Proceedings of 10th IOE Graduate Conference
Peer Reviewed
ISSN: 2350-8914 (Online), 2350-8906 (Print)
Year: 2021 Month: October Volume: 10

Parametric Analysis of Load Carrying Capacity of Circular-Cylindrical Grid Shell with Quadrilateral Grid

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Abstract

Grid shells show properties of discrete structures as well as continuous shells due to their topology. For single-layered grid shells, the major failure mode is buckling. The major factors that affect the load-carrying capacity of grid shells are grid element properties, connection property, shell geometry, and imperfections. Analytical solution of buckling problem of grid shell is achieved by establishing equivalency between a grid shell and a continuous shell and applying the analytical equation of continuous shell. Different equivalent models are used to determine equivalent properties and an analytical equation for the continuous shell is modified to accommodate those properties.

The geometry is an open circular-cylindrical grid shell subjected to normal load with simply supported boundary conditions. The analysis parameters are grid size and span to depth ratio. An analytical solution is achieved by solving the buckling equation of the continuous shell in MATLAB for the different equivalent models. Geometries are generated in Rhino6 using the Grasshopper plugin. The numerical solution is achieved by modelling geometry and grid element properties in ANSYS and performing a linear buckling analysis. The result from the analytical and numerical methods is compared. From parametric analysis, it is seen that the load-carrying capacity of grid shells decreases with an increase in grid size and span to depth ratio. The denser grid shows bending-dominated characteristics whereas the coarser grid shows membrane-dominated characteristics. For denser grid, orthotropic equivalence model, and for coarser grid equivalent volume model is suggested.

Keywords

Grid Shell, Circular-Cylindrical Shell, Load-carrying capacity, Equivalent model, Linear buckling analysis, ANSYS

1. Introduction

Shells exhibit more stiffness over plate structures due to their extra curvature. Continuous solid shells are more efficient in covering large spaces and carrying a load over other types of structures. Grid shells also have the properties of the continuous shell but are composed of grids rather than continuous solid surfaces.

1.1 Definition of Grid Shell

In the 1960s, Frie Otto and the Institute of Lightweight Structures developed grid shells as an innovative structural system. A grid shell is described by Edmund Happold as a “Doubly curved surface formed from a lattice of timber bolted together. The lattice is a mechanism with one degree of freedom” [1]. More recently grid shell is defined as ”a structure with the shape and strength of double curvature shell but made of a grid instead of a solid surface. The structure can cross large spans with very few materials. They can be made of any kind of material: steel, aluminium, and wood also” [2]. Grid shells are made of one-dimensional elements. Terms like “latticed shell” or “reticulated shell” are also used for grid shell.

1.2 Advantages of Grid Shells

Grid Shells provide great visual elegance to the structure. They are very efficient for covering large spaces. Some of the advantages of grid shells are highlighted below:

- Grid shells save material to be used due to their discrete topography.

Figure 1: Japan Pavilion [3]

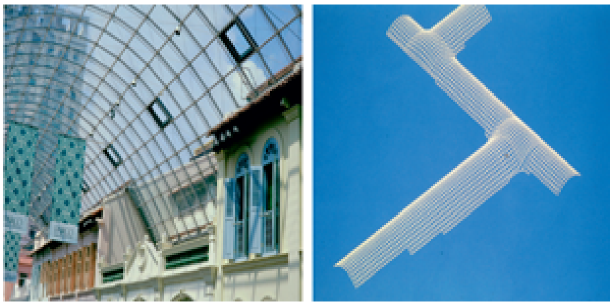

Figure 2: Roof Bugis Street Singapore [4]

- Grid Shell provide a great amount of natural lighting. The intensity of lighting can be varied by varying panel shapes and sizes.
- Grid shells are very efficient in covering existing as well as new spaces.
- Grid shells can be used as temporary structures, lightweight structures and dynamic structures as well.

2. Literature Review

Gioncu (1995) has given a detailed overview of the state-of-the-art on buckling of reticulated shells. He has mentioned fundamentally important factors in the buckling of reticulated shells which are: Form or Global Shape, Reticulation Form, Structural Element and Joint System. Various instability modes have been identified which are: Member Instability, Node Instability, Torsional Instability and Line Instability [5].

Forman and Hutchinson (1970) has presented buckling analysis of few reticulated shells with both equivalent continuum analysis and discrete analysis which is regarded as exact analysis. A shallow section of reticulated spherical shell and infinite reticulated cylindrical shell with the triangular (equilateral) grid has been analysed using both equivalent continuum and discrete analysis [6].

Sumec (1992) has performed linear stability analysis of grid shells. It has been stated that in single layer grid shell, the problem of stability comes forward than the material strength (failure). The grid shell with a triangular grid has been analysed with a continuum approach. The shape of a buckled segment of the surface has been assumed as a spherical segment. Applying the Bubnov-Galerkin method, an analytical closed-form formulation for critical pressure has been derived. The rigidity of joints (rigid & hinged) has been taken into account [7].

Mesnil (2013) has studied the influence of pre-stress on the stability of elastic grid shells. A parametric study has been conducted which focuses on both pre-buckled arch and initially flat circular elastic grid shell with different grid spacing and levels of pre-stress. Realistic values for analysis parameters have been determined from the existing projects. A pre-buckled 2D Arch has been chosen for the validation of the computational method. Firstly, a comparative analysis of the buckling capacity of the unstressed and pre-stressed arch has been prepared. Secondly, the buckling analysis, as well as the form-finding of different structures, have been performed using finite element analysis. It has been found that for high levels of pre-stress, an elastic grid shell and grid shell have the same bending mode shapes. Elastic grid shells are subject to two competing effects: the geometrical stiffens and loss of stiffness due to pre-stress [1].

Malek (2012) has done a parametric study of the buckling load of grid shells varying the topology and topography. Spherical cap and corrugated barrel vault have been chosen for analysis. For spherical cap grid shells, the effect of grid size grid shape and span to depth ratio on buckling load has been studied. For corrugated barrel vault grid shells, the effect of corrugation has been studied. Different equivalent models have been used to establish equivalency between grid shell and equivalent continuous shell. Expression for buckling of the continuous shell has been used to determine the buckling load of grid shells analytically. A 2D Arch analysis is prepared to establish the accuracy of FEM. Linear buckling analysis has been done for analysis parameters. It has been concluded that a triangular grid is better for steeper shells (spherical cap). A denser grid is recommended for the shallower shell [3].

3. Problem Definition and Research Objectives

Despite various advantages, grid shell has not been a structure used frequently in the world. The main reason for less use might be due to its complexity in structural analysis and construction process. Grid shells carry their loads mainly by compression force. It has been well established that buckling is the dominant failure mode of single-layered grid shells [8]. It is assumed that individual member of the grid shell remains straight and stable during buckling. There is a debate about whether a quadrilateral grid or triangular grid is efficient. Are grid shells bending or membrane dominated? How much singularity and imperfections affect the buckling load? The variation of load-carrying capacity with variation in topology and topography is also a subject of research.

3.1 Analysis Parameters

From the review of the previous works done on grid shells, it can be deduced that the following are the parameters that affect the load-carrying capacity of grid shells.

- Span and depth (span to depth ratio)
- Grid shape
- Grid Size

Imperfections and Joint Rigidity are the other factors that affect the load-carrying capacity. But these factors are not in the scope of this paper.

3.2 Research Objectives

The objectives of this paper are: for circular-cylindrical grid shell with quadrilateral grid

- Compare the buckling load calculated from analytical and numerical method and suggest a better equivalent continuum model approximation.
- Analyze the effect of grid size and span to depth ratio on the load-carrying capacity.
- Identify bending dominated and membrane dominated characteristics.

4. Methodology

The parameters required for analysis has been deduced from the literature. The numerical value of parameters for analysis is identified from previously built grid shells. An insight on analysis methods, their usefulness and limitations are very necessary.

4.1 Selection of Analysis Parameters

In the recent decade, many grid shell structures have been constructed around the world. The structures are built individually as per requirement. Various materials (from steel to cardboard) and geometries have been used. These grid shells have varying values of a combination of parameters. The analysis parameters and grid element properties are summarized in **Table 1**.

Table 1: List of properties of grid shell and analysis parameters

Global Geometry	Open circular-cylindrical
Span (L)	15000mm
Length (l)	30000mm
Span to depth ratio (r)	5, 14, 20
Grid Shape	Quadrilateral (Single Layered)
Grid Size/Spacing of Grid Element (s)	500mm, 1000mm, 1500mm
Grid Element (rod) Size	50mm x 50mm
Cross-Section Area of Grid Element (A)	2500mm^2
Moment of Inertia of Grid Element (I)	5.208 x 10^{5}mm^4
Torsional Moment of Inertia of Grid Element (J)	8.813 x 10^{5}mm^4
Modulus of Elasticity (E)	210000 N/mm^2
Poission‘s ratio (ν)	0.3
Modulus of Rigidity (G)	80769.23 N/mm^2

4.2 Method of Analysis

The basic process of solving a problem is identification, formulation and solution. A discrete (grid) shell is idealized as a continuous shell establishing some equivalence. Results for the continuous shell are applied to the discrete shell with those equivalencies.

In engineering, every problem may not result in a closed-form solution. With increasing complexity in

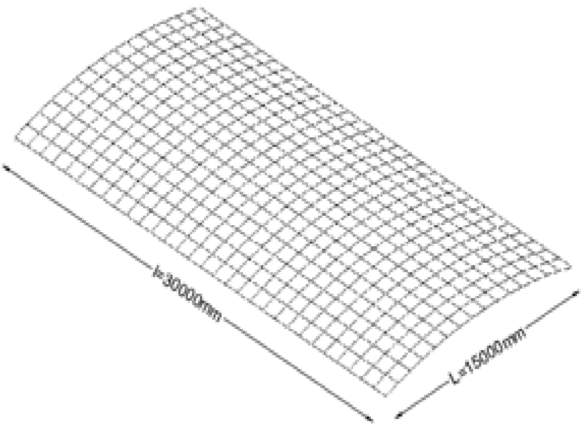

Figure 3: Geometric properties of grid shell

idealization and formulation of the problem, the exact solution becomes less feasible. Numerical methods are applied for those problems whose exact solutions are not available. FEM is a common tool for solving such complex problems. Finite element models are prepared for each combination of parameters and results are compared with analytical solutions.

4.2.1 Analytical Method

To analyse grid shells, an approach has to be formulated. The approach considered here is the continuum approach. In this approach, some equivalencies are established between a continuous and a grid shell. A grid shell is related to a continuous shell the latter being as its calculation model. A shell has to transfer the load either by membrane action or by bending action or by a combination of them. The load transfer characteristics are defined by its axial (membrane) or bending rigidity. The thickness has to be decided based on the trade-off of whether the shell has to transfer load by membrane action or bending action. C and D are the axial and bending rigidity which are given in **Eq. 1** & **Eq. 2** respectively.

$$C = \frac{Eh}{(1-\nu^2)} \tag{1}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2}$$

A relationship between the element of grid shell (rod) and shell's axial and flexural rigidities for different equivalent models is illustrated in **Table 2**.

The equivalent properties (C_{eq}, D_{eq}, ν_C & ν_D) are calculated for quadrilateral grid with variation in grid size as other parameters are constant in this analysis. Equation of buckling of the continuous shell under normal pressure loading and simply supported boundary condition has been derived by Timoshenko [11]. The equation is modified to accommodate the equivalent properties, which is given in **Eq. 3**.

Table 2: Summary of equivalent models and equivalent properties

Equivalent Models	Equivalent Properties		
	h_{eq}	C_{eq}	D_{eq}
Equivalent Volume	$\frac{A}{s}$	$\frac{Eh_{eq}}{(1-\nu^2)}$	$\frac{Eh_{eq}^3}{12(1-\nu^2)}$
Equivalent Area	$\frac{2A}{s}$	$\frac{Eh_{eq}}{(1-\nu^2)}$	$\frac{Eh_{eq}^3}{12(1-\nu^2)}$
Equivalent Moment of Inertia	$(\frac{12I}{s})^{\frac{1}{3}}$	$\frac{Eh_{eq}}{(1-\nu^2)}$	$\frac{Eh_{eq}^3}{12(1-\nu^2)}$
Split Rigidity[9]		$\frac{EA}{s}$	$\frac{EI}{s}$
Orthotropic Equivalence [10]		$C_{eq} = \frac{EA}{s}$ $\nu_C = 0$	$D_{eq} = \frac{EI}{s}$ $\nu_D = 0$

$$\phi = \frac{qa}{C_{eq}}, \alpha = \frac{D_{eq}}{C_{eq}a^2}, \lambda = \frac{\pi x}{l}$$

$$\begin{aligned}
(-\lambda^2 - \frac{1-\nu_C}{2}n^2)U_n + (\frac{1+\nu_C}{2}n\lambda + n\lambda\phi)V_n & \\
+\lambda(\nu_C+\phi)W_n &= 0 \\
(\frac{1+\nu_C}{2}n\lambda)U_n & \\
-(\frac{1-\nu_C}{2}\lambda^2 + n^2 + \alpha(1-\nu_D)\lambda^2 + \alpha n^2)V_n & \\
-(n + \alpha n\lambda^2 + \alpha n^3)W_n &= 0 \\
(\nu_C\lambda)U_n - (n + \alpha n^3 + \alpha(2-\nu_D)n\lambda^2)V_n & \\
-(1 + \alpha\lambda^4 + \alpha n^4 + 2\alpha n^2\lambda^2 & \\
+\phi(1-n^2))W_n &= 0
\end{aligned} \tag{3}$$

A program in MATLAB is written to calculate the buckling load of grid shells analytically as a matrix of grid size and continuum model for each value of span to depth ratio. The final results are presented as graphical plots of buckling load vs grid size for each value of span to depth ratio.

4.2.2 Numerical Method

For finite element analysis, geometries are generated in Rhino6 & Grasshopper with a maximum length error of 1.9%. 9 geometrical models are created for analysis parameters. The geometries are imported in

ANSYS SpaceClaim and models for further analysis are created. The element of grid shell (rod) is modelled as beam element in ANSYS Material properties & cross-section is assigned and mesh is generated with an element size of 50mm. Point force is applied at vertices (joints) in radial (normal) direction and simply supported boundary condition is applied. Linear buckling analysis is performed to calculate the buckling load of the grid shell.

5. Results

5.1 Model Verification

For the validation of a more complicated computational modelling technique required for grid shell, simpler failure mode and buckling mode of 2D Arch is analysed. 2D Arch analysis has been used in [1, 3] for verifying modelling techniques.

5.1.1 2D Arch Analysis

Buckling Load Convergence: Timoshenko [11] has given expression for buckling load for a circular arch under normal loading which is given in **Eq. 4**. Both analytical and finite element analysis results are compared and the accuracy of FEM is established.

$$q_c = \frac{EI}{R^3}\left(\frac{\pi^2}{\alpha^2} - 1\right) \tag{4}$$

The results are shown in **Figure 4**. Applying normal load to an arch in ANSYS Workbench is not possible so instead vertical load is applied. As the arch becomes shallow, the difference between vertical and normal load becomes smaller and the error between analytical and finite element results becomes smaller. The maximum error between analytical and FEM results is 9.281% for r=5 and reduces with reduction in the value of r. The error is also due to applied vertical load instead of normal load. But in the case of grid shell, a normal load can be applied.

Load Equivalency: In the analytical solution of 2D Arch, the load applied is linear pressure load. But for grid shells, the load applied is point load at joints. So, it is necessary to establish that analytical solution assuming pressure load can be used for grid shells when the applied load is point load at joints. If P is point load, N is no. of node and S is the arc length of the arch then buckling load can be determined from **Eq. 5**. For arch of r=14, the error between analytical result and the FEM result is 4.35% with 300 elements.

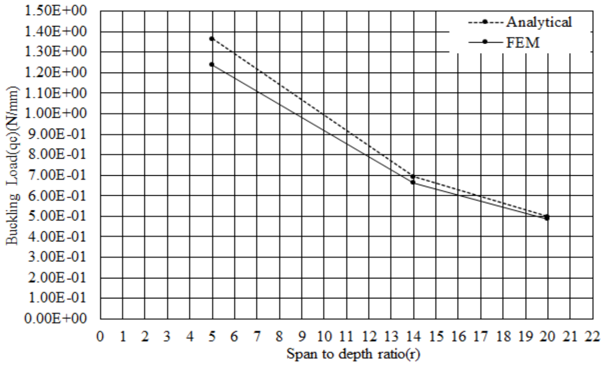

Figure 4: Comparisons of buckling load of 2D Arch

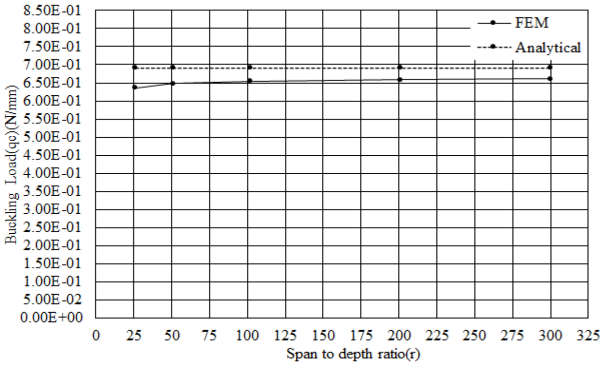

Figure 5: Load equivalence for the arch (r=14)

The result also shows FEM results for both pressure load and point load are almost the same.

$$q_c = \frac{PN}{S} \tag{5}$$

5.1.2 Continuous Shell Analysis

To proceed for further analysis, it becomes necessary to establish the analytical validity of the equation derived to calculate buckling load of continuous shell and source code written for calculating buckling load of grid shell. Also, the accuracy of FEM in solving buckling problems is established. 6 models of the continuous shell with different thicknesses are prepared in ANSYS and buckling loads of respective models are determined. The model uses shell elements and the maximum error between analytical and finite element results is 0.55% for the first buckling mode. Because the corresponding buckling load has only 0.55% error from the analytical result this error is negligible.

5.2 Analytical Results

Buckling load of grid shell calculated analytically is presented as a graphical plot from **Figure 6** to **Figure**

8. The graphical plots clearly show that the equivalent moment of the inertia model and equivalent area model gives an upper and lower bound of the behaviour of the grid shell.

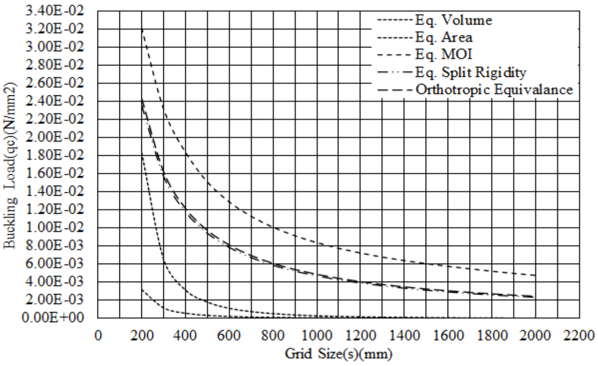

Figure 6: Analytical buckling load of grid shell (r=5)

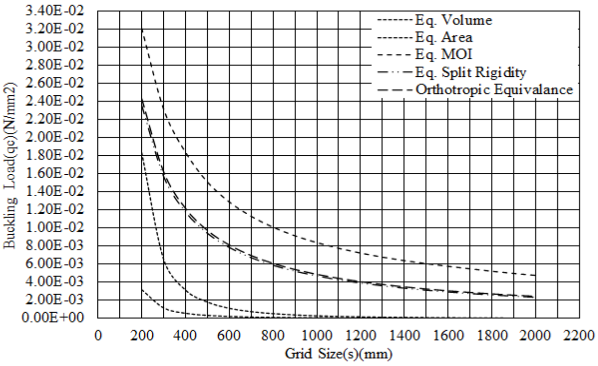

Figure 7: Analytical buckling load of grid shell (r=14)

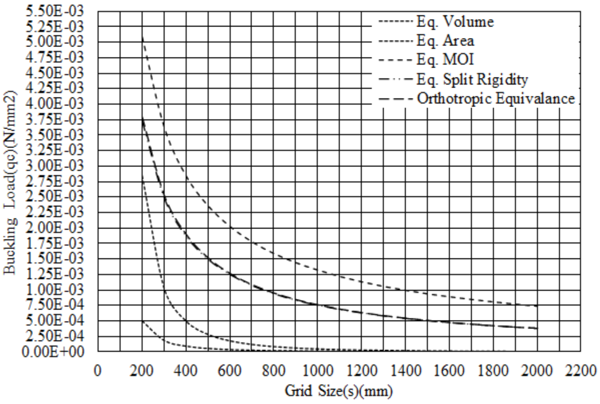

Figure 8: Analytical buckling load of grid shell (r=20)

5.3 Numerical results

The objective of this paper is to study the change in load-carrying capacity of grid shells as a function of grid size and span to depth ratio. The output of finite element analysis is presented in **Figure 9**. Buckling load as uniform pressure is calculated as $q_c = \frac{Pointload}{Area(s^2)}$. The result of finite element analysis clearly shows the buckling load of the grid shell reduces significantly with an increase in grid size. Considering the effect of span to depth ratio, the shallower shell has less buckling load.

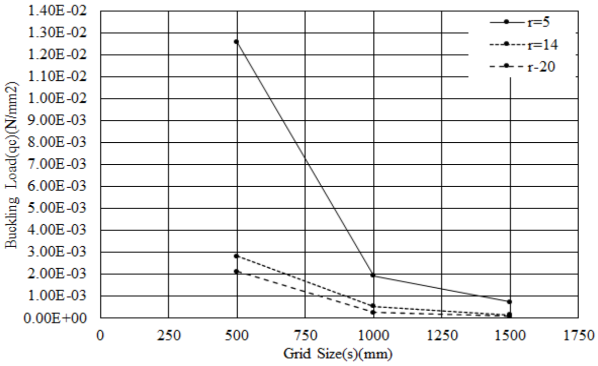

Figure 9: Numerical buckling load of grid shell

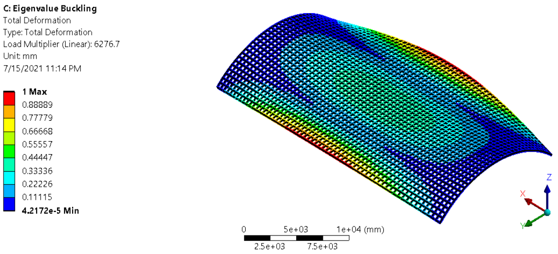

Figure 10: The deformed shape of grid shell (r=5, s=500)

5.4 Comparison of Analytical and Numerical Results

From **Figure 11** to **Figure 13** plots both numerical and analytical results for all values of span to depth ratio for the quadrilateral grid.

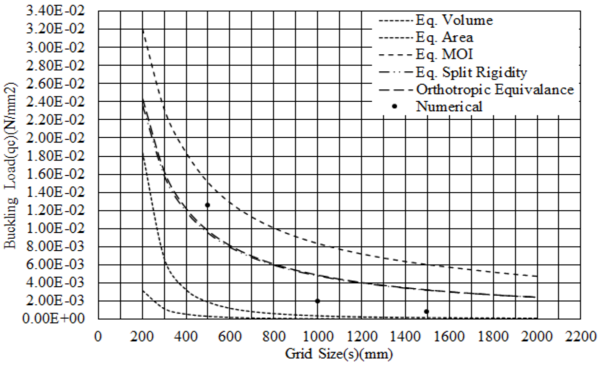

Figure 11: Comparison of buckling load r=5)

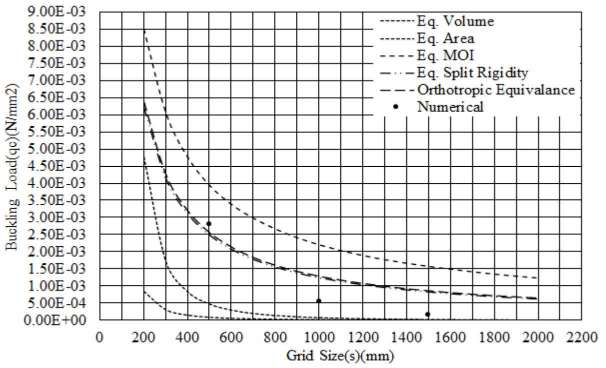

Figure 12: Comparison of buckling load r=14)

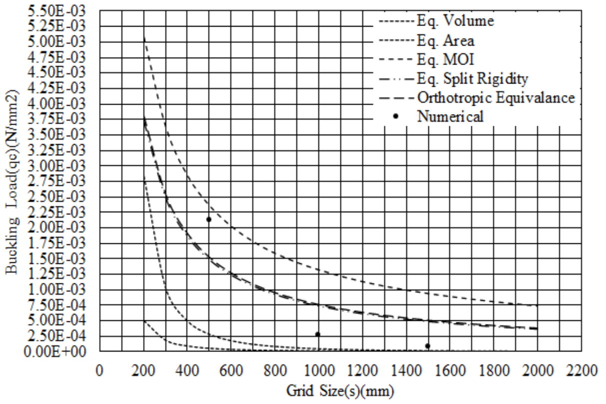

Figure 13: Comparison of buckling load r=20)

The analytical solution for the equivalent area model and equivalent moment of inertia model always gives the lower and upper bound solution respectively. All the numerical results fall well between the lower and upper bound. From the graphical plots, it is clear that the denser grid shows more bending dominated characteristics whereas the coarser grid shows more membrane dominated characteristics. A factor β_q is defined in **Eq. 6** which gives the degree of membrane characteristics. Value of $\beta_q > 1$ indicates that the behaviour of the grid shell is membrane dominated.

$$\beta_q = \frac{q_{cEq.MOI} - q_{cNumerical}}{q_{cNumerical} - q_{cEq.Area}} \tag{6}$$

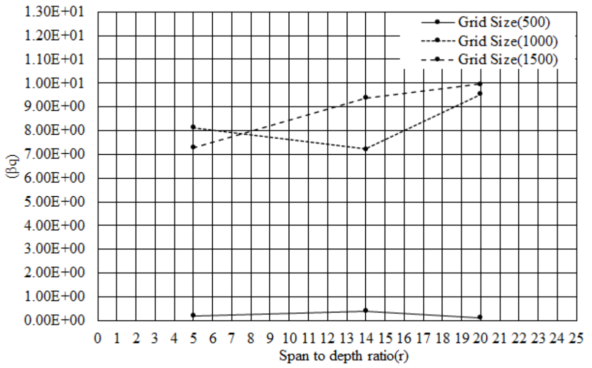

Figure 14: Degree of membrane dominance (β_q)

Figure 14 shows the variation of β_q with grid size and span to depth ratio. The value of β_q for grid size 1500mm increases with increase in span to depth ratio and for grid size 1000mm it decreases slightly and then increases but the value of β_q remains well above 1. For grid size 500mm value of β_q increases slightly and then decreases but its value remains well below 1. So, it can be concluded that for the denser grid (500mm) grid shell becomes more bending dominated with the increase in shallowness but for the coarser grid (>1000mm) grid shell becomes more membrane dominated with the increase in shallowness. The equivalent model cannot estimate the buckling load of the grid shell with considerable accuracy. But the equivalent model can yield a conservative value at the beginning of the design process which can omit the risk of changing the parameters after structural design. For coarser grid size (>1000mm) equivalent volume model is the best model to calculate the buckling load of the grid shell. For denser grid size (500mm) orthotopic equivalence model can yield a conservative value of buckling load.

5.5 Discussion

In this Paper an open circular-cylindrical shell of span (L) 15000mm, length (l) 30000mm and span to depth ratio (r) 5, 14 & 20 are considered for analysis. In the design and construction of the grid shell, its geometrical properties are described in terms of span and span to depth ratio. But in analytical solution span and radius of curvature describes the geometrical properties. For span to depth ratios 5, 14 & 20 the radius of curvature (the radius for cylindrical shell) are 10875mm, 26785,71mm & 37875mm respectively. The results can also be analysed in terms of the radius of curvature. Ratio L/a describes the type of shell as short, intermediate and long. The range of L/a with equivalent thickness from equivalent models indicates the grid shell considered is intermediate. For the intermediate shell, the value of buckling load is different for different parameters but the pattern of values remains the same for the equivalent models. As the numerical results fall well within the bound, the conclusions drawn from the result of this Paper can be applied for intermediate grid shells.

The failure mode of a single-layered gird shell is buckling. So, higher the value of buckling load higher is the load-carrying capacity of the grid shell. Grid sizes considered here are 500mm, 1000mm and 1500mm. A grid size of about 500mm is considered a denser grid whereas a grid size $\geq$ 1000mm is considered a coarser grid. As the grid size increases load-carrying capacity of the grid shell decreases. For a steeper shell a coarser grid can also result in a significantly high load carrying capacity but for a shallower shell coarser grid results in a very low load-carrying capacity. A coarse grid can be used for steeper grid shells but the use of a dense grid is suggested for shallower grid shells.

The numerical value of buckling load of grid shell falls well within the bound of membrane and bending characteristics. A denser grid shows bending dominated characteristics whereas a coarser grid shows membrane dominated characteristics. With an increase in span to depth ratio degree of bending characteristics increases for the denser grid but the coarser grid degree of membrane characteristics increases. The load-carrying capacity of grid shells decreases with increases in span to depth ratio. Different equivalent models act as a tool for calculating the load-carrying capacity of grid shells analytically. Equivalent area and equivalent moment of inertia give the lower and upper bound for it. For coarser grid equivalent volume model and, for denser grid orthotropic equivalence model can be used to calculate the load-carrying capacity of the grid shell.

6. Conclusion

The objectives of this paper are achieved by conducting a parametric analysis varying grid size and span to depth ratio and calculating the load-carrying capacity of open circular-cylindrical grid shell using both analytical and numerical methods. The process and results in this Paper are concluded in the following points:

1. Different continuum models are used to define grid shells as an equivalent continuous shell. The equation for calculating the buckling load of the continuous shell is modified to accommodate the equivalent properties of the grid shell and an analytical solution is achieved. Geometries are modelled in ANSYS and the numerical solution is achieved. Comparison of analytical and numerical results is presented in graphical plot form.

2. For coarser grid equivalent volume model and denser grid, orthotropic equivalence model can be used to calculate the load-carrying capacity of grid shell.

3. Denser grid shows bending dominated characteristics whereas the coarser grid shows membrane dominated characteristics. The bending or membrane dominance characteristics are defined based on the closeness of numerical value of buckling load with the equivalent moment of inertia model (Upper bound) and equivalent area model (Lower bound) respectively.

4. For denser grid bending dominated characteristics increases with an increase in shallowness whereas for coarser grid membrane dominated characteristics increases with an increase in shallowness.

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