# Performance Evaluation of Slow Sand Filter Using Fuzzy Rule-Based System

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### Abstract

Most chemical and biological systems are not linear in nature. To account for the non-linearity of the processes in slow sand filters, fuzzy rule-based system with classification was used to model slow sand filter in Siddhipur Water Treatment Plant, Lalitpur. Membership functions were created using regular hierarchy, and rule conclusions were generated by using the values which have the most matching degree. A total of 82 data rows of inlet turbidity, filter runtime and outlet turbidity was collected for 3 filter cycles for the modeling. Takagi-Sugeno method was used for the fuzzy inference, and outlet turbidity was used as the output variable. Both modeling and validation of the model was done. The modeling showed accuracy of 75.6 % over all the data rows. For validation, the model showed accuracy of 81.25 %. The observed and inferred outlet turbidities showed a value of mean absolute error of 0.2561. Open-source software FISPRO was used for fuzzy modeling.

# Keywords

filtration, sand, model, fuzzy, grey box

# 1. Introduction

Slow sand filtration is the one of the oldest water filtration technology to improve the physical, chemical and biological qualities of water. It performs by the combination of several mechanisms, most important of which straining and microbiological action. In the absence of biological processes, it has been established that the process mechanism of slow sand filters is the same as that of rapid sand filters [1]. Modeling of slow sand filters would provide a better understanding of slow sand filters, but very less work has been done.

Fuzzy rule-based modeling has been used to model total coliform (TC) removal efficiency [2] recently by taking sand grain size, sand bed depth and filtration rate as input variables. Fuzzy rule-based modeling is a non-linear and has been used extensively in other fields. It has been used in medical diagnosis [3], water quality [4] and many industrial applications [5]. The fuzzy rule-based modeling was done by using the software FISPRO [6], an open-source software for fuzzy inference.

In Fuzzy Rule-Based Modelling, the relationships

between various variables (input variables and output variables) are connected with the aid of if-then rules of the form "IF antecedent proposition THEN consequent proposition" [2]. Based on the crisp or fuzzy form of the consequents, the models are either: linguistic fuzzy models where the consequent is of fuzzy form, or Takagi-Sugeno (TS) fuzzy models [7] where the consequent is of crisp form.

The Mamdani system [8] gives a fuzzy output (linguistic fuzzy model) which has to be defuzzified into a crisp value for further interpretation. In Takagi-Sugeno models, no defuzzification is necessary. Using a weighted average of the rules of the consequents, a crisp result is obtained [7].

# 2. Methodology

Fuzzy rule-based modeling was used to model outlet turbidity of slow sand filters located in Siddhipur Water Treatment Plant, Lalitpur. The model was created by defining inputs, a set of fuzzy rules, a method for fuzzy interpretation and output. The input variables used are inlet turbidity and filter runtime, and the outlet turbidity was used as the output. A total of 82 data rows were used which were collected for three filter cycles. The filter was designed with dimension of 12 m length, 8 m width and 3 m height and the filtration rate was 0.19 m/hr.

Takagi-Sugeno system [7] was the first major work to have introduced fuzzy inference system design using a data-driven approach [8]. In FISPRO, with the opening of data in a text comma separated values format, first a fuzzy inference system (FIS) is generated without rules. The input parameters, inlet turbidity and filter runtime are partitioned regularly with respect to the data available to generate the membership functions. The generated FIS has as many variables as the data file has columns. Here it must be noted that the output variable conclusions are generated as crisp values, which are simply taken as the majority class  $c^i$  in observed outputs, i.e., outlet turbidity. This subset  $c^r$  for the  $r_{th}$  rule, is selected for each rule.

The final rules generated using the FISPRO are shown in Figure 1.

Rule	IF Inlet	And Filter Runtime	THEN Outlet
1	low	low	1
1	IOW	IOW	1
2	low	medium	1
3	low	high	2
4	medium	low	2
5	medium	medium	2
6	medium	high	4
7	high	low	5
8	high	medium	2
9	high	high	5

**Figure 1:** Fuzzy rules created for modeling

By the rules generated, it can also be concluded that the outlet turbidity is minimum i.e., 1 NTU in two different conditions:

a. if the inlet turbidity is low and filter runtime is lowb. if the inlet turbidity is low and filter runtime is medium

Similarly the outlet turbidity is maximum i.e., 5 NTU in two different conditions:

a. if the inlet turbidity is high and filter runtime is lowb. if the inlet turbidity is high and filter runtime is high

The activation levels for max operator after aggregation is given by equation (1).

$$w^{i} = max_{r}(w^{r}(x))|c^{r} = i$$
(1)

In equation (1),  $w^i$  is the membership value, at the required input values  $x_{i1}$  which was inlet turbidity and  $x_{i2}$  which was the filter runtime. The Sugeno operator calculated the inferred value of outlet turbidity using defuzzification operation, given by equation (2) [7].

$$y_i = \frac{\sum_{i=1}^m w^i c^i}{\sum_{i=1}^m w^i} \tag{2}$$

Classification method was used for the output, since our data contained discrete classes of turbidities from 1 NTU to 5 NTU. Rules were generated using the FIS menu in FISPRO, which creates all the rules with the maximum matching degree using a threshold weight of 0.1. The rules are written in the form of if-then statements. This initializes the rule conclusions to a default value of 1. After rules and output variable conclusions are generated, Sugeno operator was used as given by equation (2) for inference. The results were rounded to the nearest whole number value finally. After the model was created, it can be used to infer the outlet turbidity for any value of inlet turbidity and filter runtime. This can be done by importing the created model into FISPRO in the field, and thus the outlet turbidity is predicted.



Figure 2: Fuzzy rule-based modeling process

16 data rows were collected after the study for input turbidities and filter runtime to test the accuracy of the model.

# 3. Results

Before any fuzzy inference, construction of membership functions of inlet turbidity and filter

runtime was necessary. The membership functions were created using regular algorithm, where average of the minimum and the maximum value of the input variable was used to find the mid-point. Using the mid-point, triangular membership functions were generated for both inlet turbidity and filter runtime.



**Figure 3:** Membership function graph for inlet turbidity (NTU)

The inlet turbidities ranging from 3 NTU to 8 NTU were referred as low inlet turbidities, from 3 NTU to 13 NTU were medium turbidities and from 8 NTU to 13 NTU were high inlet turbidities. For example, inlet turbidity of 5 NTU 40 % low and 60 % medium, as can be seen from the membership at that value.



**Figure 4:** Membership function graph for filter runtime (hrs)

The filter runtime from 0 hrs to 420 hrs was referred to as low filter runtime, from 0 hrs to 840 hrs was referred as medium filter runtime and from 420 hrs to 840 hrs was referred to as high filter runtime. The data menu of FISPRO was used for fuzzy rule-based modeling. During the inference, a blank threshold of 0.1 was used, and links between rules and data was activated. The inference showed an error in 20 data points and coverage of 100%. The maximum error encountered in the predicted outlet turbidity was 1 NTU.

Using FISPRO, a 'confusion matrix' was generated which gives a cumulative matching degree of the inferred outputs with observed ones. The matching degree which varies from 0 and 1, is computed using the intersection of the inferred possibility distribution with output fuzzy sets, and also considering distribution width. The result is cumulated over all the data file items, and a matrix is generated.

**Table 1:** Inferred and observed outlet turbidities fromFuzzy Rule-Based Model

Inf/Oha	1	2	2	4	5
III/ODS	1	Z	3	4	3
	NTU	NTU	NTU	NTU	NTU
1 NTU	24	4	0	0	0
2 NTU	9	24	0	0	0
3 NTU	0	4	12	0	0
4 NTU	0	0	2	2	1
5 NTU	0	0	0	0	0

The table shows that for all observation that 1 NTU were inferred, 24 data points were well classified but 4 data points are misclassified. Similarly, for all observation that 2 NTU were inferred, 24 data points were well classified but 9 data points were misclassified and were inferred to 1 NTU class. This was majorly due to large number of data points consisting of outlet turbidity as 1 NTU and 2 NTU.

For all observation that 3 NTU were inferred, 12 data points were well classified but 4 data points belonging to 2 NTU class were inferred to 3 NTU class. Additionally, for all observation that 4 NTU were inferred, 2 data points were well classified but 2 data points belonging to 3 NTU classed were inferred to 4 NTU class and 1 data point belonging to 5 NTU were inferred to 4 NTU class.

The inferred and observed outlet turbidity with respect to filter runtime are shown in Figure 5.



**Figure 5:** Inferred outlet turbidity (NTU) plotted against observed outlet turbidity (NTU)

A mean absolute error (MAE) of 0.2561 was obtained, which was small in value, with respect to the minimum value of observed outlet turbidity i.e., 1 NTU.

For testing the model, 16 data rows were collected after the study. The results after model was run on the data collected are shown in Table 2.

Table 2:	Testing	of Fuzzy	Rule-Based	Model
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Inlet	Filter	Observed	Predicted
Turbidity	Runtime	Outlet	Outlet
(NTU)	(Hrs)	Turbidity	Turbidity
		(NTU)	(NTU)
3	0	1	1
4	192	1	1
10	240	3	3
9	360	2	2
5	408	1	1
7	432	2	2
11	456	3	3
8	480	1	2
6	504	2	2
4	528	2	2
13	552	3	3
12	576	3	3
6	600	3	2
5	624	2	2
8	696	2	3
10	720	4	4

This showed that out of 16 observations, the model gives accurate results on 13 data rows. This corresponds to accuracy of 81.25 %. This validates fuzzy rule-based system for further use to predict outlet turbidity for slow sand filters.

# 4. Conclusions

Since physical and biological systems do not generally follow linear and exponential trends, the use of statistical method is very difficult. Such non-linear systems are easily handled by fuzzy models, on the basis of data available. The model generated was fairly accurate and gave the right classification result 75.6 % of the times. The model can be made more accurate with more data points collected. Fuzzy rule-based model can be used with more number of inputs and more number of output, the selection of which are possible using expert knowledge in the field the modeling is pertinent to. The major advantages of fuzzy-rule based models are:

a. uncertainities in the input can be incorporated into the modeling process

b. linguistic information can be quantified suing experience and expert judgement

c. very flexible models due to availability of different inference methods

In addition to the clear advantages of fuzzy rule-based system, the implementation of this model in the field is easy and effective. With the installation of FISPRO, the model can be imported. After that, a comma seperated file with three columns of 'Inlet', 'Filter Runtime' and 'Outlet' can be created easily, and the data rows are filled with the observations and opened in FISPRO. The data menu of FISPRO has an 'INFER' option, which can be used to import the prediction results as a text file. The results can be obtained for any random values of inlet turbidity and filter runtime. With more observations being available, the accuracy of the model also increases.

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