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Probabilistic Load Flow of Integrated Nepal Power System using Point Estimate Method

Soni Kumari Sah [a], Nava Raj Karki [b]

[a, b] *Department of Electrical Engineering, Pulchowk Campus, IOE, Tribhuvan University, Nepal*
Corresponding Email: [a] sonisah206@gmail.com, [b] nrkarki@ioe.edu.np

Abstract

Load flow analysis is an important tool used for planning, operation, protection and determining the steady state operation of electric power systems. Probabilistic load flow considers input uncertainties and aims to overcome the limitations of deterministic load flow in case of consideration of uncertainties of input parameters. This paper presents probabilistic load flow analysis of Integrated Nepal Power System (INPS) considering uncertainty of generation and load parameters using 2m+1 scheme of Point Estimate Method (PEM). PEM replaces the probability distribution of the random input variable with a finite number of discrete concentrations in such a way to preserve required probabilistic information of random input variable. Results of PEM obtained for IEEE-14 bus test system and INPS are presented and also compared against those obtained from Monte Carlo Simulation technique for checking the accuracy and suitability of PEM.

Keywords

Probabilistic Load Flow (PLF), Point Estimate Method (PEM), Monte Carlo Simulation (MCS), Integrated Nepal Power system (INPS)

1. Introduction

In power system, uncertainties mainly occur due to variation of loads, generation outage, faults and failure in power system networks. So, the input parameters of load flow behave as random variable and it is desirable to obtain system output parameters for all possible range of variation of these input parameters using load flow analysis. Deterministic load flow methods consider a particular specific set of input parameters like load, generation, line parameters, etc to find system output parameters. To incorporate all system uncertainties through deterministic load flow approach, it becomes necessary to run them for all possible sets of system states which turns out to be impossible for present network system. Hence, from practical point of view probabilistic methods of load flow is convenient approach to incorporate power system uncertainties and produce desired results in short time.

In literature, there are mainly three probabilistic methods to deal with uncertainties: Monte Carlo Simulation (MCS), analytical methods and approximate methods.

MCS [1, 2] is most accurate method to solve probabilistic load flow and used as benchmark method to compare other methods. It randomly generates input variables and deterministic load flow is run at generated input values. Its main drawback is that it requires large number of iterations to converge. Another way to solve probabilistic load flow (PLF) problems are analytical methods including linearization, multi-linearization involving convolution techniques, cumulants and quadratic PLF. These methods use probability density function (PDF) or statistics of the input random variable and need some assumptions to solve the dependency between the input random variables and the non-linearity of power equations [3] so the complexity and accuracy of problems increases with size of power system.

Approximate methods provide an approximation of statistical properties of random output varibales. First-order second-moment methods (FOSMM) and point estimate methods (PEM) are two methods in this group. FOSM[4] linearizes the power flow equations by using the first two terms of the Taylor expansion series and approximate the mean and standard deviation of the output variables directly. Its main drawbacks is the linearization requirement and its inability to produce more statistical data than only the mean and standard deviation of the output variables.

Of all mentioned methods, PEM is one of the most used methods to solve probabilistic problems [3, 5, 6] due to various reasons: it runs deterministic load flow equations like MCS with lower computation burden, it does not require complete knowledge of density functions of random input variables since it approximate these functions with statistical moments and gives approximation of raw moments of output variables. Using raw moments, PDF and CDF curves of output parameters can be obtained using Gram-Charlier expansion method [7, 8].

2. Mathematical Model

2.1 Point Estimate Method

The PEM is an approximate method, so based on knowledge of the random nature of the input variables, approximate random behavior of the output variables is found. This approximation is achieved on the basis of a weighted sum of the results achieved by a number of deterministic load flow (DLF) run for strategic states with regards to the input variables. PEM approximate strategic states using few statistical moments depending upon type of scheme used. This paper uses $2m+1$ scheme of point estimate method. Comparative performance of Hong's PEM schemes is done in [6] and shows benefits of $2m+1$ scheme. This scheme gives results as accurate as $4m+1$ with only $2m+1$ times run of DLFs but is faster than $4m+1$. In comparison to $2m$ scheme it only require one more load flow, but results are more accurate and do not depend on input variables. The random input variables of PEM can be denoted as $x_l(l = 1, \ldots.., m)$. Probabilistic output Z can be expressed as Equation (1) such that F (which then represents a regular DLF) is the function expression relating the input and output variables.

$$Z = F(x_1, x_2, \ldots..x_m) \tag{1}$$

According to [6] each input variable x_l will produce K points(concentrations). The k^{th} concentration $(x_{l,k}, w_{l,k})$ of a random variable x_l can be defined as a pair composed of a weight $w_{l,k}$ which accounts for relative importance of the input variable in output and a location $x_{l,k}$ which is the k^{th} value of variable x_l at which the function F is evaluated. The location $x_{l,k}$ to be determined is given by Equation (2).

$$x_{l,k} = \mu_{x_l} + \zeta_{l,k}\sigma_{x_l} \qquad l = 1, .., m \qquad k = 1, .., K \tag{2}$$

In $2m+1$ scheme $(K = 3, \zeta_{l,3} = 0)$, three locations are considered for each variable x_l and the third location $x_{l,3}$ is set to be equal to the mean value of x_l. The standard locations and weights of x_l are:

$$\begin{aligned} \zeta_{l,k} &= \frac{\lambda_{l,3}}{2} + (-1)^{3-k}\sqrt{\lambda_{l,4} - \frac{3}{4}\lambda_{l,3}^2}, \text{ for } k = 1, 2 \\ \zeta_{l,3} &= 0, \text{ for } k = 3 \end{aligned} \tag{3}$$

$$\begin{aligned} w_{l,k} &= \frac{(-1)^{3-k}}{\zeta_{l,k}(\zeta_{l,1} - \zeta_{l,2})} \text{ for } k = 1, 2 \\ w_{l,3} &= \frac{1}{m} - \frac{1}{\lambda_{l,4} - \lambda_{l,3}^2} \end{aligned} \tag{4}$$

Once the location and weight for all input variables are found for $k = 1, 2$, the j^{th} order raw moment of the output vector Z is found using Equation (5).

$$E(Z^j) = \sum_{l=1}^{m}\sum_{k=1}^{K} w_{l,k}\,(Z_{l,k})^j \tag{5}$$

Where, $Z_{l,k}$ is the function expression evaluated at the mean of all variables except x_l, whose mean is replaced by location $x_{l,k}$, i.e. $Z_{l,k} = F(\mu_{x_1}, \mu_{x_2}, \ldots.x_{l,k}, \ldots \mu_{x_m})$.

For $k = 3$, function F will be evaluated once with all variables at their expected values $\mu_{x_l}(l = 1, 2, \ldots\ldots.m)$, with the corresponding weight w_0 equal to the sum of the third-location weights of all variables:

$$w_0 = \sum_{l=1}^{m} w_{l,3} = 1 - \sum_{l=1}^{m} \frac{1}{(\lambda_{l,4} - \lambda_{l,3}^2)} \tag{6}$$

Hence the required number of evaluations of F is $2m+1$, even though three locations are used for each of the m input variables.

To overview overall performance of PEM the following error indices are defined for each random output variable x_l:

$$\varepsilon_\mu^x = \left|\frac{\mu_{MCS}^x - \mu_{PEM}^x}{\mu_{MCS}^x}\right| \times 100\% \tag{7}$$

$$\varepsilon_\sigma^x = \left|\frac{\sigma_{MCS}^x - \sigma_{PEM}^x}{\sigma_{MCS}^x}\right| \times 100\% \tag{8}$$

Where, μ_{MCS}^x and σ_{MCS}^x are the mean and standard deviation respectively obtained from MCS and are used as reference values. Similarly, μ_{PEM}^x and σ_{PEM}^x

are the mean and standard deviation respectively obtained from PEM. Similarly, the average error indices $\bar{\varepsilon}_{\mu}^{X}$ and $\bar{\varepsilon}_{\sigma}^{X}$ are defined as the mean values of ε_{μ}^{x} and ε_{σ}^{x} respectively for each set X of variables. In this case X may refer to voltages (V), angle (δ), active power injections (P_i), reactive power injections (Q_i), active power line flow (P_{ij}), or reactive power line flow (Q_{ij}).

2.2 Deterministic Load Flow Method

Deterministic load flow (DLF) is called several times in probabilistic load flow method. Among various DLF technique available in literature Newton Raphson (NR) method is best suitable for transmission network and is selected in this work. In load flow analysis, four quantities, namely the voltage magnitude V, voltage angle δ, net real power injections P and net reactive power injections Q are associated with each bus in the system. Current of all buses in vector form can be obtained by $I = Y_{bus} \times V$, where Y denotes the admittance matrix describing the network typology, I and V are vectors of injected bus currents and bus voltages respectively. The power flow equations are a set of non-linear equations given by Equation (9) and Equation (10).

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad (9)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad (10)$$

Among several available strategies for solving such non-linear equation NR method is the most efficient and practical method. The NR method is an iterative method which takes an initial estimate of the state variables expanding the power flow equations in Taylor's series. The linear set is then solved and a new estimate for the state variables is obtained. This process is iterated until an acceptable value of mismatch between specified and calculated values of powers is reached. The linear equation set is formulated as Equation (11).

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \bar{V} \end{bmatrix} \quad i.e. \Delta W = J \, \Delta X \qquad (11)$$

Where, J denotes the Jacobian matrix of the state variables, ΔW represents the power mismatch vector and ΔX represents the incremental change to be made to the state variables before moving to the next iteration. The solution of the load flow problem is obtained after running a sufficient number of iterations such that the mismatch ΔW is within an acceptable value.

3. Methodology & Results

In this work deterministic load flow is done using Newton Raphson (NR) method and then considering probabilistic data of system probabilistic load flow is done using $2m+1$ point estimate method. The flow chart of PEM is shown in Figure 1.

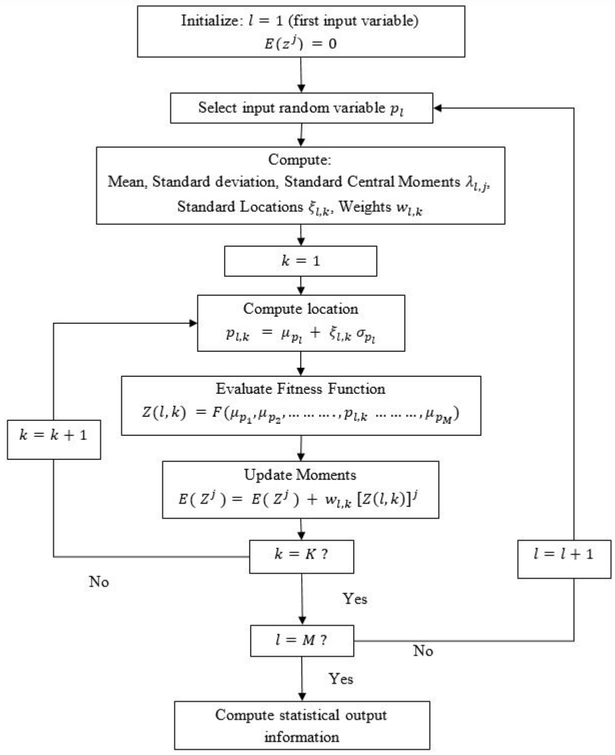

Figure 1: Flow chart of PEM

Studies are done on IEEE-14 bus and INPS network, and the results obtained are compared with each other in terms of accuracy to verify the suitability of the selected method. Loads and generations of the network are considered as the probabilistic data of the system.

The deterministic and probabilistic data for IEEE-14 bus system are obtained from [9] and [10] respectively. System details of the network considered for study is shown in Table 1. The generations, loads and branch details for INPS network are obtained from [11, 12, 13]. Probabilistic data for INPS network are determined as follows:

- Generation : Generation plants are modeled as binomial distribution. Forced outage rate (FOR) of each unit for particular generation plants is calculated through data collection from Nepal Electricity Authority (NEA) generation houses. Different generating plants have different FOR. Using FOR statistical measures for generations are obtained.

- Load : Loads of INPS does not follow perfectly normal distribution. Active and reactive demand of buses are obtained from respective substation and then required probabilistic measures are calculated using [14].

Table 1: Test system detail

Description	IEEE-14 bus	INPS
Number of buses & branches	14 & 20	83 & 98
Bus voltage in kV	1, 11, 33, 132	66, 132, 220
Number of Slack, PV & PQ buses respectively	1, 4 & 9	1, 34 & 48
Total active and reactive load respectively	259.04 MW and 73.50 MVar	1401.287 MW and 1050.965 MVar

Table 2: Average error indices

Details		Bus system	
		IEEE-14 bus	INPS
V	$\bar{\varepsilon}_\mu$	0.0023	0.0015
	$\bar{\varepsilon}_\sigma$	23.07	21.64
δ	$\bar{\varepsilon}_\mu$	0.0237	0.2687
	$\bar{\varepsilon}_\sigma$	1.76	0.57
P_{flow}	$\bar{\varepsilon}_\mu$	0.1044	0.1904
	$\bar{\varepsilon}_\sigma$	4.98	1.99
Q_{flow}	$\bar{\varepsilon}_\mu$	0.2700	0.3219
	$\bar{\varepsilon}_\sigma$	16.91	9.50
P_{gen}	$\bar{\varepsilon}_\mu$	0.0061	0.0831
	$\bar{\varepsilon}_\sigma$	0.69	0.97
Q_{gen}	$\bar{\varepsilon}_\mu$	0.0680	0.1561
	$\bar{\varepsilon}_\sigma$	16.10	18.76

As per literature, it is well known that Monte Carlo Simulation (MCS) method gives most accurate result. So, in this work results of probabilistic load flow are first obtained using MCS method and then by point estimate method. Accuracy and efficiency of PEM is verified by comparing with MCS results in terms of average error indices as shown in Table 2.

Table 2 shows mean and standard deviation average error indices of PEM in comparison with MCS method for different output variables in case of IEEE-14 bus and INPS network. It can be observed that average error indices do not increase significantly even when the total number of random input variable increases from 23 in case of IEEE-14 bus system to 140 in case of INPS network. This is because in $2m+1$ point estimate method concentrations do not depend on number of random input variables. Also, from the above result it can be concluded that PEM gives result similar to MCS technique and number of times deterministic load flow have to be called is very less thus requiring minimum time in case of PEM in comparison to MCS method. Voltage magnitude of all the buses obtained from PEM is shown in Figure 2.

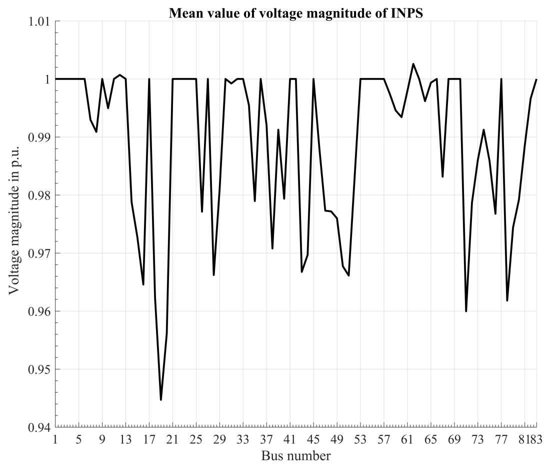

Figure 2: Voltage magnitude of buses of INPS network

Probabilistic output results of INPS network using PEM are plotted in terms of probabilistic distribution function (PDF) and cumulative density function (CDF) curves to obtain the information that can be obtained from probabilistic load flow method. Figure 3 and Figure 4 shows PDF and CDF curve of voltage magnitude of bus-19 respectively having mean and standard deviation value of 0.9447 p.u. and 0.0052 p.u. respectively. Figure 3 shows distribution of voltage magnitude is approximately normal as maximum density occur at mean value. From CDF curve, it can be said that probability of voltage magnitude less than equal to this mean value i.e. 0.9447 p.u. is 50.28 %. It can be noted that probability of occurrence of lowest value of voltage magnitude i.e.$\leq$ 0.9291 p.u. is only 0.15 % which shows variation in voltage magnitude values with obtained mean and standard deviation is within its $\pm$ 10 % limit.

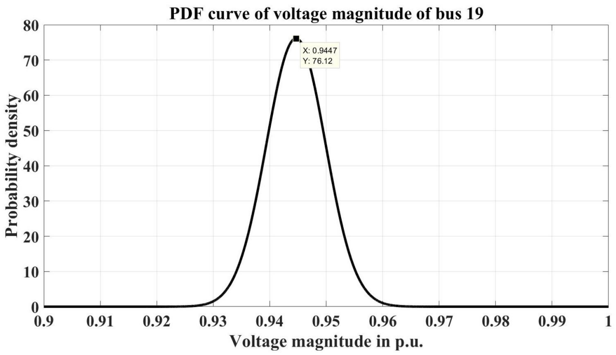

Figure 3: PDF curve of voltage magnitude of bus-19

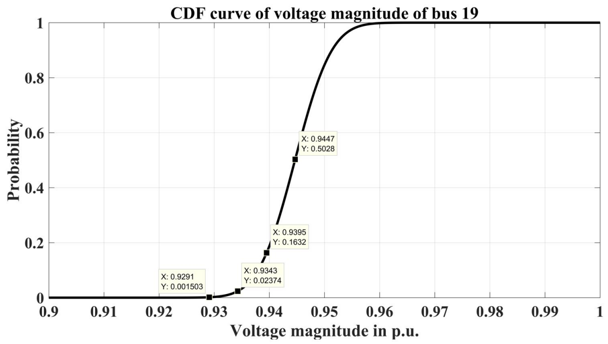

Figure 4: CDF curve of voltage magnitude of bus-19

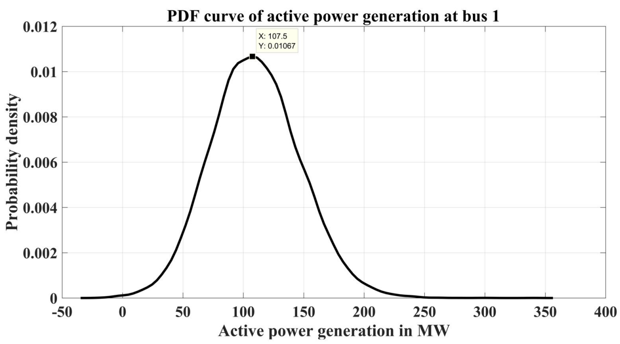

Figure 5: PDF curve of active power generation at bus-1

Figure 5 and Figure 6 shows PDF and CDF curve of active power required to be generated by slack bus i.e. bus-1 to balance the total active power of INPS network respectively. The mean and standard deviation of the active power generation at bus-1 are 110.001 MW and 36.359 MW respectively. From the PDF curve it can be observed that distribution of active generation at bus-1 is not symmetrical, and is right skewed. The maximum probability density occurs at 107.5 MW which is left to the mean value i.e. 110.001.

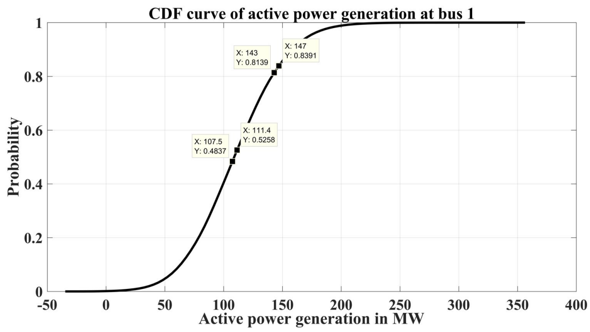

Figure 6: CDF curve of active power generation at bus-1

From the CDF curve, cumulative probability for different values of generation is shown in Table 3.

Table 3: Cumulative probability of active power generation at bus-1

Generation in MW	% Probability ($\leq$)
107.500	48.37
110.001	51.07
143.00	81.39
147.00	83.91

From Table 3, it can be observed that for only 51.07% of the time power less than and equal to 110.001 MW is required to be generated in the system but from deterministic load flow concept for 100% of the time this amount of power is only required to be generated. So if we design the capacity of slack bus as per the deterministic concept without any additional margin then there will be 48.93% chances of failure of the system. From Table 3, it can be also be observed that probability of exceeding generation requirement of 143.00 MW is 18.61%. Since the installed capacity of slack bus is 144 MW, approximately 17.98% of the times system requires more import power than current import value to balance the system. Hence, it can be seen that a wide range of information can be obtained from probabilistic load flow method which is not possible in case of deterministic load flow method.

Figure 7 shows CDF curve of active power loss of branch-16 (Kaligandaki to Butwal) which has highest active power loss, mean value of 7.937 MW and standard deviation of 1.932 MW. From this CDF curve it can be observed that the probability of having power loss in between 5.918 MW and 9.821 MW is 0.6622. So, percentage loading with its capacity in terms of probability for this line is analyzed further.

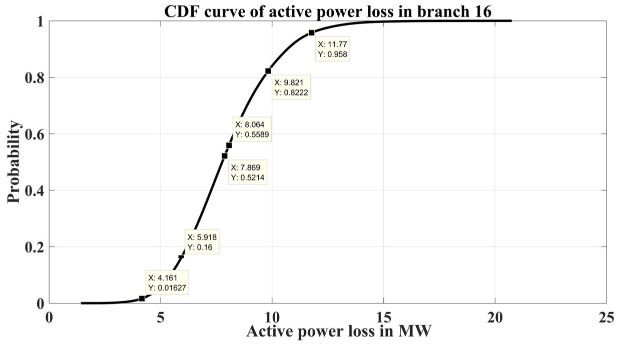

Figure 7: CDF curve of active power loss in branch-16

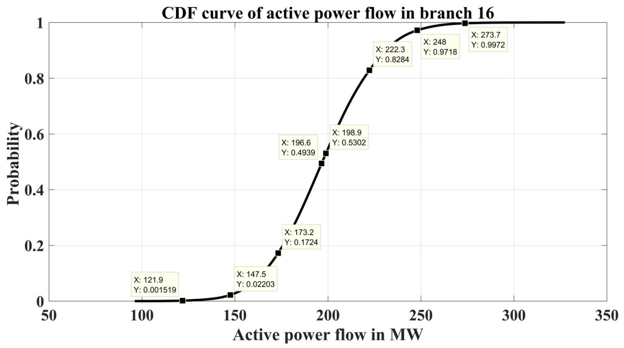

Figure 8: CDF curve of active power flow in branch-16

From PEM results mean and standard deviation value of active power flow in branch-16 is 197.637 MW and 25.167 MW respectively. Figure 8 shows less and equal to probability for different values of active power flow. Using CDF, probability of branch loading with given capacity 247.94 MW can be obtained and is shown in Table 4.

Table 4: Cumulative probability of active power flow in branch-16

Flow in MW	% Capacity Loading	% Probability ($\leq$)
196.6	79.3	49.39
197.6	79.7	51.03
222.3	89.7	82.84
248.0	100.0	97.18
273.7	110.4	99.72
173.2	69.9	17.24
147.5	59.5	2.203
121.9	49.2	0.15

Probability of branch flow $\leq$ 197.6 MW (mean value) is approximately 51.03% but from deterministic concept this flow is 100%. It can be seen that at mean value branch is 89.7% loaded so it can allow more flow within its limit. At 248 MW flow, line is 100% of its capacity but there is still 2.82% chance to exceed this capacity resulting in overloading of line. Overloading of line might result in tripping of line. There is 2.54% probability that branch 16 gets overloaded by 10.4%. All these probability information can't be obtained from deterministic load flow approach. Table 5 show active and reactive power losses of the network considered in this study obtained from deterministic, MCS and PEM method. From the result it can be observed that total power loss in reality i.e. load is variable is higher in comparison to deterministic case where load are considered as constant.

Table 5: Active and reactive power losses

Test system	IEEE-14 bus		INPS	
Power Losses	Active (MW)	Reactive (MVAR)	Active (MW)	Reactive (MVAR)
Deterministic	13.385	25.805	70.940	33.096
MCS	13.492	26.236	72.619	37.908
PEM	13.494	26.247	72.597	37.854

4. Conclusion

In this work the difficulties that arise by consideration of load and generation variation with time is solved by proposing the point estimate method, which is one of the probabilistic load flow technique. With probabilistic load flow study, uncertainties of input parameters is considered in terms of statistical parameters and their impact on output parameters is observed in terms of statistical parameters. This paper obtains probabilistic load flow results of two different size system IEEE-14 bus and INPS network taking uncertainty of load and generation. Results with PEM is compared with MCS results to measure its efficiency in terms of average error indices and is found that $2m+1$ scheme of PEM gives results similar to MCS with less computational burden. Probabilistic results of INPS obtained in terms of CDF and PDF helps to analyze the system operation and requirement from planning point of view which is inhibited in deterministic approach.

Branch parameters variation can also be incorporated for analysis and planning purposes. The studied technique can be implemented in other network as well as in distribution network also.

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