ISI is only due to the previous symbol, however, in a multipath environment the ISI can be due to several other symbols. ICI is the interference experienced by an OFDM symbols subcarriers. To minimize ISI, a guard interval of sufficient length is generally employed between OFDM symbols; however, a guard interval does not prevent ICI from occurring. To combat ICI, the guard interval is replaced by a cyclic prefix of the last L samples of the OFDM symbol. In effect, the cyclic prefix behaves much like a guard interval in preventing ISI and similarly makes the OFDM symbol appear periodic to the receiver which assists in reducing ICI in the convolution operation of the FFT block in the receiver [5].

c) V-BLAST

It has been shown that if a wireless channel is rich in multipath scattering it is capable of producing large capacities with a relatively low number of bit errors. Various techniques have been developed to take advantage of this property including BLAST. This architecture takes advantage of Space Division Multiplexing (SDM) or Space Division Multiple Access (SDMA). SDM is inherent to a MIMO system because multiple antennas are being used to transmit data across the wireless channel at the same frequency. To ensure error-free decoding is possible multiple receive antennas are required, again, an inherent property of a MIMO system. There are two common encoding methods for MIMO spatial multiplexing, horizontal and vertical encoding. In horizontal encoding each data stream is independently encoded and transmitted by different antennas. Vertical encoding uses a single encoder to spread information across all antennas. V-BLAST utilizes the horizontal encoding method. The term “vertical", as mentioned, does not refer to the encoding method used but it refers to the method in which the detection at the receiver is performed. The receivers, due to the effect of multipath, will receive the signals radiated from all n_i transmit antennas. In comparison to other multiple access techniques, such as Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA), and Code Division Multiple Access (CDMA), V-BLAST ensures that the entire bandwidth of the system is used all the time by all transmitting antennas, each transmitted signal occupies the entire bandwidth, and the total bandwidth utilized is only a small fraction in excess of the symbol rate. To achieve a sufficient level of de-correlation at the receiver, V-BLAST requires its operation to be conducted in a multipath environment. These conditions allow the receiver to distinguish signals occupying the same channel space. V-BLAST and OFDM can be combined in frequency selective fading channels to achieve a high rate of data transmission [6].

The research in detection algorithms for MIMO systems have focused on attempting to achieve optimal error performance while remaining practical for implementation purposes. Various decoding algorithms have been proposed for MIMO systems such as ZF, MMSE and ZF-SIC. The general premise for each decoding method is to develop a means to counter the effects of the channel given that the received signal can contain delayed versions of the same signal due to multipath. Optimizing this detection process with real measurements of the CSI can greatly reduce the complexity of decoding which OFDM introduces into a MIMO system.

i) V-BLAST-ZF

Zero forcing receivers are a simple linear receiver with low computational complexity. It minimizes interference but suffers from noise enhancement's receiver works best with high SNR level. Zero Forcing implements matrix (Pseudo) inverse (+). The ZF estimated receive signal is given by:

$$\hat{X} = (H^H)^{-1} H^H .X = H^+ X$$

Where the zero forcing decoding matrix is as follows:

$$S_{ZF} = (H^H H)^{-1} H^H \quad (2)$$

Where superscript H denotes Hermitian transpose [1].

ii) V-BLAST MMSE

At very high SNR level decorrelator completely suppress the interference, therefore it provide better performance at higher SNR level. Now in low SNR level condition the maximal ratio combining receiver provide better performance. Therefore in order to design an optimal receiver it is necessary to converge these two advantages in a single receiver. In MMSE receiver this two features are optimally combined. MMSE receiver is another type of linear detector which minimizes the mea squared error between the transmitted symbols. MMSE detector helps to jointly minimized both the noise and interference or we can say that the MMSE detector seeks to balance between cancellation of the interference and reduction of noise enhancement. Therefore MMSE detector outperforms the ZF detector in the presence of noise. MMSE receiver gives a solution of:

$$\hat{X} = (\frac{1}{SNR} I + H^H H)^{-1.} H^H X \quad (3)$$

The above linear equalization algorithm is based on multiplying the received vector by a detection matrix and then decoding the symbols separately. Another approach in VBLAST receiver design is successive interference cancellation to achieve better performance at the cost of much higher complexity [2, 6].

d) Beamforming

Beamforming or spatial filtering is a signal processing technique used in sensor arrays for directional signal transmission or reception. This is achieved by combining elements in a phased array in such a way that signals at particular angles experience constructive interference while others experience destructive interference. Beamforming can be used at both the transmitting and receiving ends in order to achieve spatial selectivity [7].

The improvement compared with omnidirectional reception/transmission is known as the receive/transmit gain (or loss).Beamforming can be used for radio or sound waves. It has found numerous applications in radar, sonar, seismology, wireless communications, radio astronomy, acoustics, and biomedicine. Adaptive beamforming is used to detect and estimate the signal-of-interest at the output of a sensor array by means of optimal (e.g., least-squares) spatial filtering and interference rejection [7, 8].

Beamforming techniques can be broadly divided into two categories:

a) conventional (fixed or switched beam) Beamformers

b) adaptive Beamformers or phased array

Conventional Beamformers use a fixed set of weightings and time-delays (or phases) to combine the signals from the sensors in the array, primarily using only information about the location of the sensors in space and the wave directions of interest. In contrast, adaptive Beamforming techniques generally combine this information with properties of the signals actually received by the array, typically to improve rejection of unwanted signals from other directions. This process may be carried out in either the time or the frequency domain. Adaptive Beamforming is LMS style Beamforming which used pilot symbol as to get feedback to the weight vector block to update the weight vector co-efficient adaptively to get more perfect result by mitigating co-channel interference. In null steering Beamforming, the main signal beam is steered according to receiver antenna position. Some of frequently used adaptive Beamforming techniques are follows:

i) Least Mean Square (LMS)

The Least Mean Square (LMS) algorithm is an adaptive algorithm, which uses a gradient-based method. It uses the estimates of the gradient vector from the available data. It incorporates an iterative procedure that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error compared to other algorithms. LMS algorithm is relatively simple it does not require correlation function calculation nor does it require matrix inversions. Our proposed scheme is based on the Beamforming technology that tries to eliminate the interference of signals in a particular direction. Steepest Descent is one of the leading algorithms used for Beamforming. The adaptive implementation of this algorithm can be done by LMS algorithm. This is one the simplest and easiest algorithm for implementation. But it lack fast convergence property. In LMS algorithm the correlation matrix and cross correlation matrix is not used instead using their instantaneous values respectively. In order to train the adaptive weights of the Beamformer requires training sequence of known symbols d_n

The LMS algorithm can be summarized as follows.

$$W_{n+1} = W_n - \mu(x_n x_n^H W_n - x_n d_n)$$

$$W_{n+1} = W_n - \mu x_n (x_n^H W_n - d_n)$$

$$W_{n+1} = W_n - \mu x_n e_n \qquad (4)$$

Where $e_n = x_n^H W_n - d_n$ represents the error signal for the nth training step. The convergence of the LMS algorithm can be determined by the factor μ, step size and can be keep as small as possible to get better convergence [9].

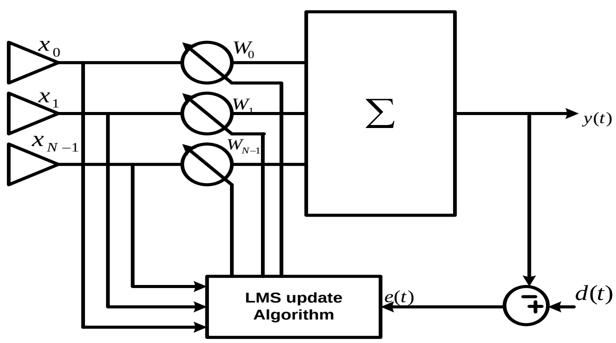

Figure 4: Adaptive Beamforming System

ii) Recursive Least Square (RLS)

The capacity of the RLS can improve due its faster convergence. This advantage is based on the factor that the error at any point of time is independent of the statistical properties of the signal. The algorithm updates the autocorrelation matrix for the next instant with the aid of the autocorrelation matrix calculated for the present instant. The main drawback of the RLS algorithm is that it suffers from computational complexity. The RLS weight update equation can be written as

$$W(n+1) = W(n) + \alpha(n) * h(n) \qquad (5)$$

The RLS algorithm is used to reduce the error using the following equation:

$$E(p) = \sum_{K=0}^{P} \lambda^{p-k} |e(p)|^2 = \sum_{K=0}^{P} \lambda^{p-k} \left| d(k) - W_p^H x_k \right|^2 \qquad (6)$$

Here λ is a constant term having value $\lambda \leq 1$ and so the error term is exponentially weighted toward the recent data samples. The reason for the fast convergence of the RLS is due to the fact that that RLS technique whitens the input data by using the inverse correlation matrix of the data. Despite the initial presentation of the weight update includes the correlation matrix inversion, recursive computations can be employed to update the value of the correlation matrix of the input step by step [9, 10].

3. System Model

First, generate a binary data stream in random manner. This generated data stream is convolutional encoded so that at the receiver side it can be decoded with less bit error rate. The encoded bits are modulated by QPSK/16 QAM then preprocess for the OFDM. First the modulated data are converted into parallel data stream and then added a cyclic prefix so that the Inter-symbol Interference (ISI) can be minimized between OFDM symbols also the Guard Interval is also created so that the Inter Carrier Interference (ICI) between OFDM subcarrier is minimized. Now it is converted into time domain by using Inverse Fast Fourier Transform (IFFT) since the channel can detect the time domain signal. Now the time domain signal are passed through a Eigen Beamforming which create a steering signal to the desired direction which makes a signal with more directive in desired direction. The signals are then passed through a AWGN with Rayleigh fading channel. At the receiver side, the antennas with beamformer processed signal are passed to Fast Fourier Transform (FFT) and then remove cyclic prefix and then passed to the V-BLAST detector with ZF with channel estimate of RLS and LMS. The detected signal is passed to the parallel to serial converter and demodulated. Finally, signal are decoded by Viterbi decoder and compare the bit that are transmitted and bit that are received. The proposed system model is shown in figure below:

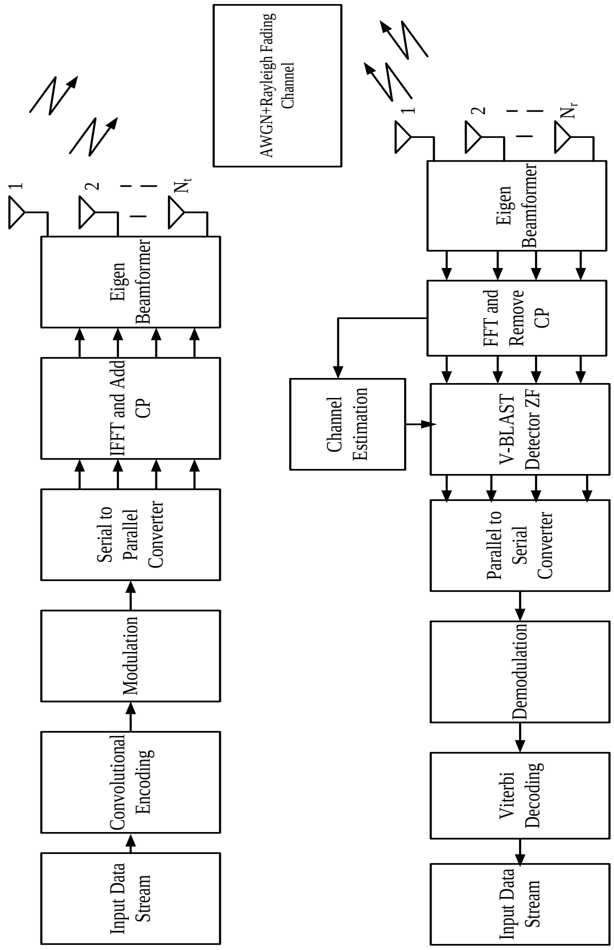

Figure 5: Block diagram of Proposed System

5. Simulation Parameters

Table 1: Simulation Parameters Used

Parameters	Specification
Number of Iteration	10^5
FFT Size	64
Guard Interval	¼
Number of Carrier	64
Signal Constellation	QPSK and 16QAM
Channel Model	Rayleigh with
Pilot Type	Block
Encoding	Convolutional
Decoding	Viterbi
BLAST Architecture	Vertical-BLAST
V-BLAST Nulling	ZF
Beamforming Type	Adaptive
Beamforming	LMS and RLS

4. Results and Discussions

The plot is in between BER and SNR and hence compared the various adaptive Beamforming algorithms with different spatial degree of MIMO. The plots are simulated in Rayleigh fading channel with V-BLAST-ZF detection for MIMO-OFDM.

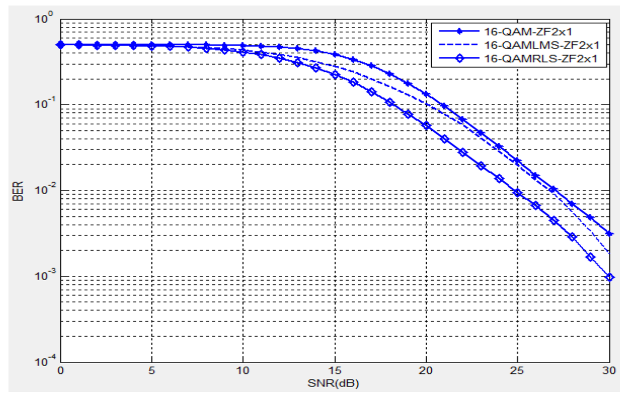

Figure 6: SNR vs. BER for V-BLAST -ZF MIMO-OFDM with (RLS & LMS) and Without Beamforming for 16QAM using 2x1 Antenna

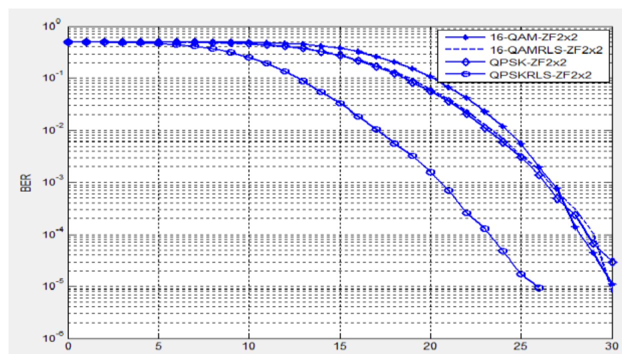

Figure 7: SNR vs. BER for V-BLAST-ZF MIMO-OFDM With (RLS & LMS) and Without Beamforming for 16QAM and QPSK Modulation using 2X2 Antenna

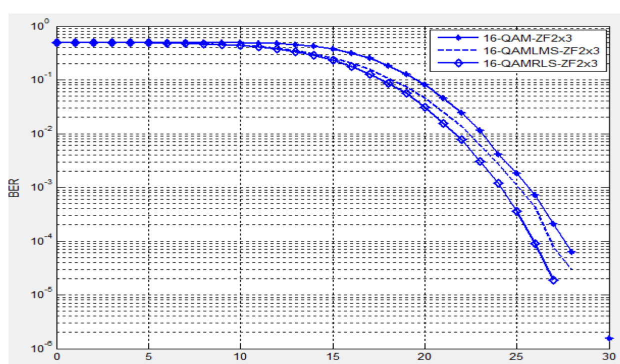

Figure 8: SNR vs. BER for V-BLAST-ZF MIMO-OFDM for with (RLS & LMS) and Without Beamforming for 16QAM using 2X3 Antenna

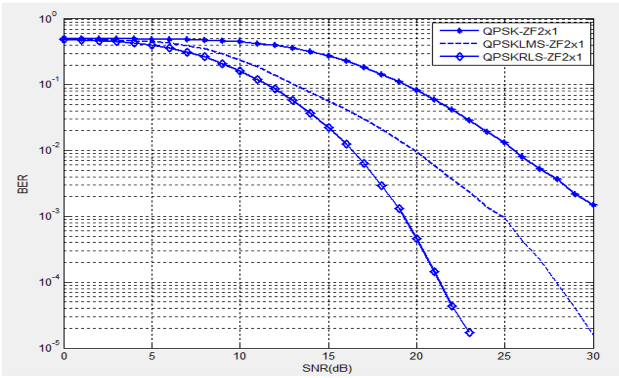

Figure 9: SNR vs. BER for V-BLAST-ZF MIMO-OFDM for with (RLS & LMS) and Without Beamforming for QPSK using 2X1 Antenna

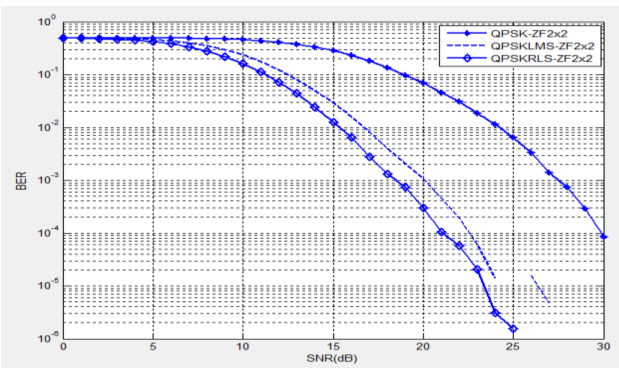

Figure 10: SNR vs. BER for V-BLAST-ZF MIMO-OFDM for with (RLS & LMS) and Without Beamforming for QPSK using 2X2 Antenna

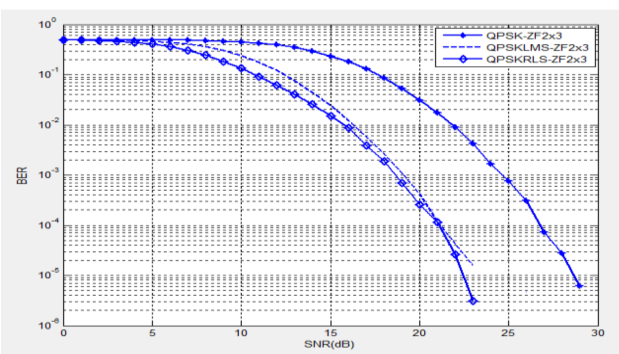

Figure 11: SNR vs. BER for V-BLAST-ZF MIMO-OFDM for with (RLS & LMS) and Without Beamforming for QPSK using 2X3 Antenna

Figure 6 shows the BER Analysis for V-BLAST MIMO-OFDM for 16QAM with and without Beamforming using 2X1 Antenna. This show, BER for lower SNR values are same for both with Beamforming and without Beamforming but for higher SNR value Beamforming produces pretty good BER results than without Beamforming. For BER value of 10^{-2}, SNR value drops 3dB than that of system without using Beamforming.

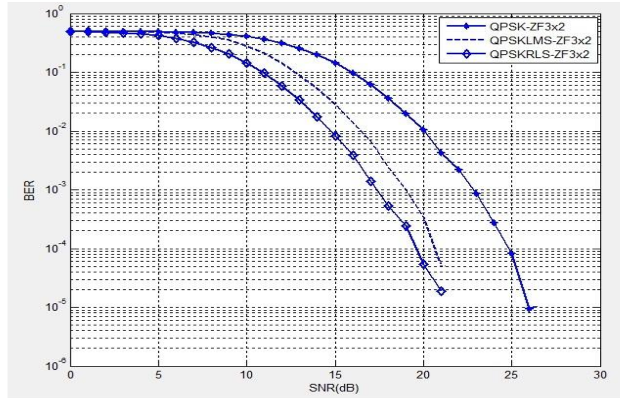

Figure 12: SNR vs. BER for V-BLAST-ZF MIMO-OFDM for with (RLS & LMS) and Without Beamforming for QPSK using 3X2 Antenna

Figure 7 shows BER performance of proposed system significantly outperform for RLS algorithm. For higher value of SNR, the BER performance will increases from 10^{-1} to10^{-2} for SNR value of 20dB. Figure 8 shows the BER Analysis of V-BLAST MIMO-OFDM for 16QAM with and without Beamforming using 2X3 Antenna. This show, BER performance is good for system with using Beamforming. Figure 9 shows BER performance for RLS algorithm significantly outperforms over the V-BLAST MIMO-OFDM without using Beamforming. Approximately 10dB of SNR value is dropped for BER of 10^{-2} with using RLS algorithm but for LMS algorithm only 5dB of SNR value is dropped for BER of 10^{-2}. Figure 10 shows the BER Analysis of V-BLAST MIMO-OFDM for QPSK with and without using Beamforming using 2X2 Antenna. This show, BER performance for RLS algorithm significantly outperforms over the system without using Beamforming. Approximately 10dB of SNR value is dropped for BER of 10^{-3} with RLS algorithm. Figure 11 shows the BER Analysis of V-BLAST MIMO-OFDM for QPSK with and without Beamforming using 2X3 Antenna. This show, BER performance for RLS algorithm significantly outperforms over the system without using Beamforming. Approximately 12dB of SNR value is dropped for BER of 10^{-3} with RLS algorithm. Figure 12 shows the BER Analysis of V-BLAST MIMO-OFDM for QPSK with and without Beamforming using 3X2 Antenna. This show, BER performance for RLS is better than system performance without using Beamforming

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6. Comparison Table

Table 2: BER for different SNR with and without Beamforming for QPSK Modulation

SNR in dB	BER (QPSK) For 2x2		
	Without Beamforming	With LMS	With RLS
5	0.4962	0.4649	0.4144
10	0.4558	0.2538	0.1337
15	0.2526	0.0278	0.0083
20	0.0388	3.75×10^{-4}	1.1875×10^{-4}
25	9.0938×10^{-4}	0	0
30	6.25×10^{-6}	0	0

Table 3: BER for different SNR with and without Beamforming for 16QAM modulation

SNR in dB	BER (16 QAM) For 2x2		
	Without Beamforming	With LMS	With RLS
5	0.4988	0.4898	0.4876
10	0.4875	0.4361	0.4250
15	0.3447	0.2547	0.2241
20	0.0774	0.0563	0.0360
25	0.0022	0.0042	9.7188×10^{-4}
30	4.6875×10^{-6}	2.656×10^{-5}	0

Table 4: Range for SNR required for BER 10^{-3} for QPSK and 16 QAM modulations with and without Beamforming

QPSK			16 QAM		
Without Beam-forming	With LMS	With RLS	Without Beam-forming	With LMS	With RLS
27dB	19dB	17dB	26dB	25dB	24dB

7. Conclusions

It is observed that BER performance of Beamforming algorithm shows good result over the system without using Beamforming. Among LMS and RLS adaptive algorithm RLS performs better results over the LMS however with slow convergence while running in MATLAB. The BER performance is good for QPSK modulation than 16 QAM and finally as the number transmitter and receiver increases the BER performance of the system is also increases. In conclusion, we can enhance the BER performance of the MIMO-OFDM system approximately 10dB by use of Beamforming.

References

[1] Anubhuti Khare, Manish Saxena and Vijendra Singh Mandloi, "Performance Analysis of V-BLAST Based MIMO-OFDM System with Various Detection Techniques", International Journal of Engineering and Advanced Technology, Vol.1, Issue-2, December 2011.

[2] Shreedhar A Joshi, Dr.Rukmini T S, Dr. Mahesh H, "Analysis of V-BLAST Techniques for MIMO Wireless channels with different modulation techniques using Linear and Non Linear Detection", International Journal of Computer Science, Vol.1,Issue 1, November 2011.

[3] Dhruv Malik and Deepak Batra," Comparison of Various Detection Algorithms in a MIMO Wireless Communication Receiver", International Journal of Electronics and Computer Science Engineering, Vol. 1, No. 3, Page 1678-1685

[4] Suchitra Varade and Kishore Kulat, "BER Comparison of Rayleigh Fading, Rician Fading and AWGN Channel using Chaotic Communication based MIMO-OFDM System", International Journal of Soft Computing and Engineering, Vol. 1, Issue 6, January 2012.

[5] Najoua Achoura and Ridha Bouallegue, "Channel Estimation for MIMO-OFDM beamforming systems", IJCSNS,Vol.8,No.3,March 2008

[6] Gurpreet Singh, Priyanka Mishra and Rahul Vij, "BER Analysis of V-BLAST MIMO Systems under Various Channel Modulation Techniques in Mobile Radio Channels", International Conference on Computer Technology and Science, Vol. 47, 2012

[7] Zijian Wang, Wen Chen and Jun Li, "Efficient Beamforming for MIMO Relaying Broadcast channel with Imperfect Channel Estimation", IEEE Transections on Technology,Vol. 61,No.1,January 2012

[8] Bishwarup Mondal and Robert W. Health, "Performance Analysis of Quantized Beamfoming MIMO Systems", IEEE Transections on Signal Processing, Vol.54, No.12,December 2006

[9] Syed. Shameem and Dr. Habibullah Khan, "Performance Comparison of LMS, SMI and RLS Adaptive Beamforming Algorithms for Smart Antennas", International Journal of Computer Science and Telecommunication, Vol.3, Issue 2, April-June 2012.

[10] Ravi Kumar and Rajiv Saxena, "Capacity Analysis of MIMO Spatial Channel Model using Novel Adaptive Semi Blind Estimation Scheme", Journal of Wireless Networking and Communications Vol.2, Issue 4, Page 66-74, 2012