Anisotropic Damage Model for Concrete Subjected to Tension-Tension Fatigue Loading

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Abstract: Stiffness degradation and inelastic deformation are the essential features of concrete that develop due to the formation of multitude of microcracks in the fatigue environment. Microcracking, which is anisotropic in nature, destroys the bond between material grains, and affects the elastic properties resulting in the reduction of material stiffness. This paper presents an anisotropic fatigue damage model for plain concrete subjected to cyclic tension. The model is developed, in strain space, using the general framework of internal variable theory of continuum thermodynamics. It is argued that within the damage surface of given strain states the unloading-reloading cycles (fatigue loading) stimulate the nucleation and growth microcracks in concrete, which will result in stiffness degradation and inelastic deformation, and hence material is termed as damaged. Damage is reflected through the fourth-order stiffness tensor involving a damage parameter whose increment is governed by the consistency equation associated with a cycle dependent damage surface in strain space. The model is capable of predicting stiffness degradation, inelastic deformation and strength reduction under fatigue loading and compared against experimental result.

Keywords: Fatigue; anisotropic; damage; Concrete; Thermodynamics; stiffness; microcracks.

1. Introduction

Reinforced concrete structures such as bridges, hydraulic foundations, pressure vessels, crane beams are subjected to long term cyclic loading. The effect of cyclic loading is to develop permanent damage in the concrete materials as a result of which failure happens under the stress having value less than the ultimate strength of concrete. Concrete, a heterogeneous material comprising the mixture of cement, sand and aggregate, exhibits several mutually interacting inelastic mechanisms such as microcrack growth and inelastic flow even under small amplitude of cyclic load when applied in large number of cycles. As a consequence, concrete does not guarantee endurance fatigue limit like metal as described in Miner's hypothesis [1].

The presence of permanent damage at fatigue failure has been documented by a number of investigations. [2] developed fatigue damage model for ordinary concrete subjected to cyclic compression based on mechanics of composite materials utilizing the concept of dual nature of fatigue damage, which are cycle dependent and time dependent damage. The model was capable of capturing the cyclic behavior of plain concrete due to progressive creep strain with the increase in number loading cycles. [3] used accelerated pavement testing results for carrying out cumulative fatigue damage analysis of concrete pavement. In [3], they reported that Miner hypothesis does not accurately predict cumulative fatigue damage in concrete. The experimental work of [4] clearly showed that increase of damage in the material takes place in about last 20% of its fatigue life. [5] presented a theoretical model to predict the fatigue process of concrete in alternate tension-compression fatigue loading using double bounding surface approach described in strain-energy release rate by constructing the damage-effective tensor.

In the past few years, a number of damage constitutive models have been published to model the observed mechanical behavior of concrete under monotonic and cyclic loading ([6], [7], [8], [9], and [10]). The need for such models arises from the physical observation that two dominant microstructural patterns of deformation in concrete are inelastic flow and microcracking. The inelastic flow component of deformations is modeled by using plasticity theories whereas the nucleation and propagation of microcracks and microvoids is incorporated in the constitutive models with the use of damage mechanics theories. The progressive development of cracks and microcracks alters the elastic properties (degradation of elastic moduli) due to energy dissipation and concrete material becomes more compliant.

This paper presents a class of damage mechanics theory to model the fatigue damage and failure of concrete caused by multitude of cracks and microcracks whereby anisotropic damaging behavior is captured through the use of proper response function involving damage parameter in material stiffness tensor. The increment of damage parameter is obtained from consistency equation in cycle dependent damage surface in strain space. The model is also capable of capturing the inelastic deformations that may arise due to misfits of crack surfaces and development of sizable crack tip process zone.

2. Formulation

In this paper, it is assumed that a continuum damage mechanics approach can be taken to describe the constitutive relation for concrete and that the fatigue loading is of low frequency so that the thermal effects could be ignored. For isothermal process, rate independent behavior and small deformations, the Helmholtz Free Energy (HFE) per unit volume can be deduced from [11] and shown as follows:

$$\mathrm{A}(\boldsymbol{\varepsilon},\mathrm{k}) = \tfrac{1}{2}\boldsymbol{\varepsilon} : \mathbf{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^{\mathbf{i}} : \boldsymbol{\varepsilon} + \mathrm{A}^{\mathrm{i}}(k) \qquad .....(1)$$

(1)

Where **E**(k) represents fourth order elastic stiffness tensor which depends on the state of microcracking (damage), $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{\sigma}^{\mathbf{i}}$ denotes the stress tensor corresponding to inelastic damage. The term $A^i(k)$ represents surface energy of microcracks [12], and k is the cumulative fatigue damage parameter. The colon (**:**) represents the tensor contraction operation.

For an inelastic damaging process, a constitutive relation between the stress and strain tensors can be established utilizing fourth order material's stiffness tensor as

$$\boldsymbol{\sigma} = \frac{\partial A}{\partial \boldsymbol{\varepsilon}} = \mathbf{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^{i}(k) \qquad .....(2)$$

The rate form of Eqn (2) with respect to cyclic number N is given by

$$\begin{aligned}\dot{\boldsymbol{\sigma}} &= \mathbf{E}(k) : \dot{\boldsymbol{\varepsilon}} + \dot{\mathbf{E}}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^{i}(k) \\ &= \dot{\boldsymbol{\sigma}}^{\mathbf{e}} + \dot{\boldsymbol{\sigma}}^{\mathbf{D}}(k) + \dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)\end{aligned} \qquad .....(3)$$

Where $\dot{\boldsymbol{\sigma}}^{\mathbf{e}}$ is the stress increment in the absence of further damage in the material**,** $\dot{\boldsymbol{\sigma}}^{\mathbf{D}}$ is the rate of stress-relaxation due to further microcracking (elastic damage), and $\dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)$ designates the rate of stress tensor corresponding to irrecoverable or permanent deformation due to microcracking.

It is further assumed that damage during fatigue loading alters elastic properties and affects the stiffness tensor. For small deformation, the following decomposition of the fourth-order stiffness tensor, E, is adopted

$$\frac{\partial^2 A}{\partial \boldsymbol{\varepsilon} \partial \boldsymbol{\varepsilon}} = \mathbf{E}(k) = \mathbf{E}^{\mathbf{0}} + \mathbf{E}^{\mathbf{D}}(k) \qquad .....(4)$$

Where $\mathbf{E}^{\mathbf{0}}$ is stiffness of the undamaged or virgin material and $\mathbf{E}^{\mathbf{D}}(k)$ denotes the overall stiffness degradation caused by damage during fatigue loadings. Further, $\dot{\mathbf{E}}(k)$ and $\dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)$ are expressed as fluxes in the thermodynamic state sense and are expressed in terms of evolutionary equations as

$$\dot{\mathbf{E}}^{\mathbf{D}} = -\dot{\mathrm{k}}\mathbf{L} \quad \text{and} \quad \dot{\boldsymbol{\sigma}}^{i} = \dot{\mathrm{k}}\mathbf{M} \qquad .....(5)$$

Where **L** and **M** are, respectively, fourth and second order response tensors that determine the directions of the elastic and inelastic damage processes. Following the Clausius-Duhem inequality, utilizing the standard thermodynamic arguments [13] and assuming that the unloading is an elastic process, a potential function

$$\Psi(\boldsymbol{\varepsilon},k) = \tfrac{1}{2}\boldsymbol{\varepsilon} : \mathbf{L} : \boldsymbol{\varepsilon} - \mathbf{M} : \boldsymbol{\varepsilon} - \tfrac{1}{2}\, p^2(\boldsymbol{\varepsilon},k) = 0 \qquad .....(6)$$

is obtained as damage surface which establishes the onset of material inelasticity and stiffness deterioration. In Eqn (6), $p(\boldsymbol{\varepsilon},k)$ is interpreted as the damage function given below as

$$p^2(\boldsymbol{\varepsilon},k) = 2\left[h^2(\boldsymbol{\varepsilon},k) + \frac{\partial A^i}{\partial k}\right] \qquad .....(7)$$

for some scalar valued function $h^2(\boldsymbol{\varepsilon},k)$. It should be noted that as long as the function $p^2(\boldsymbol{\varepsilon},k)$ is defined, the individual terms on the R.H.S of Eqn (7) need not to be identified.

To progress further, specific forms of response tensors **L** and **M** must be specified. Since damage is highly directional, the response tensors should be formulated to address such directionality. In determining a proper form for the response tensor, the strain tensor is decomposed into positive and negative cones. The positive and negative cones of the strain tensor hold the corresponding positive and negative eigenvalue of the system, that is, $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{+} + \boldsymbol{\varepsilon}^{-}$. Here, $\boldsymbol{\varepsilon}^{+}$ and $\boldsymbol{\varepsilon}^{-}$ represent the positive and negative cones of the strain tensor, respectively. Based on the experimental observations and results for concrete materials, damage is assumed to occur in the cleavage mode of cracking as shown schematically in Figure 1.

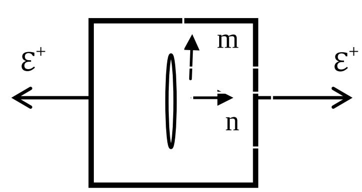


Figure 1. Schematic representation of mode I damage (crack opening in tensile mode) under cyclic tension.

For cleavage cracking mode, the following forms of response tensors are postulated for **L** and **M**

$$\mathbf{L} = \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} \qquad .....(8)$$

$$\mathbf{M} = \beta \boldsymbol{\varepsilon}^{+} \qquad .....(9)$$

The substitution of response tensors **L** and **M** from Eqns (8) and (9) into Eqn (6) leads to the final form of the damage surface

$$\Psi(\boldsymbol{\varepsilon}, k) = \tfrac{1}{2}\boldsymbol{\varepsilon} : \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} : \boldsymbol{\varepsilon} - \beta \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon} - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0 \qquad ....(10a)$$

For uniaxial tensile loading the damage surface of Eqn (10a) is rewritten as

$$\Psi(\boldsymbol{\varepsilon}, k) = \tfrac{1}{2}\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+} - \beta\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+} - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0$$

$$= \tfrac{1}{2}\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}(1-2\beta) - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0 \quad ...(10b)$$

The damage function p(k) obtained from uniaxial tensile test for concrete materials based on the experimental results of [14] is given in [15] as

$$\mathrm{p}(k) = \varepsilon_{\mathrm{u}} ln\left(\frac{\mathrm{E}^{0}}{\mathrm{E}^{0} - \mathrm{k}}\right) \qquad .....(11)$$

And for elastic damaging process, ($\beta = 0$), the limit damage surface reduces to

$$\mathrm{p}(k) = \varepsilon_{\mathrm{u}} \qquad .....(12)$$

Where ε_{u} represents the strain corresponding to uniaxial tensile strength of concrete, which is used as the reference strain and hence the result of a conventional uniaxial tensile test is needed to establish ε_{u}.

3. Fatigue Damage Model

Fatigue loading is understood as the repeated or fluctuating strains acting in a material due to which progressive permanent structural change occurs in the form of cracks or flaws and the material fails at stresses having a maximum value less than the tensile strength of the material. In this paper, it is assumed that within the damage surface of the given strain state, the unloading-reloading cycles (Fatigue loadings) increase damage in concrete due to the growth of microcracks leading to inelastic deformations and stiffness degradation, which eventually reduces the ultimate strength of the concrete. To achieve this, the damage surface $\Psi(\boldsymbol{\varepsilon}, k)$ is modified to predict the increase in damage in the material with increasing number of cycles of loading as

$$\tfrac{1}{2}\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}(1-2\beta)X(N) - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0 \qquad .....(13)$$

Where X(N) is a function that depends on the number of loading cycles and adopted to increase of damage with increasing number of cycles. We propose a power function for X(N) as

$$X(N) = N^{A} \qquad .....(14)$$

Here, N represents the number of loading cycles, and A is a material parameter. Utilizing Eqns (11) through (14), we obtain the cumulative fatigue parameter k as

$$k = E^{0}\left[1 - \frac{1}{exp\left(\frac{\sqrt{(1-2\beta)N^{A}\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}}}{\varepsilon_{\mathrm{u}}}\right)}\right] \qquad ...(15)$$

By differentiating Eqn (15) with respect to N, the increment of damage in one cycle is obtained as

$$\dot{k} = \frac{dk}{dN}$$

$$= \frac{AN^{\frac{A}{2}-1} E^{0}\sqrt{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}(1-2\beta)}}{2\varepsilon_{u} exp(-\sqrt{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}(1-2\beta)N^{A}/\varepsilon_{u}^{2}})} \qquad (16)$$

Finally, the rate of damage parameter, $\dot{\mathrm{k}}$, must be used in the simple constitutive relation of the form given by Eqn (3) in uniaxial tensile stress path for representing inelastic deformation, stiffness degradation and strength reduction due to fatigue cycles. Substituting Eqns (8), (9) and (16) into Eqn (5) and then substituting Eqns (4) and (5) into Eqn (3) yields

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}(k) : \dot{\boldsymbol{\varepsilon}} - \dot{k}\left(\frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} : \boldsymbol{\varepsilon} + \beta\boldsymbol{\varepsilon}^{+}\right) \qquad .....(17)$$

Eqn (17) is the rate of stress tensor for uniaxial tension-tension fatigue loading where it is assumed that unloading is elastic process. When $\beta = 0$, the process is classified as elastic-damaging, in which stress-strain curve returns to origin upon unloading of the material. In fact, damage incurred in concrete cannot be considered perfectly elastic. The unloaded material shows some residual strains due to the development of sizable crack tip process zone and misfits of the crack surfaces.

For uniaxial tension, Eqn (17) becomes

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}} - \left[\frac{A N^{\frac{A}{2}-1} E^0 \sqrt{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+ \eta} \left((1+\beta) \right)}{2\varepsilon_u exp\left(-\sqrt{\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+ \eta N^A}{\varepsilon_u^2}} \right)} \right] \boldsymbol{\varepsilon}^+ \quad ...(18)$$

where $\eta = 1 - 2\beta$.

4. Numerical Examples

The proposed model contains two material parameters (A and β). The damage parameter, k, which is regarded as the stiffness degradation in the proposed model, is computed by measuring stiffness at three different stages of loading cycle. The kinematic parameter, β, is determined by measuring the permanent deformation during one of the cyclic loadings. Due to the scarcity of experimental data in the literature for the measurement of these material parameters in the numerical simulation, analyst's judgments are required to obtain numerical results.

The model prediction of modulus reduction with increasing number of cyclic loading is shown in Figure 2. Figure 3(a) shows the decrease of maximum stress level (S-N curve) in cyclic tension-tension loading. Figure 3(b), on other hand, shows corresponding experimental result by [5]. As may be seen, the model captures the relevant features of cyclic response.

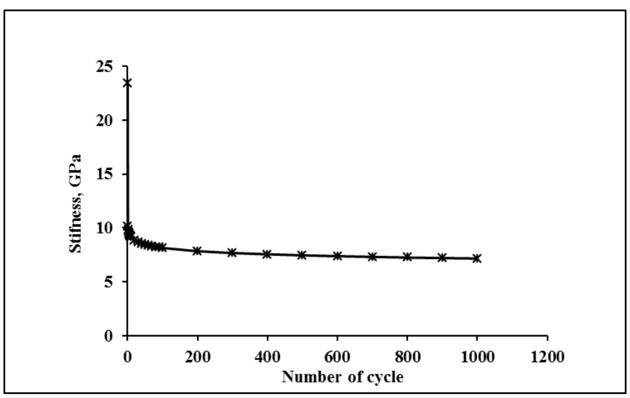
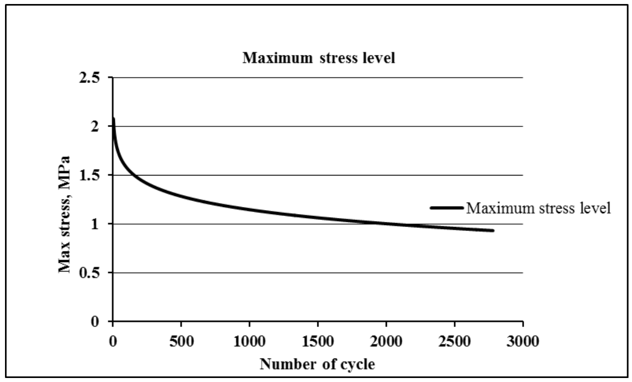
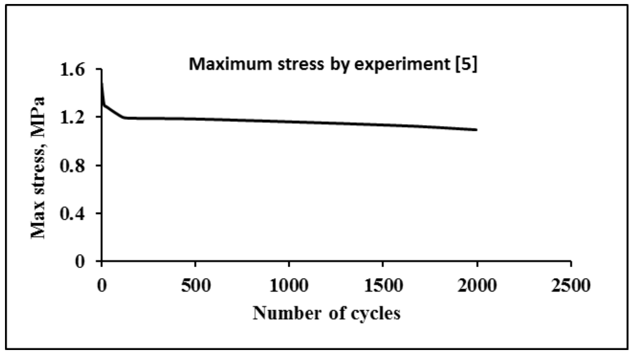


Figure 2: Model prediction of stiffness reduction with number of cyclic loading.



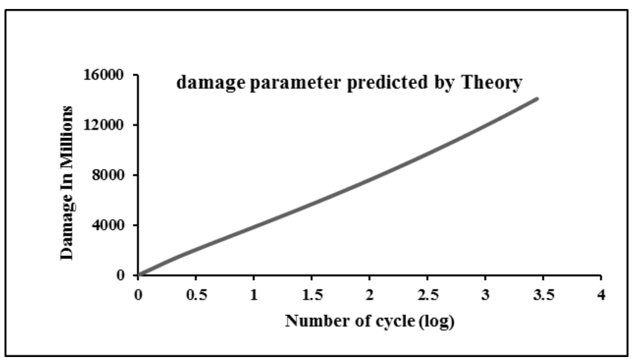
(a)



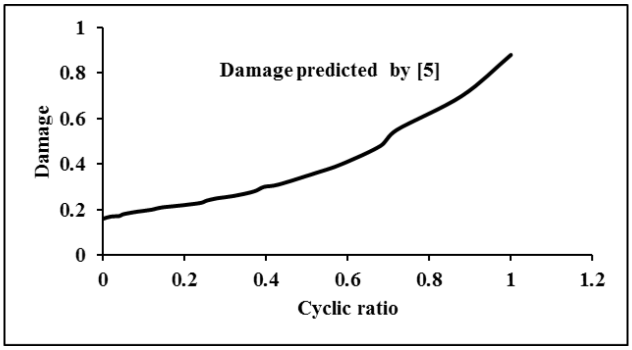
(b)

Figure 3: Reduction in maximum stress level during cyclic tension.
Prediction of the theory, (a). Experimental data [5], (b)

Figure 4(a) shows the increase in damage with increasing loading cycles. The experimental work of [5] is also shown in Figure 4(b) for comparison. The theoretical model also captures the similar trend of increasing damage with the cyclic loading as observed in the experiment [5]. For numerical simulation, the following constant were used, A = 0.1 and β = 0.15. Parameter A is estimated by comparing predicted results and experimental results over a range of applied strains.



(a)



(b)

Figure 4: Variation of damage with the number of cyclic loading. Prediction of the theory, (a). Experiment [5], (b)

Figures 5 and 6 depict the cyclical stress-strain behavior of concrete material in tension. In Figure 5, no permanent deformations are predicted to remain upon unloading of concrete material however progressive damage accumulation takes place in each loading cycle due to degradation of elastic modulus. This is the ideal case of elastic perfectly damaging behavior which can be achieved by letting $\beta = 0$ with the assumption that crack surfaces close perfectly upon unloading. Heterogeneous materials like concrete exhibits permanent deformations. Figure 6 shows the versatile behavior of the model where the stiffness degradation and permanent deformation are illustrated simultaneously.

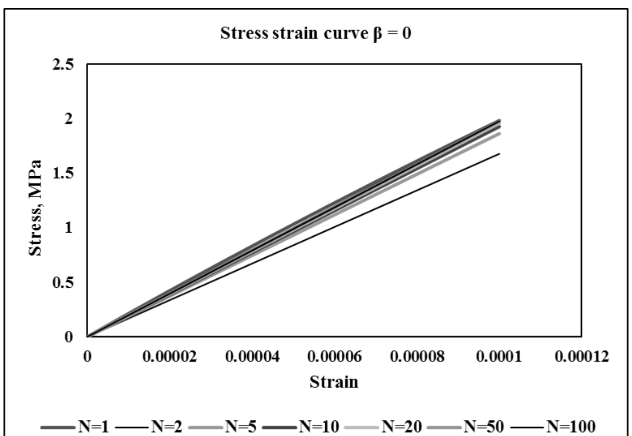


Figure 5: Theoretical cyclic stress-strain behavour of concrete during elastic damaging process (β = 0).

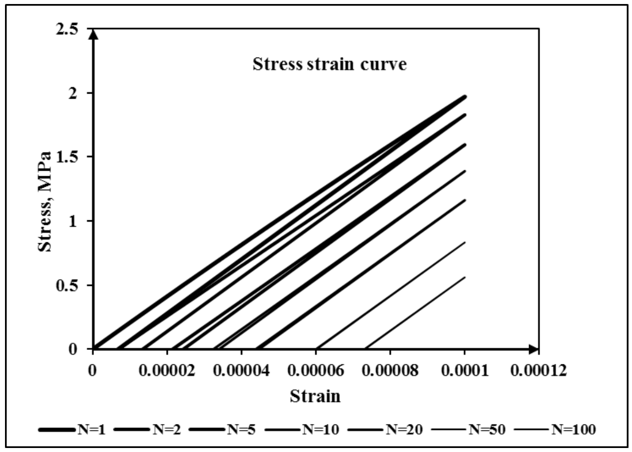


Figure 6: Theoretical cyclic stress-strain behavour of concrete during inelastic damage accumulation.

5. Conclusion

Continuum damage mechanics approach for modeling anisotropic inelastic behaviour of concrete materials during low frequency tension-tension fatigue loading is presented utilizing the framework of continuum thermodynamics. Since the fatigue damage in concrete during the fatigue process is mainly due to development of microcracks and microvoids, a cycle dependend damage surface is employed in the formulation of this strain based theoretical model. Fatigue damage evolution law together with the damage response functions were used in the constitutive relation to demonstrate the capability of the model in capturing the essential features of concrete material, such as stiffness degradation and the inelastic deformations, under fatigue loading environment.The comparision of the model with the available experimental data is also shown in the paper.

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