A Case Study of Dodhara-Chadani Distribution Feeder for Multi-Objective Optimization of PV Penetration

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Abstract

Electrical power involving Distributed Generation (DG) is being the hot cake to serve consumers demand. While supplying demand, power quality, voltage stability, loss reduction, etc. of the Distribution System Network (DSN) are of prime concern. In this work, Modal Analysis determines candidate buses for solar PV placement and different methodology for multi-objective (MO) Penetration Optimization of Solar PV in a RDS are analyzed. Random Penetration Optimization, MO Penetration Optimization using Genetic Algorithm (MOGA) and Non-Dominated Sorting Genetic Algorithm-II (NSGA-II) are discussed for sizing optimally the solar PV in IEEE-33 Bus System with expected improved performances like Active Loss Reduction (ALR), Voltage Deviation Index (VDI), Voltage Stability Index (VSI), etc. The results of the optimization tools are analyzed and hypothesis testing is performed to check whether the obtained results are statistically significant. Penetration optimization of Dodhara-Chadani (DoC) Distribution Feeder of Dodhara Substation is performed taking multiple objectives for deterministic Sources and deterministic loads. The effect of uncertainty is also analysed to determine the best locations for PV penetration. Other different cases are also analysed.

Keywords

Modal Analysis, Penetration Optimization, Multi- Objective Genetic Algorithm (MOGA), Non-Dominated Sorting Genetic Algorithm-II (NSGA-II).

1. Introduction

In the context of rising global concerns for environmental conservation, growing demand and costly upgradation and or expansion planning of DSN, DG is being the important component. The performance indices of prime concern in a DSN are power quality, voltage stability/ profile, VDI, loss reduction, etc. The optimized Penetration of RE into power distribution system, considering these various factors, forms a multi-objective Optimization problem. In Nepalese context, Solar is the most abundant, prominent, viable and free source of RE[1]. То achieve sustainable development in power sector, Government of Nepal (GoN) has prioritize generation mix to achieve 5-10 percent share of power generation from solar and mix it into the grid system [1]. Power Purchase Agreement (PPA) has already been signed on 2017 for 61MW grid tied solar to be installed at 21 different locations within Nepal. And in days ahead to come, the demand of solar integration to the grid will be increasing. Thus, there is an increasing scope of

grid integration of Solar PV in Nepalese Power Distribution System. Normally, A non-optimal DG penetration can create conditions like exceeding voltage limits, exceeding thermal limits, increasing power losses, protection dysfunction against short circuits, etc. And,integrating more proportion of PV on DSN reduces losses, improves voltage profile and reduces pollution but brings the DSN to operational limits and several power quality issues. Thus, research area is proliferated for the placement and optimal penetration sizing of DG into distribution system for the improvement in different performance indices.

Though the integration of the DG units in power distribution systems has numerous advantages, the complexity is increased [2]. The installation of DG in DSN brings changes in Apparent system losses, voltage profile and system performances [3]. Modal Analysis deals with voltage stability of DSN to determine the Eigen values which indicatively explains the mode of the system and through stability margin, suggest which bus of the system is weak [4, 5].Multi-Objective Optimization is an optimization technique involving more than one conflicting objective function to get a compromise solution sets [6]. Different algorithm eg. SA, PSO, ABC, TS, ACO, GA, MOGA, NSGA-II, etc.can be used. The authors in [3] only discussed on Power Loss minimization with DG Penetration using MATLAB inbuilt GA Toolbox module optimization. In [7], only two objectives: minimizing active power losses and maximizing voltage stability is discussed for most favorable Multi DG locating and sizing in a RDS based on NSGA-II and fuzzy logic based amalgamation and in [8, 9], Harris Hawks Optimization and improved decomposition based evolutionary algorithm (I-DBEA) in order to determine optimal DG size and location at various power factors (p.f) is proposed to minimize APL and VDI, and increase VSI. In paper [10], though three objectives are taken, it discussed L-index for voltage stability margin (VSM). This paper is mainly concerned to optimize PV Penetration in a Radial Distribution System involving Multi-Objectives.

Section II briefly explains the methodology used, Section III formulates the proposed MOO, Section IV depicts the Results and Discussion and Section V summarizes the conclusion of this paper.

2. Methodology

The proposed methodology is divided into two parts: (a) Distribution system load flow and Modal analysis, and (b) Penetration or sizing optimization and determination of performance indices.

2.1 Load Flow in a Radial Distribution System

As explained in [3], iterative techniques is used for the load flow in a RDS. Active power loss in any branch 'j' is given by 1.

$$P_{Loss,j} = R_j \cdot \frac{P_{i+1}^2 + Q_{i+1}^2}{|V_{i+1}|^2} \tag{1}$$

Total active power of the RDS is given by 2.

$$TPL = \sum_{i=1}^{n-1} P_{Loss,j} \tag{2}$$

Similarly, VDI, VSI and DGPL is given by 3, 4 and 5.

$$VDI = \sum_{i=1}^{n} (V_{i,0} - V_{i,1})^2 = \sum_{i=1}^{n} (V_{Spec.} - V_{i,1})^2 = \sum_{i=1}^{n} (1 - V_{i,1})^2$$

$$VSI_{i} = V_{i-1}^{4} - 4(P_{i}R_{(i-1)(i)} + Q_{i}X_{(i-1)(i)})V_{i-1}^{2}$$

$$-4(P_{i}X_{(i-1)(i)} - Q_{i}R_{(i-1)(i)})V_{i-1}^{2}$$
(4)

(2)

$$DGPL = \frac{S_{DG}}{S_{LOAD}} * 100\%$$
(5)

2.2 Selection of Candidate buses

Modal Analysis is used to select the weak/candidate buses[5]. With linear approximation, Modal Analysis calculates the eigen value and eigenvector from power flow Jacobian Matrix. Every eigen value represent a mode of V-Q change and gives information about weak part of the system. From reduced Jacobian Matrix, Equation 6 is obtained.

$$\Delta V = \sum_{i} \frac{\xi_{i} \eta_{i}}{\lambda_{i}} \triangle Q \tag{6}$$

Where, ξ_i , λ_i and η_i represents i_{th} column of Right, diagonal and left Eigen Vector Matrix respectively. The sign and magnitude of λ_i is the indicator of power system stability. If λ_i is positive, the system is stable and if λ_i is negative, then unstable. And, λ_i =0 indicates the condition of voltage collapse.

2.3 Multi-Objective Optimization (MOO)

The sizing of DG units shall be obtained by the MOO algorithm . The inputs to the optimization tool are the locations determined by Modal Analysis and the output are the power rating at each location.

2.3.1 MOO using MOGA

While implementing optimization using GA, selection and reproduction process takes place within the optimization tool and finally, an optimal solution is obtained from the population using algorithm as in [11].

2.3.2 MOO using NSGA-II

NSGA-II algorithm has elitism, better and faster non-dominated sorting, improvised crowding distance, constraint handling capability, etc. and algorithm as in [6]. MOGA and NSGA-II differs in the method of ranking. The input to the algorithm are No. of objectives, Population size, Maximum no. of iteration S_{max} , etc.

2.4 Overall methodology

The detail flowchart explaining the overall methodology is shown in Figure 1.



Figure 1: Flowchart Explaining Overall methodology

2.5 Selecting Best Compromize Solution

After obtaining various equally optimal solutions, the best compromised solution is chosen from the equally likely optimal solutions using different techniques. Aggregated sum method is used in this paper. Each objective has their own weight. The ultimate aim shall be to reduce the installation and operating cost of the DG unit.

2.6 Hypothesis Testing

The testing of validity of the assumption about the population parameter is done using Z-test[12]. It is an important parametric test for large samples ($s \ge$ 30). Let, the two populations obtained from the two optimization tools have size, mean, standard deviation n_1, μ_1, σ_1 and n_2, μ_2, σ_2 , then

$$H_0: \mu_1 = \mu_2 \tag{7}$$

i.e., there is no significant difference between 2 tools for convergency, and

$$H_1: \mu_1 \neq \mu_2 \tag{8}$$

i.e. there is significant difference between 2 tools for convergency.

Under null hypothesis, the test statistics is given by

$$Z = \frac{(X_1^- - X_2^-) - (\mu_1 - \mu_2)}{S.E.of(X_1^- - X_2^-)}$$
(9)

$$S.E.of(X_1^- - X_2^-) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
(10)

At level of significance, $\alpha = 0.05$, if calculated value of Z > Tabulated value of Z, H_o is rejected i.e. H_1 is accepted and vice-versa.

3. Problem Formulation

3.1 Overall Multi-Objective Optimization Problem

Three different objectives include; minimizing APL, minimizing VDI and maximizing VSI. VSI value nearer to 1 is better and VSI value equal to 0 indicates condition of voltage collapse ($0 \le VSI \le 1$). The overall MOO problem is:

$$min.MOF = min.f_1 + min.f_2 + min.f_3$$

= $\alpha_1 * min.P_{Loss} + \alpha_2 * min.VDI +$
 $\alpha_3 * min.VSI function$
= $\alpha_1 * \sum_{i=1}^{k-bus} P_{Loss(i,i+1)} + \alpha_2 * \sum_{i=1}^{k-bus} (V_{i,0} - V_{i,1})^2$ (11)
 $+ \alpha_3 * \frac{1}{(1 + VSI_{Max})}$

Subjected to various constraints for any bus i where $\sum_{i=1}^{3} \alpha_i = 1, \alpha_i \varepsilon[0, 1]$. α_i is called the weighting factor. In this paper, $\alpha_1 = 0.3$, $\alpha_2 = 0.3$ and $\alpha_3 = 0.4$ are taken.

3.2 Constraints

3.2.1 Inequality Constraints

It includes Bus Voltage Limit $[V_i^{min} \le V_i \le V_i^{max}(i = 1, 2, 3, ..., N_{bus})]$, Generation Limits $[P_{DGi}^{min} \le P_{DGi} \le P_{DGi}^{max}, Q_{DGi}^{min} \le Q_{DGi} \le Q_{DGi}^{max}]$ and Line Thermal Limit $[S_k \le S_{kmax}$ (for k=1,2,...,N_{Branch})].

3.2.2 Equality Constraints

For any bus 'i', $P_G + P_{DG} = P_D + P_L$ and $Q_G + Q_{DG} = Q_D + Q_L$.

3.2.3 DG Capacity Constraints

$$0 \le \sum_{i=1}^{N_{DG}} P_{DGi} \le \sum_{i=1}^{N} P_{Loading}$$
(12)

Where, $\sum_{i=1}^{N} P_{Loading}$ is the total active power load of the DSN.

4. Results and Discussion

4.1 Considerations

Standard IEEE 33 bus system has been taken as a test system [3]. 72-Bus Dodhara-Chadani Distribution Feeder is used for the application purpose. Matlab© R2016a is used for optimization and Electrical Simulation Software is used for the simulation purpose.The computer specification includes processor of Intel (R) Core (TM) i7-6500U CPU@2.50GHz.

4.1.1 72-Bus Dodhara-Chadani (DoC) Distribution Feeder

As shown in Figure 2, DoC feeder having 72-Bus, 9 laterals lie in the Sudurpashamin Pardesh of Nepal in Kanchanpur district. The total nominal feeder load is 2.4228 MW. Rabbit and Weasel ACSR conductor is used as the line conductor and have own line and bus data.



Figure 2: SLD of 72-Bus DoC Feeder

Summer season having the maximum load is taken for analysis. A minimum loading (1324.32 kW) at 1 pm which is 60% loading to that of absolute load (2207.2 kW) at 8 pm is taken as the time for PV integration to the grid.

4.2 Simulation for the Base Case System

Base case load flow was performed on both IEEE 33-Bus System and 72- Bus DoC Feeder. The results

obtained from both the cases were nearly similar as shown in Table 1. Bus voltages were below the tolerable voltage limits, branch losses near the substation is more, VSI at the farther end is much less than 1 as shown in Figures 4,5 and 6.

4.3 IEEE 33-Bus System

4.3.1 Penetration of Solar Photovoltaic on Candidate Buses

When fixed but random equal size of Solar PV was penetrated at candidate buses of IEEE 33-Bus System, the optimum Solar PV penetration occurred when (850, 890)KW is injected at candidate buses. In between, VDI and APL are minimum and VSI is maximum (closer to 1). The optimal Solar PV penetration limit shall be within 73% to 110% as shown in Figure 3.



Figure 3: ALR, VDI, VSI and DGPL at different scenario of PV Penetration (in kW)

4.3.2 Penetration Optimization using MOGA

Taking initial population Size '10', No of Generations '100', Cross over Probability, $P_c = 0.9$, Mutation Probability, $P_m = 0.09$, MOGA was independently run for 35 times to obtain convergence graph as an average of 35 independent runs. The optimized size of PV that can be penetrated without violating the constraint limits were 0.968MW, 0.813MW, 0.323MW and 0.452MW at candidate buses 18, 33, 22 and 25 respectively. Figure 4, 5 and 6 shows the plot of bus voltages, branch APL and bus VSI before and after PV penetration respectively.

4.3.3 Penetration Optimization using NSGA-II

Taking similar values for optimization as in MOGA, NSGA-II optimization algorithm was also run for 35 times to obtain the average convergence graph. The optimized size of PV that can be penetrated were

Base Case	Candidate Buses	Corresponding Eigen Values	Minimum Bus Voltage	Total APL	VSI at Weak Bus	
IEEE-33	18,33,18,	0.0163, 0.047, 0.1499,	0.91p.u at	0.2027MW (51.8kW	0.6950	
Bus System	22,25	0.1747, 0.2744	Bus No. 18	at Branch 2)	at bus no.18	
72-Bus	68,49,56,	0.0079,0.0380,0.0907,	0.8869p.u at	0.17905MW (15.27kW	0.6188	
DoC Feeder	30,25	0.1365,0.2253	Bus No. 68	at Branch 4)	at bus no.68	

 Table 1: Base Case Simulation



Figure 4: Bus Voltages before and after PV Penetration using MOGA



Figure 5: Branch APL before and after PV Penetration using MOGA

1.018MW, 0.477MW, 0.551MW and 0.591MW at candidate buses 18, 33, 22 and 25 respectively. Similar nature of plot as in Figure 4, 5 and 6 is obtained with slight modification in the values.

4.3.4 Comparison of the cases for IEEE 33-Bus System

Table 2 shows the optimized values obtained from MOGA and NSGA=II. Authors in [3] had taken only one objective function of APL minimization. While achieving this objective, the penetration level was 62.60%. However, while taking multi-objectives, the sizes to be penetrated at candidate buses got



Figure 6: Bus VSI before and after PV Penetration using MOGA

decreased. It is seen that, the DGPL for IEEE 33-Bus System while using MOGA is 58.50% but it is 60.33% while using NSGA-II. For the similar condition, MOGA with small sized PV penetrated can improve the system indices significantly. Both the optimization tools were run for 35 times and taking fitness function as the average of 35 runs of the convergence graph, standard deviation, variance (V) and coefficient of variance (C.V.) were obtained. It was seen that, while using MOGA, it was 129.1, 5.0454 and 0.0505 whereas while using NSGA-II, it was 492.7, 18.17 and 0.1817. The smaller value of C.V implies that MOGA is consistent. In terms of loss reduction, optimized PV size and minimum fitness value, MOGA optimization tool is better. However, statistically speaking, hypothesis testing suggests that z=1.7815 < 1.96 implies there is no significant difference between MOGA and NSGA-II for convergency.

4.4 72-Bus DoC Distribution Feeder

4.4.1 Penetration Optimization using MOGA

The MOGA algorithm was independently run for 35 times and similarly, optimized size of PV that can be penetrated without violating the constraint limits were 0.572MW, 0.194MW, 0.074MW, 0.160MW and 0.214MW at candidate buses 68, 49, 56, 30 and 25

Optimi zation Tool	Objective Function	Weak Buses	PV Sizes (Kw)	APL (Kw)	VSI at Weak Bus	VDI	Fitness Value	APL Reduc tion (%)	V Improve ment (%)	VSI Improve ment (%)
Base Case	-	-	-	207.81	0.6920	0.1202	62.79	-	-	-
Single Obj. GA [6] MOGA	Total APL reduction Minimize APL/VDI; Maximize	18,33, 22,25 18,33, 18,22, 25	609,254, 936,936 855,813, 113,323,	81.03 95.87	0.8130	- 0.0228	- 29.17	60.02% 53.87	- 9.74	- 17.49
	VSI	25	432							
NSGA-II	Minimize APL/VDI; Maximize VSI	18,33, 18,22, 25	752,477, 266,551, 590	100.14	0.8165	0.0202	29.98	51.81	10.00	17.99

 Table 2: Multi-Objective Optimization of PV sizes for IEEE 33-Bus System at 5 candidate buses using

 Optimization Tools

respectively. DGPL of 50.11% was obtained.

4.4.2 Penetration Optimization using NSGA-II

Similarly, the optimized size of PV that can be penetrated without violating the constraint limits were 0.487MW, 0.425MW, 0.086MW, 0.192MW and 0.027MW at candidate buses 68, 49, 56, 30 and 25 respectively. Before penetration, branch 1 had branch current of 136A. However, after penetration optimization using NSGA-II, it is typically 78A, well within limit. Figure 7, 8, 9 and 10 shows the plot of bus voltages, branch APL, bus VSI before and after PV penetration and the plot of fitness function respectively. All the performance indices were improved after PV penetration at DoC Feeder. In this case, DGPL of 50.23% was obtained.



Figure 7: Bus Voltages before and after PV Penetration using NSGA-II



Figure 8: Branch APL before and after PV Penetration using NSGA-II

4.4.3 Comparison of different cases for 72-Bus DoC Feeder

Table 3 shows the optimized values obtained from MOGA and NSGA-II with different values of the performance indices. For the similar condition, MOGA with small sized PV penetrated can improve the system indices significantly. Both the optimization tools were run for 35 times and taking fitness function as the average of 35 runs of the convergence graph, standard deviation and coefficient of variance (C.V.) were obtained. The smaller value of C.V implies that MOGA is consistent.GA algorithm is effective for power loss minimization whereas NSGA-II algorithm is effective for voltage deviation reduction and VSI improvement. However, statistically speaking, hypothesis testing suggests that z=0.1522 < 1.96 implies there is no significant difference between

Optimiza tion Tool	PV Nos. at weak Buses	Weak Buses	PV Size (Kw)	Total PV Size (Kw)	Fitness Value	APL (Kw)	VSI at Weak Bus	VDI	APL Reduc tion (%)	RPL Reduc tion (%)	V Improve ment (%)
Base Case	-	-	-	-	54.27	179.06	0.6188	0.468	-	-	-
MOGA	1 PV	68	1183	1183	18.22	59.34	0.8337	0.058	66.86	66.86	41.02
MOGA	5 PV	68,49, 56, 30,25	572,194, 74,160, 214	1214	17.96	58.46	0.8367	0.051	67.35	67.35	41.69
NSGA-II	1 PV	68	1196	1196	18.23	59.36	0.8358	0.055	66.85	66.85	41.33
NSGA-II	5 PV	68,49, 56, 30,25	487,425, 86,192, 27	1217	18.00	58.70	0.8370	0.051	67.22	67.22	41.77

 Table 3: Multi-Objective Optimization of PV sizes for DoC Feeder at 5 candidate buses using

 Optimization Tools



Figure 9: Bus VSI before and after PV Penetration using NSGA-II

MOGA and NSGA-II for convergency.

Paper [13] had proposed the empirical relation for the Penetration Limit of PVDG that may be injected in any distribution system. Without provoking the line overloads, the expression is given by Equation 13.

$$p_{PV} = 2 * P_{Load} + (1 - S_{Load}) \tag{13}$$

The obtained results in each case of the optimization is lower than the value depicted by the empirical relation. As stated in [1], a greater number of PV DGs can be hosted by a LV network with the value of penetration level tending to 110% if they are uniformly distributed over shorter lengths. The obtained hosting capacity are well below the stated penetration level. The simulation of 72-Bus Dodhara-Chadani Feeder using electrical software also produces the similar results as obtained by the optimization algorithm.



Figure 10: Plot of the Fitness Function



4.4.4 Effect of PV Penetration at different no. of candidate buses

Figure 11: Effect of PV penetrated at different no. of candidate buses

Modal analysis, on the basis of eigen values gives the weakest part of the system. For 72-Bus DoC Feeder, the candidate buses in increasing order of Eigen values from smaller to larger values are 68, 49, 56, 30, 25, 72, 30, 39, 33, 58.... As shown in Figure 11, graph was

plotted to see the effect of PV penetrated at different no. of candidate buses, one to many. It was seen that, beyond 5 nos. of candidate buses, the effect on the size of PV penetrated was almost constant. So, as the effect is similar, it is not wise to take more than 5 nos. of candidate buses for PV penetration.

4.4.5 Effect of Uncertainty of Solar PV at the Feeder Location

To consider the effect of uncertainty of Solar PV, as explained in [14], the realtime solar irradiance and temperature data from province no. 7 was collected, solar PV was modelled with uncertainty added using Beta PDF function to obtain the total PV size including uncertainty that can be available within a year. Taking the size of solar PV throughout the year as input, the optimization tool gave the best location for the solar penetration on 72-Bus DoC feeder. The best location for PV penetration obtained were buses 54, 41, 68, 27 and 14 respectively.Summer season was throughout taken for the analysis.

5. Conclusion

In this paper, both MOGA and NSGA-II optimization tools were used to optimize the penetration of Solar PV at IEEE 33-Bus System as well as Dodhara Chadani Feeder. Simulation results exhibit that the penetration level of Dodhara Chadani Distribution Feeder using MOGA is 50.11% and using NSGA-II, it is 50.23%. Statistical analysis and hypothesis testing implies that the results obtained from MOGA is sufficient to obtain the penetration sizing. It can be concluded that PV Penetration in a RDS can be optimized with multi-objectives by placing Solar PV at all or any candidate buses. Injecting numerous PVs at candidate buses causes increased penetration than that of the case of injecting single PV at the weakest bus. However, conclusion holds under assumption that both load and PV sources are deterministic. This work can be helpful for Distribution System Planner for Solar PV penetration optimization in any real RDS. The work can be extended to load and source variation including aspects like frequency control, reactive power control, transient analysis of a DSN.

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