

Effect of Trailing Edge Profile on Performance of Francis Turbine for Micro Hydropower

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Abstract

To develop a Francis turbine runner blade, analytical calculations and CFD simulations were performed. The Bovet method was used to design a single blade runner. Ansys BladeGen V16.0 was used to construct the blade geometry. Ansys Meshing was used to create the grid model in Turbogrid which used fine mesh quality and a tetrahedron type mesh. For the steady state analysis, a SST turbulence model was chosen. More over three different modification was done on trailing edge profile apart from the parabolic trailing edge obtained from Bovet Method. Elongation on hub region resulted 1.28 percent increase in hydraulic efficiency of runner. Circumferential Velocity and pressure Contour was extracted from CFX result to analyze the increase in efficiency. Mesh independence test was also carried out to choose the best number of elements and node to reduce computation time. In addition to this performance analysis of optimized runner was done on off design condition by applying 70, 80, 90 and 110 percent of designed flow rate.

Keywords

Francis Turbine, Micro Hydropower, Trailing Edge, Hydraulic Efficiency

1. Introduction

Following a series of tests on Boyden's inward flow turbine, British-American engineer James B. Francis used the designs of Howd and Poncelot inward-flow wheels to create his own model. He tested his model and reported his findings in 'Lowell Hydraulic Experiments', which is regarded as one of American society's best contributions to hydraulic engineering. Despite the fact that it does not resemble his model, the contemporary Francis turbine is named after him because of his achievements [1]. The modern Francis turbine is a mixed flow reaction turbine with a number of components, each of which serves a distinct purpose in the system. The runner, blades, spiral casing, stay vanes, guiding vanes, and draft tube are the primary components of a Francis turbine. The Francis turbine is the world's most commonly used turbine. Francis Turbines are said to create 60% of the world's hydroelectricity, making them the most widely utilized hydro-turbine in the world, according to GE Hydro, a leading manufacturer of hydro turbines [2]. As demonstrated in the figure below, the Francis turbine has a wide range of applicability in terms of head, flow, and power, therefore its speed

number ranges from 0.2 to 1.25. For certain characteristic characteristics, a turbine type can be selected based on a speed number, which is a dimensionless quantity. The head required for this type of turbine is typically between 20 to 800 meters, with a medium flow requirement [3]. Francis turbines are well-known for their efficiency and ability to predict it to a high degree of accuracy. Despite its excellent performance, Francis turbines are frequently troubled with cavitation and erosion issues. It is also difficult to design, manufacture, and maintain due to its complicated structure and large number of parts. Furthermore, Francis turbines are not recommended for areas with a high head and flow variance [3]

2. Francis Turbine in Micro Hydropower with Relevance to Nepal

Hydropower facilities with a capacity of 5kW to 100kW are classified as micro hydropower plants by the Nepalese government. The majority of micro hydro plants are of the 'run of river' variety, which means they use the river's flow rather than storing water. As a result, there is no need to build dams or other huge civil structures. If civil structures are

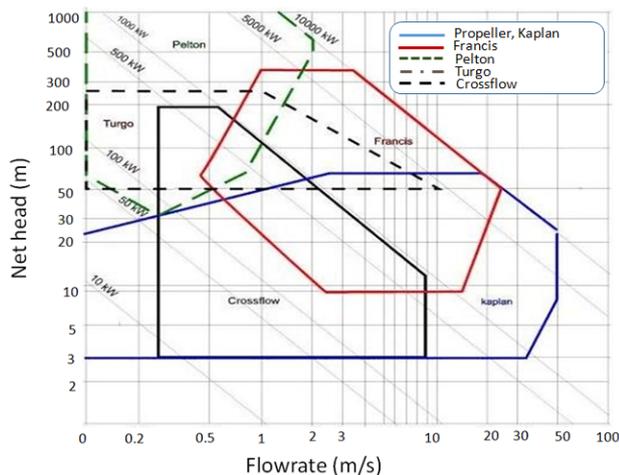


Figure 1: Selection chart for radial turbines by net head-flow[4]

present, they help to convey water to the penstock and regulate the water level at the plant’s intake [5]. Micro hydro is regarded as one of the most environmentally friendly energy solutions because it has no negative impact on the environment, unlike huge hydropower operations [6].

Nepal had been harnessing hydropower for mechanical power and food processing through traditional water wheels known as ‘Ghattas.’ The first adventure of Micro Hydro in terms of energy generation began in Nepal in 1962, when a Swiss-funded initiative developed and installed a 5kW propeller turbine at Godavari Fish Farm [5]. AEPC is a government institution that was founded in November 1996 under the Ministry of Science and Technology of the Nepalese government to promote renewable energy in Nepal, especially Micro Hydro. The foundation of these institutions aided the growth of micro hydro and thus now Nepal has over 3300 micro hydro, electrifying over 350,000 families in rural areas [5].

The adoption of a 33kW Micro Francis turbine, which was much smaller than the actual cross-flow turbine in size, increased production by 6kW, according to AEPC’s 2009 annual success report which was the case at Handi Khola, Sindhupalchok, [7]. NHE, Butwal, has produced a 92kW Francis turbine, which is a scaled-down version of the 4.2MW Francis turbine used in the Jhimruk Hydropower Project [5]. Nepal has 83,000MW of theoretical hydroelectric potential, but only a few have been tapped. Because grid extension is difficult in Nepal’s hilly and Himalayan regions, micro hydro is the greatest

alternative for rural electrification. According to a preliminary survey of the earthquake-affected MHP, more than half of the locations were eligible for Francis turbines, however other turbines with efficiency of 50% to 60% were employed [8]. In Nepal, there are several places ideal for Francis turbines, and using Francis turbines in these locations may have provided more power as Handi Khola, Sindhupalchok. Moreover, The Nepal Electricity Authority’s policy has encouraged developers to choose more efficient turbines

3. Design of Francis Turbine

In the literature, there were numerous studies on Francis turbine design. Blade-design approaches are classified as direct, in-verse, direct method, Bovet method, and others [9].

Calculating the set of system parameters, the turbine shape, that will result in the desired system behavior of the runner efficiency or flow field characteristics is an inverse design method. Tan et al introduced one of the first 3D inverse design approaches for turbomachinery, assuming inviscid and incompressible flow as well as a simplified meridional channel geometry for heavily loaded, infinitely thin blades in an annular cascade configuration [10]. Sebastián Leguizamón did research on open source implementation and validation of a 3D Inverse design Method for Francis Turbine runner. More-over K Daneshkah did parametric design of Francis Turbine runner by means of a three dimensional inverse design Methods[11]. More importantly, there are some mathematical discrepancies, particularly in the final form of the equations in the curvilinear coordinate system, which make it difficult to execute this method independently [10].

In direct method the designing process starts with the inlet conditions, and various dimensions of the runner are calculated based on the inlet conditions, and various dimensions of other components are calculated based on the runner dimension. Francis runner blade should be built using the direct method with multiple iterations adjusting the various energy distribution parameters [12]. The direct design method always requires designer experience and is more laborious because it does not provide any guidance to the designer to control the shape of the blade [9]. Korakianitis [12] has worked on designing airfoils by direct method.

Both strategies mentioned have respective benefits and drawbacks. When the whole geometry is known, it is reasonably easy to fulfill mechanical and geometric requirements with the direct method; however, obtaining the appropriate distribution of pressure or velocity throughout the profile with this method is usually tedious. On the other hand, with an inverse technique, where the velocity is provided and the geometry is calculated, it can be difficult to achieve an appropriate geometry [9].

The Bovet method, which uses empirical equations to get parameters of Francis type hydro turbine runner, is one of the most dominant design approaches for Francis turbine runner [13]. The dimensionless specific speed value is the major parameter used in the Bovet method to determine the overall dimensions of the turbine. Miloş and Bârglâzan conducted research by optimizing the Francis turbine design utilizing CAD techniques [14]. To calculate meridional channel and blade splines, the authors employed the Bovet technique. The authors choose the parabola arcs approach for obtaining the leading and trailing edge profiles. For the preliminary design of the Francis Turbine, Kocak, E. et al [4], used the Bovet and Conformal mapping method. Fatma, A. et al. employed MATLAB codes to identify the key dimensions of the Francis turbine, particularly for low-head systems, and applied a CFD analysis for optimization [15]. Using CFD, Thapa et al [16] looked into optimizing sediment erosion on a Francis runner blade by creating new types of profiles by using commercial software called "Khoj."

Dadi Ram Dahal had used K-epsilon turbulence model for design and numerical analysis of simplified Francis turbine for micro hydropower applications as k-epsilon turbulence model requires less computational effort, it is used for both the initial solution and the comparison study. Because of its robustness, good accuracy, and cost-effectiveness, the K-epsilon model is widely used [17]. The Reynolds averaged Navier-Stokes equations were calculated using the k- ω SST turbulence model by Kocak. The residual values of convergence criteria were set to 10⁻⁶ grade while a repeated solver was used [4].

3.1 Bovet Method for Meriodional Plane

Bovet Method was choosed to design the base reference runner. The Bovet approach determines the crucial dimension of the runner as well as the blade's meridional shape. The dimensions of the runner are

calculated using an empirical formula derived from a statistical examination of a large number of Francis turbines. All of the characteristic parameters become dimensionless by employing R_{2e} as the nominal radius for the sake of simplicity. All of these measurements are in relation to the runner reference radius r_{2e}, which is set to 1. The features of turbine operation at optimal speed matching to the best efficiency point are used to design the blade. All of the blade's dimensions are calculated using two characteristics parameters, as given below.

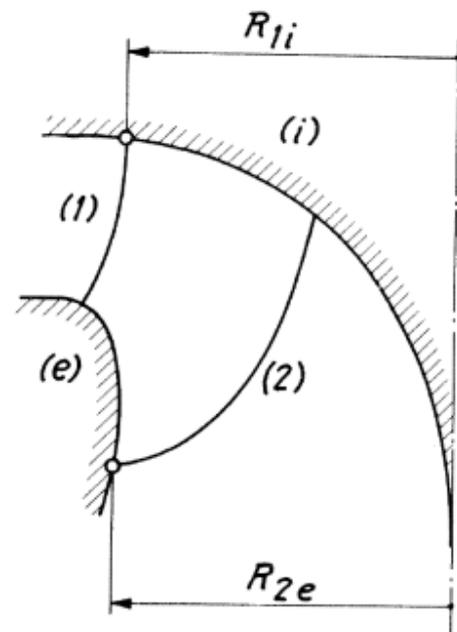


Figure 2: Characteristic Dimension of flow channel [13]

It can be observed that the Bovet technique uses these primary variables to calculate dimensionless specific rotational speed.

$$n_0 = \phi_{2e}^{\frac{1}{2}} \psi_{2e}^{\frac{-3}{4}} = \frac{\omega^2 \sqrt{\frac{Q}{\pi}}}{(2gH)^{\frac{3}{4}}} \quad (1)$$

The Bovet technique can calculate runner channel parameters as a function of 'n₀' in a dimensionless form. The Figure 3 represents progressive change in the form of runner passage as the function of specific speed. The method employs a specified radius, which is the distance between the runner's rotational axis and the intersection of the shroud and the blade's trailing edge. Specific radius can be calculated from following equation.

Table 1: Equations of meridional plane

$b = \frac{B_0}{R_{2e}} = 0.8(2 - n_0)n_0$	$r_{oi} = \frac{R_{oi}}{R_{2e}} = y_{mi}$ $= 0.68 + \frac{0.16}{n_0 + 0.08}$
$l_{li} = 3. + 3.2(2 - n_0)n_0$	$l_e = 2.4 - 1.9(2 - n_0)n_0$

$$R_{2e} = \left(\frac{Q}{\pi \omega} \right)^{\frac{1}{3}} \quad (2)$$

Where, φ_{2e} is flow coefficient and ψ_{2e} is energy coefficient .Bovet choose $\varphi_{2e} = 1.72$ and $\psi_{1e} = 0.27$. Table 1 Equations of meridional plane is given by[13].

The runner channel meridional plane can be formed after these calculations are completed. The leading and trailing edges of the runner blade were then intersected with the hub and shroud, respectively. As a result, the meridional profile of the runner blade was placed on the meridional plane of the runner channel. The profile of a runner blade from leading to trailing edges is known to be the most critical design element. Bovet accomplishes this by separating the runner blade with streamlines that have the same flow rate as each other, which is the main property of these streamlines. According to recent research, the angle of the leading and a high-efficiency runner. Using velocity triangles, the design approach estimates radiuses and angles at intersection points of streamlines with trailing and leading edges.

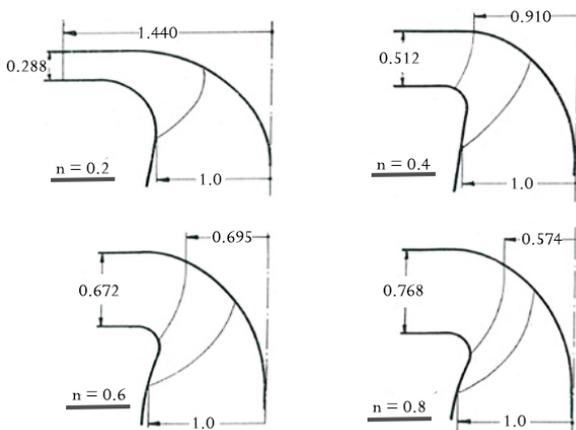


Figure 3: Progressive change in form of runner passage as a function of specific speed n_0 [13]

CFD simulation is used to assess a Francis type turbine runner that was designed utilizing the Bovet technique in this work. The major dimension of the runner based

Table 2: Design Parameters

Parameter	Symbol	Values	Units
Head	H	16	m
Discharge	Q	0.1	m ³ /s
Hydraulic efficiency	η_h	91	%
Wire Power	P_w	14.75	kW
Acceleration due to gravity	g	9.81	m/s ²
Rotational Speed	N	1500	RPM
Dimensionless specific speed	n	0.38	

on head and flow is determined using data from Damile MicroHydro in Pharping, Kathmandu.

After obtaining the meridional profile, the same equation used to create the hub and shroud curves is applied to generate three more streamlines, assuming that these curves are bounding streamlines. However, each streamline’s boundary condition is distinct, and this is achieved by interpolating the needed points between the hub and shroud curves. Figure 4 represents Meridional profile of runner.

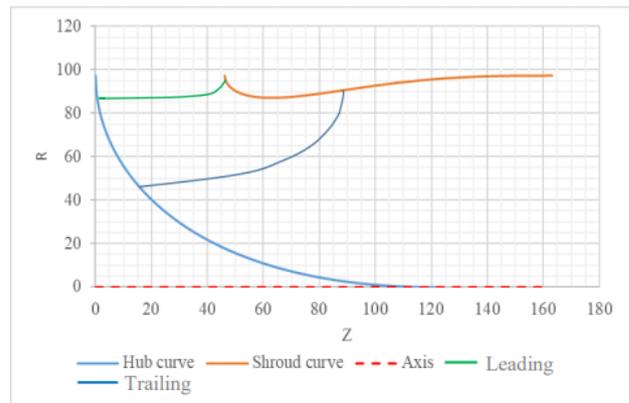


Figure 4: Meridional profile of runner.

4. Modeling of blade geometry

All meridional curves and streamlines are supplied in a suitable format to the Bladegen blade module of ANSYS 16.0 for blade and runner geometry development. First, the beta angles over the leading and following edges of each streamline are calculated. After that, for each stream line, a linear beta angle distribution is assumed from the leading to the trailing edge. Furthermore, the thickness is thought to be

Table 3: Radius And Blade angle distribution on leading and trailing edge

R ₁ (m)	β ₁ (degree)	R ₂ (m)	β ₂ (degree)
0.097134	35.27	0.09088	15.1097
0.092151	44.341	0.0717	18.8914
0.089434	50.986	0.0602	22.1768
0.087622	56.381	0.05157	25.4464
0.086036	61.833	0.0439	29.2038

decreasing linearly from 5 mm at the leading edge to 2 mm at the trailing edge. The Figure 5 represents the 3D view of runner.

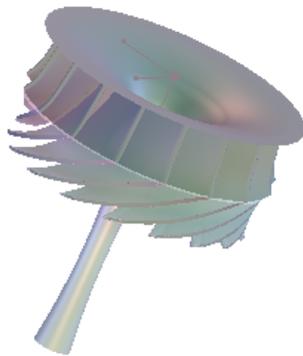


Figure 5: 3D geometry of runner

4.1 Mesh Generation

The architecture, quality, and types of mesh play a critical impact in the overall accuracy of the simulation, as well as the time and convergence of the output. Structured meshes, such as hexahedral meshes, are more efficient in terms of accuracy, CPU solving time, and memory allocation. Meshing of the runner is done using the ATM optimized features of ANSYS Turbo Grid. Total nodes of 142110 and total elements of 114884 was generated by making topology unsuspending. Boundary layer refinement control was done according to proportionality of mesh size along with global size factor 1. The Figure 6 represents meshing in 3D view of runner.

4.2 Boundary Conditions

Analysis type: Steady State Analysis
 Fluid and particle Definition: Water
 Reference pressure: 1 atm
 Turbulence model: SST model
 Flow direction: Cylindrical Components
 Axial angle (0), radial angle as (-0.32613 radian), and

ANSYS
R16.0

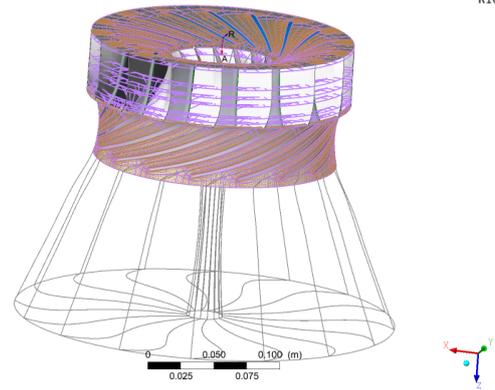


Figure 6: Meshing in Runner

tangential value (-0.945324 radian).
 Mass flow rate: Mass flow inlet P-static outlet with 99.7 kg/s
 Convergence Criterion: 0.0001 residual Wall function: automatic

4.3 Parametric Study

The parameter study used is one way in which effect of changing one parameter at a time is studied. The different trailing edge profile is studied taken by keeping all other parameters constant. The tip of trailing edge is taken ellipse but its orientation from hub and shroud was changed by using bezier curve of polynomial with order 2. The Figure 7 represents different trailing edge modification done on the runner.

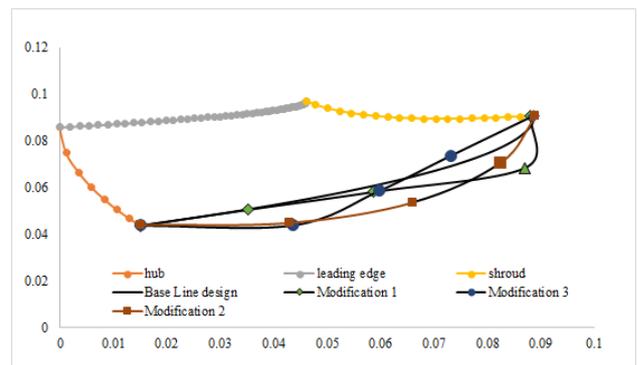


Figure 7: Different Trailing Edge Modification

4.4 CFD Analysis

The mesh generated in Turbogrid was analyzed by Using CFX and result for every profile was generated. Here convergence control is done by 100 iterations with double precision. In fact, this model retains the

properties of the $k-\omega$ model close to the wall, but gradually blends into the $k-\omega$ model away from the wall.

4.5 Mesh Independence

Mesh independence tests were performed on several meshes with coarse, medium, and fine mesh as goal mesh sizes. For near-wall elements, the $y+$ technique was utilized with Reynold’s number of 1,000,000. Following the mesh independence study, 1,14,884 elements were chosen for further investigation. The Figure 8 represents mesh independence test carried out.

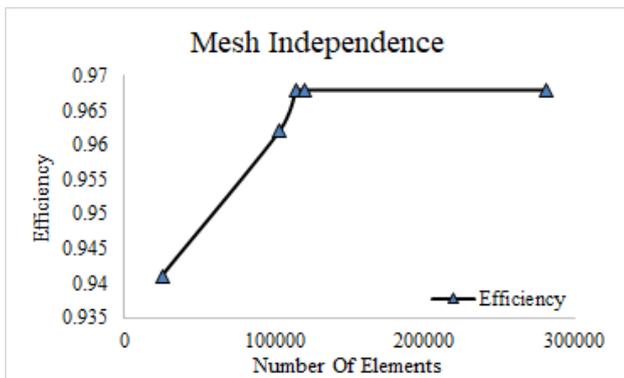


Figure 8: Mesh Independence

5. Analytical Solution

The efficiency of runner with parabolic trailing edge (i.e base trailing edge profile) was calculated by using Euler head. The Figure 9 represents inlet and outlet velocity triangles.

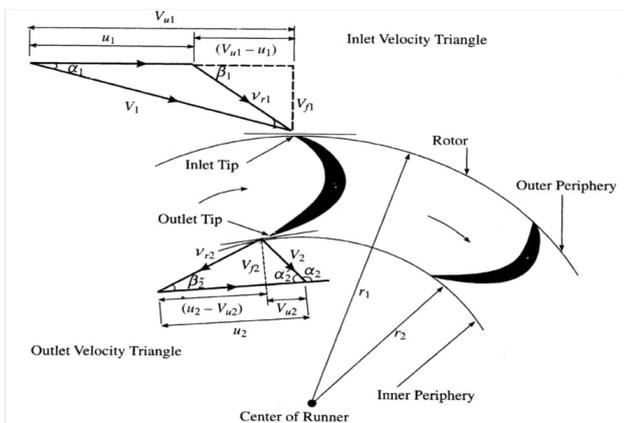


Figure 9: Inlet and Outlet Velocity Triangles [18]

Rotational speed (N) = 1500 RPM
 Mean Beta angle at trailing edge (β_{mean}) = 49.813782°
 Mean Radius at leading edge (R_{mean}) = 0.09047

$$\text{Blade Velocity at inlet (U)} = \frac{2\pi R_{mean} N}{60} = \frac{2\pi * 0.09047 * 1500}{60} = 14.2109 \text{ m/s}$$

$$\text{Flow component of velocity at inlet (V}_{f1}) = \frac{Q * K}{2\pi * R_{mean} * B} = \frac{0.1 * 0.95}{2\pi * 0.09047 * 0.0461} = 4.0108 \text{ m/s}$$

Where K= vane thickness coefficient and is usually in order of 0.95

$$\text{From Velocity diagram, } V_{U1} = U_1 - \frac{V_{f1}}{\tan\beta_1} = 14.2109 - \frac{4.0108}{\tan(49.813782)} = 10.82 \text{ m/s}$$

$$\text{Hydraulic Efficiency } (\eta_h) = \frac{V_{U1} * U_1}{gH} = 97.99 \%$$

6. Results and Discussion

First of all efficiency of baseline design was compared with analytical solution. And after runner was optimized by selecting best trailing edge profile. The optimized runner is subjected to off design flow rate to compare its efficiency and power extraction.

6.1 Comparison of Efficiency of baseline design with analytical solution

First from the Ansys CFX result we got 95.55% efficiency and from analytical method we obtained 97.99% efficiency for reference trailing edge profile. CFD simulation analysis are more accurate and precise than analytical method because in analytical method we assume whole amount whirl component of velocity perpendicular to the blade but in real flow due to flow losses the whole transformation of velocity does not occurs. The velocities from CFX does not assembly exact to a correct velocity triangle , a distortion parameter is always present [19]. Moreover, there are present of secondary vortices in real flow which decreases the runner efficiency but that is not taken consideration while obtaining efficiency from analytical method.

6.2 Comparison of velocity contour of different Trailing edge Profile

The Figures 10 and 11 represents velocity vector on Meridional surface view of baseline design and modification 3. From comparison of different circumferential velocity plot over entire blade runner what we see that there is unused velocity at hub region which was not captured by our other trailing edge profile. So when we elongate our trailing edge on hub region, unused velocity which in turns give torque gets captured and gives rise to hydraulic efficiency. Thus, we were able to recapture residue momentum that was present on the trailing edge side of the hub

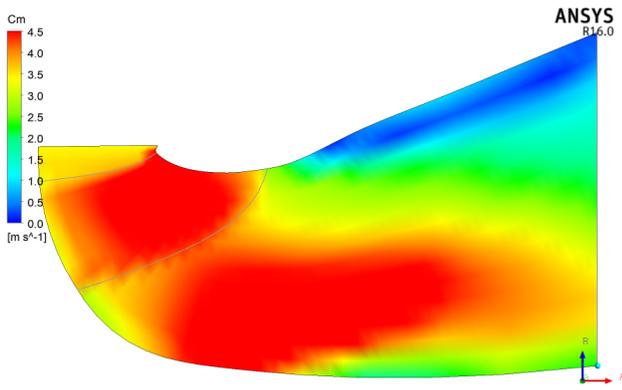


Figure 10: Circumferential Velocity of Baseline design

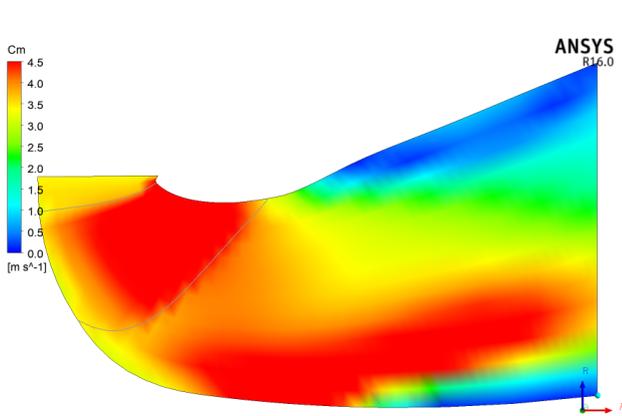


Figure 11: Circumferential Velocity of Modification 3

region. The Figure 12 histogram represents different efficiency achieved during trailing edge modification.

6.3 Pressure Contour

The Figures 13 and 14 represents pressure contour of base line design and modification 3 and what we see that better control of pressure distribution from hub to shroud is seen in modification 3 as compared to baseline design. As in reaction turbine work done on the runner is due to change in inlet and outlet pressure along change in kinetic energy of the flowing fluid. So, the value difference in maximum and minimum pressure of modification 3 is higher compared to other trailing edge modification which may be the root cause for increasing hydraulic efficiency of runner.

6.4 Performance analysis of optimized case

The analysis of optimized runner obtained through modification of trailing edge is subjected through off

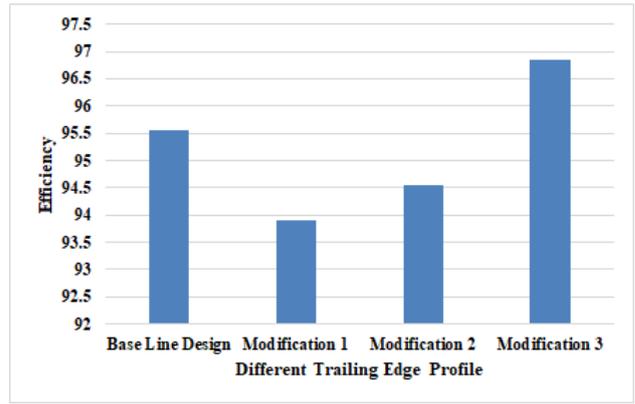


Figure 12: Efficiency Vs Different trailing edge profile

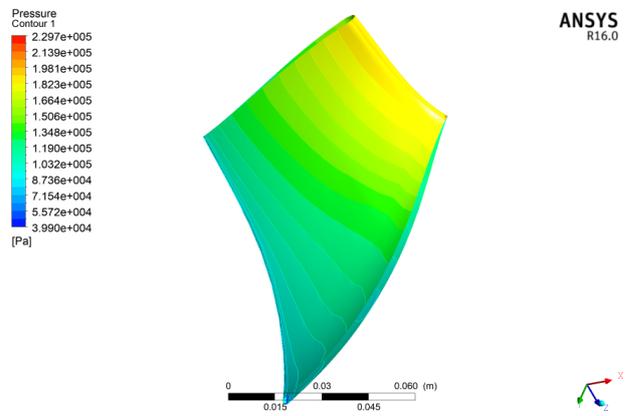


Figure 13: Pressure Contour Of Modification 3

design scenario by applying different discharge case. The Figure 15 represents efficiency and power extraction on different discharge subjected.

Thus maximum efficiency and maximum shaft power was extracted at 100 percent discharge and seems decreasing when discharge was altered. The off design study was done by altering 70%, 80%, 90% , 110% of designed flow rate.

7. Conclusion

The meridional plane of a Francis turbine runner is developed using the Bovet method in this paper. The Bovet method is a simple method for determining the dimensions of a blade in the meridional plane that relies on empirical relationships. At the preliminary stage, an excel application is utilized to calculate all of the blade’s key dimensions and meridional profile, as well as the leading and trailing edges. The turbine profile and parameters are afterwards determined using MATLAB code. The streamline on meridional

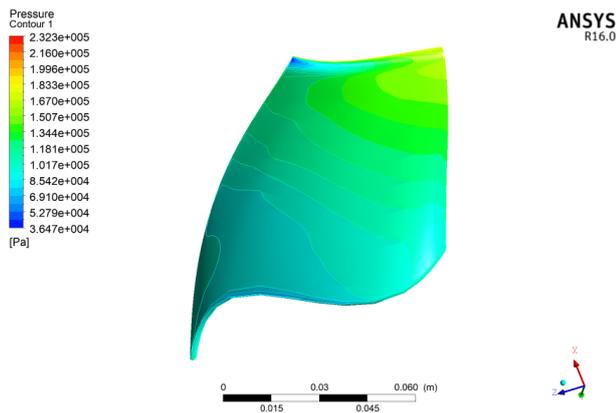


Figure 14: Pressure Contour of Baseline design

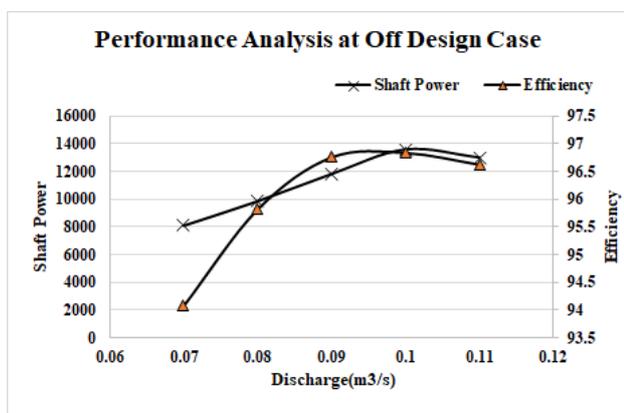


Figure 15: Performance Analysis at Off Design Case

profile is drawn once all the parameters and dimensions have been determined. Because the design and 3D modeling of a blade is difficult, the Bladegen function of ANSYS blade modular was used to obtain the domain and model of the runner. The efficiency of the runner, pressure distributions on blades, and circumferential velocity on the blade surface are all computed using CFD models. Total 1.28% difference in turbine runner efficiency was determined numerically after the modification of trailing edge when compared to committed efficiency calculated by using the Bovet method. According to the findings of the study, the Bovet design approach can be utilized to pre-design an efficient runner. Following the preliminary design of the runner achieved by the Bovet approach, modifications to the runner design can be made based on the findings of CFD simulations, in order to improve the runner’s efficiency and performance. The Bovet approach is proved to be a reliable way for pre-designing Francis turbine runner in this study. The most critical component affecting turbine efficiency is the turbine runner. CFD simulations have emerged as a viable

and innovative tool for developing high-efficiency runners. Despite the necessity of model turbine test validation, trustworthy CFD simulation results can be used to directly validate prototype turbine performance characteristics.

To improve the Bovet design approach in future research, the trailing edge tip can be changed to achieve high runner efficiency. Furthermore, blade thickness, angles, and profiles can all be tweaked.

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